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TOCM-VAM METHOD VERSUS ASM METHOD IN TRANSPORTATION PROBLEMS

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ABSTRACT. Reinfeld and Vogel (1958) developed a method known as VogelâĂŹs Approximation Method (VAM), which is the most efficient solution procedure for more than five decades, for obtaining an Initial Basic Feasible Solution (IBFS) for the transportation problems (TPs) as it provides a very good IBFS. Maharajan and Meenakshi (2004) extended the Total Opportunity Cost Matrix (TOCM) of Kirca and Satir (1990) by using VAM procedure on the TOCM, called TOCM-VAM method. It yields a very efficient initial solution Abdul Quddooset al. developed a new method called ASM method (July 2012) and Revised Version of ASM method (June 2016) for obtaining the best IBFS for TPs with minimum effort of mathematical calculations. In this paper, by comparing the performance of the ASM and TOCM-VAM methods, we have tried to demonstration that the ASM method is the most excellent one for finding an IBFS for any TP. To verify the performance of the methods, 50 classical benchmark instances (30 of balanced category and 20 of unbalanced category) from the literature have been tested. Simulation results authenticate that the ASM method has produced optimal solution directly to 40 TPs, whereas TOCM-VAM method has produced optimal solution directly to only 27 TPs. Therefore, it is acknowledged that the ASM method produces the best IBFS, in the sense that, which is either optimal directly or very close to optimal solution. Hence, it is smart to apply only the ASM method to find IBFS for TPs. Further, the most attractive feature of this method is that it requires only uncomplicated arithmetical and logical calculations and hence any one can easily understand and apply it far better than any other method. Also, this method will be more cost-effective for those decision makers who are dealing with logistics and supply chain problems.

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1. Introduction

Transportation problems have been widely studied in Operations Research and Computer Science. They play an important role in logistics and supply-chain management for reducing the shipping cost and improving the service. In 1941 Hitchcock [5] developed the basic transportation problem along with the constructive method of solution and later in 1949 Koopmans [8] discussed the problem in detail. Again in 1951 Dantzig [4] formulated the transportation problem as linear programming problem and also provided the solution method. During 1960s, quite few methods such as North West Corner (NWC) Method, Least Cost Method (LCM) and Vogel's Approximation Method (VAM) [6, 13–15] have been established for finding the IBFS.

In the recent years several methods have been projected by several researchers to find the optimal solution for TPs. directly. But no method is attaining optimal solution directly to all TPs. Among them, in July 2012, Abdul Quddoos et al. [2] proposed a new method, named ASM method, based on making allocations to zero entry cell of reduced cost matrix, for finding an optimal solution directly for a wide range of TPs. In October 2012, Mohammad Kamrul Hasan [10] proposed that direct methods (including ASM method) for finding optimal solution of a TP do not reflect optimal solution continuously. Murugesan [11] confessed and recognized the statement of Mohammad Kamrul Hasan by testing the ASM method for various benchmark problems. Meanwhile by doing further research, Abdul Quddoos et al. [1] encountered a few problems in which ASM method does not directly provide optimal solution to each and every problem, but provides a best IBFS, which is very close to optimal solution.

One basic problem encountered was the unbalanced TP (UTP) in which an IBFS, not optimal but very close to optimal, was obtained. To overcome this problem, in July 2016, Abdul Quddoos et al. [1] presented a Revised Version of the ASM method, which provides optimal solution directly for most of the problems, and if not, it provides best IBFS. Murugesan et al. [12] established Abdul Quddoos et al. claim by testing 30 benchmark instances of balanced category and 20 of unbalanced category. Again by our further research we have observed that Kirca and Satir (1990) [7] first introduced the concept of Total Opportunity Cost Matrix (TOCM) and applied the Least Cost Method with some tie-breaking policies on the TOCM to determine the feasible solution of the TP. Mathirajan and Meenakshi (2004) [9] extended TOCM of Kirca and Satir by using VAM procedure on the TOCM (called the VAM-TOC, also same as the TOCM-VAM). According to the authors, this approach yielded the optimal solution and about 80% of the time it yielded a solution very close to the optimal (0.5% loss of optimality).

In this paper, we have studied performance of the Revised Version of ASM method (hereafter it is simply called as ASM method) as well as the TOCM-VAM method and tried to expose that the ASM method is the best one for finding only IBFS and TOCM-VAM method is the next better method for finding the IBFS for TPs.

The paper is organized as follows: Following the brief introduction in Section 1, in Section 2.1 and 2.2 step-by-step algorithms of TOCM-VAM and ASM are presented. In Section 3, one benchmark problem from balanced type is illustrated by the methods of TOCM-VAM method as well as by the ASM method. Classical benchmark TPs from balanced category and unbalanced category of different sizes from some reputed journals published by several authors are shown in Section 4. Section 5 demonstrates the comparison of the results of ASM method with TOCM-VAM method for 30 classical benchmark instances of balanced type and 20 of unbalanced type .Finally, in Section 6 conclusions are drawn.

Balanced and Unbalanced Transportation Problem

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in all destinations, and is called unbalanced, otherwise.

Feasible Solution (F.S)

A set of nonnegative allocations $X_{ij} \geq 0$, which satisfies the row and column restrictions of a TP is known as a Feasible Solution to the TP.

Basic Feasible Solution (BFS)

A feasible solution to a m-sources and n-destinations BTP is said to be a Basic Feasible Solution if the number of positive allocations in it is exactly (m + n -1). In this case, it is called Non-Degenerate Basic Feasible Solution (NDBFS);otherwise, it is called Degenerate Basic Feasible Solution (DBFS).

Optimal Solution

A feasible solution (not necessarily basic) of a TP is said to be optimal if it minimizes the total cost of transportation. There always exists an optimal solution to a balanced TP.

Optimality Test Optimality test can be performed only if the solution is a non-degenerate one. Otherwise, optimality test cannot be performed. In case of the later, it can be made non-degenerate by adding enough number of positive allocations at suitable cells.

For performing optimality test, two methods namely, Stepping Stone Method and MODI Method [6,13–15] are usually used, in which MODI Method is mostly used.

Row Opportunity Cost Matrix (ROCM) For each row of the given balanced TP, the smallest cost of that row is subtracted from each element of the same row. The resulting matrix is called the ROCM.

Column Opportunity Cost Matrix (COCM)

For each column of the given balanced TP, the smallest cost of that column is subtracted from each element of the same column. The resulting matrix is called the COCM.

Total Opportunity Cost Matrix (TOCM) The TOCM is obtained by adding the ROCM and the COCM.

2. METHODOLOGY

As comparative study of ASM method with TOCM-VAM method is carried out, in this section, we describe only the algorithm of the said two methods.

2.1. **Algorithm of TOCM-VAM.** A systematic procedure for TOCM-VAM due to Mathiraj et al. [9] proceeds as follows:

Step 1: Balance the given transportation problem if either (total supply $>$ total demand) OR (total supply $<$ total demand).

Step 2: Obtain the Total Opportunity Cost Matrix (TOCM).

Step 3: Apply VAM on TOCM and obtain feasible allocation.

Step 4: Compute the total transportation cost for the feasible allocations obtained in Step3 using the original balanced-transportation cost matrix.

2.2. **Algorithm of the ASM method.** The stepwise procedure of ASM method by Abdul Quddoos et al. [1] is carried out as follows.

Step-1: Construct the transportation tableau from given TP. Check whether the problem is balanced or not. If the problem is balanced, go to Step 4, otherwise go to Step 2.

Step-2: If the problem is not balanced, then anyone of the following two cases may arise:
a) If total supply exceeds total demand, introduce an additional dummy column to the transportation table to absorb the excess supply. The unit transportation cost for the cells in this dummy column is set to âĂŸMâĂŹ, where M $>$ 0 is a very large but finite positive quantity.

or

b) If total demand exceeds total supply, introduce an additional dummy row to the transportation table to satisfy the excess demand. The unit transportation cost for the cells in this dummy row is set to 'M', where M>0 is a very large but finite positive quantity.

Step-3:
a) In case (a) of Step 2, identify the lowest element of each row and subtract it from each element of the respective row and then, in the resulting tableau, identify the lowest element of each column and subtract it from each element of the respective column and go to Step 5.

or

b) In case (b) of Step 2, identify the lowest element of each column and subtract it from each element of the respective column and then, in the resulting tableau, identify the lowest element of each row and subtract it from each element of the respective row and go to Step 5.

Step-4: Identify the lowest element of each row and subtract it from each element of the respective row and then, in the resulting tableau, identify the lowest element of each column and subtract it from each element of the respective column.

Step-5: In the reduced tableau, each row and each column contains at least one zero. Now, select the first zero (say zero) and count the number of zeros (excluding the selected one) in the row and column and record as a subscript of selected zero. Repeat this process for all zeros in the transportation tableau.

Step-6: Now, choose the cell containing zero for which the value of subscript is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in Step 5, choose the cell of that zero for breaking tie such that the sum of all the elements in the row and column is maximum. Supply maximum possible amount to that cell.

Step-7: Delete that row (or column) for further consideration for which the supply from a given source is exhausted (or the demand for a given destination is satisfied). If, at any stage, the column demand is completely satisfied and row supply is completely exhausted simultaneously, then delete only one column(or row) and the remaining row (or column) is assigned a zero supply (or demand) in further calculation.

Step-8: Now, check whether the reduced tableau contains at least one zero in each row and each column. If this does not happen, repeat Step 4, otherwise go to Step 9.

Step-9: Repeat Step 5 to Step 8 till all the demands are satisfied and all the supplies are exhausted.

3. Numerical Illustration

The above said algorithms for finding an IBFS of TPs are illustrated by the following benchmark problem from the literature.

3.1. **Illustration: (Aminur R. Khan, 2012, [3]).** Consider the following cost minimizing BTP with three sources and four destinations:

Table 3.1: The given BTP

Sources	D1	D2	D3	D4	Supply
S1	6	1	9	3	70
S2	11	5	2	8	55
S3	10	12	4	7	90
Demand	85	35	50	45	

3.1.1. *Solution by the TOCM-VAM Method.* First, the given BTP is solved using the procedure of TOCM-VAM. The IBFS is obtained as shown in Table 3.2.

Table 3.2: Allocation table due to TOCM-VAM

Sources	D1	D2	D3	D4	Supply
S1	35 / 6	35 / 1	9	3	70
S2	5 / 11	5	50 / 2	8	55
S3	45 / 10	12	4	45 / 7	90
Demand	85	35	50	45	

Writing the Allocation Values:

X_{11} = 35, X_{12} = 35, X_{21}= 5, X_{23} = 50, $X31$ = 45, X_{34} = 45 and all other X_{ij} = 0. Note that the generated solution is a non-degenerate one as it contains exactly six (m+n-1 = 3+4-1= 6) allocations.

Computing the Total Transportation Cost:

Z = (35 $\times$ 6) + (35 $\times$ 1) + (5 $\times$ 11) + (50 $\times$ 2) + (45 $\times$ 10) + (45 $\times$ 7) = 210 + 35 + 55 + 100 + 450 + 315 = \$1165.

By checking the condition for optimality by MODI method, it is found that the generated solution by TOCM-VAM is not an optimal one. By applying the MODI method, this solution has been improved towards optimality with Z = \$1160 in a single iteration. The optimal solution due to the MODI method is shown in Table 3.3.

Table 3.3: Optimal allocation table due to MODI method

Sources	D1	D2	D3	D4	Supply
S1	40 / 6	30 / 1	9	3	70
S2	11	5 / 5	50 / 2	8	55
S3	45 / 10	12	4	45 / 7	90
Demand	85	35	50	45	

Writing the Optimal Allocation Values:

X_{11} = 40, X_{12} = 30, X_{22}= 5, X_{23} = 50, $X31$ = 45, X_{34} = 45 and all other Xij = 0. Note that the generated solution is a non-degenerate one as it contains exactly six allocations.

Computing the Total Minimum Transportation Cost:

Z = (40 × 6) + (30 × 1) + (5 × 5) + (50 × 2) + (45 × 10) + (45 × 7) = 240 + 30 + 25 + 100 + 450 + 315 = $1160.

3.1.2. ***Solution by the ASM Method.*** Next, the given BTP is solved using the algorithm of ASM method. The IBFS is obtained as shown in Table 3.4.

Table 3.4: (Alternative Optimal) Allocation table due to ASM

Sources	D1	D2	D3	D4	Supply
S1	6	[30] 1	9	[40] 3	70
S2	11	[5] 5	[50] 2	8	55
S3	[85] 10	12	4	[5] 7	90
Demand	85	35	50	45	

Writing the Allocation Values:

X_{12} = 30, X_{14} = 40, X_{22}= 5, X_{23} = 50, X_{31} = 85, X_{34} = 5, and all other X_{ij} = 0. Note that the generated solution is a non-degenerate one as it contains exactly six allocations.

Computing the Total Transportation Cost:

Z = (30 × 1) + (40 × 3) + (5 × 5) + (50 × 2) + (85 × 10) + (5 × 7) = 30 + 120 + 25 + 100 + 850 + 35 = $1160.

By applying the MODI method, this solution has been checked for optimality and we have found that the obtained solution is an optimal one only. Also, it is noted that the generated solution by ASM method is an alternative optimal solution to the given BTP. The alternative optimal solution is shown in Table 3.4.

Further, it is observed that fo the illustrated problem, the ASM method produced the optimal solution directly, where as the TOCM-VAM method produced a near optimal solution only.

4. Numerical Examples

To justify the efficiency of the testing methods we have solved a good number of classical benchmark problems from balanced and unbalanced categories in different sizes, from various literature and books, which are listed in Table 4.1 and Table 4.2 respectively.

5. Result Analysis

For evaluating the performance of the ASM, TOCM-VAM and VAM methods, simulation experiments were carried out on balanced and unbalanced categories of TPs. The main purpose of the experiment was to evaluate the effectiveness of the IBFSs obtained by ASM, TOCM-VAM and VAM methods by comparing them with optimal solutions. Effectiveness indicates closeness level which is the least iteration number between IBFS and the optimal solution.

5.1. **Analysis for Balanced Case.** The assessment of the results for 30 classical benchmark problems of balanced case (Refer Table 5.1) has been studied in this research to measure the effectiveness of the ASM method over TOCM-VAM method. This assessment is shown in following Table 5.1.

From Table 5.1, we discover that VAM has produced optimal solution to 10 BTPs, TOCM-VAM has produced optimal solution to 23 BTPs, whereas ASM has produced optimal solution to 26 BTPs. Among the identified four challenging problems (Problem Nos. 21, 22, 25 and 26) to the ASM method, three problems have the same near optimal solution by the ASM and TOCM-VAM methods and one (Problem No. 26) has better near optimal solution by the ASM method than the TOCM-VAM method.

5.2. **Analysis for Unbalanced Case.** The evaluation of the results for 20 classical benchmark problems of unbalanced case (Refer Table 5.2) has been studied in this research to measure the effectiveness of the ASM method over TOCM-VAM method. This comparison is shown in following Table 5.2.

Table 5.1 Comparison of results obtained by focused methods for BTPs

Pr. No.	VAM	TOCM-VAM	ASM	Opt. Soln.	Pr. No.	VAM	TOCM-VAM	ASM	Opt. Sol.
1.	5600	5600	5600	5600	16.	2657000^	2655600	2655600	2655600
2.	955^	880	880	880	17.	92^	83	83	83
3.	59	59	59	59	18.	1500^	1390	1390	1390
4.	28	28	28	28	19.	859^	799	799	799
5.	475^	435	435	435	20.	285	285	285	285
6.	80^	76	76	76	21.	316	322†	322*	316
7.	470^	410	410	410	22.	3663^	3513†	3513*	3458
8.	859^	809	809	809	23.	779^	743	743	743
9.	476^	417	417	417	24.	112	112	112	112
10.	1220^	1165†	1160	1160	25.	1104^	1103†	1103*	1102
11.	68	68	68	68	26.	2224^	2224†	2213*	2202
12.	390	390	390	390	27.	112	116†	112	112
13.	355	355	355	355	28.	2130^	2130†	2070	2070
14.	114^	111	111	111	29.	1930^	1900	1900	1900
15.	199^	183	183	183	30.	2310^	2170	2170	2170

Note: The near optimal solutions due to VAM, TOCM-VAM and ASM methods are denoted by the symbols $\wedge$, †, and $*$ respectively.

Table 5.2 Comparison of results obtained by focused methods for UTPs

Prbm No.	VAM	TOCM-VAM	ASM	Opt. Soln.	Prbm No.	VAM	TOCM -VAM	ASM	Opt. Sol.
1.	2164000	2164000	2164000	2146750	11.	965	965	950□	950
2.	880	840∨	840□	840	12.	8150	8350	7750□	7750
3.	779	813	779	743	13.	740	740	710□	710
4.	1010	965	960□	960	14.	9200‡	9200∨	9200□	9200
5.	1555	1465∨	1465□	1465	15.	1745	1695	1695	1650
6.	5020	4780	4720□	4720	16.	195	195	195	193
7.	620	630	606□	606	17.	16400	16400	15500□	15500
8.	76	76	79	75	18.	6000	6000	5600□	5600
9.	1510	1540	1305□	1305	19.	12250	11950	11500□	11500
10.	2566	2361∨	2361□	2361	20.	17060	17060	17060	17050

Note: The optimal solutions due to VAM, TOCM-VAM and ASM methods are denoted by the symbols ‡, V, and□ respectively.

From Table 5.2, we discover that the ASM method has produced optimal solution to 14 UTPs, TOCM-VAM has produced optimal solution to only 4 UTPs, whereas VAM has produced optimal solution to only one problem. Among the identified six challenging problems (Problem Nos. 1, 3, 8, 13, 16 and 20) to the ASM method, four problems, (numbered as 1, 15 and 16) have the same near optimal solution by the ASM and TOCM-VAM methods and one problem (numbered with 3) has better near optimal solution by the ASM method than the TOCM-VAM method and one problem (numbered with 8) has better near optimal solution by the TOCM-VAM than the ASM method.

5.3. **Effectiveness of ASM over TOC-VAM.** The overall analysis of the results produced by the VAM, TOCM-VAM and ASM methods reflect their efficiency. The efficiency of the three methods on 30 BTPs is shown in Table 5.3.1 and that of on 20 UTPs is shown in Table 5.3.2 and hence that of on 50 TPs is shown in Table 5.3.3.

Table 5.3.1 Effectiveness of VAM, TOCM-VAM and ASM Methods on BTPs

Method	No. of Problems Tested	No. of Problems produced Optimal Solution directly	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
ASM	30	26	86.67%	04	13.33%
TOCM-VAM	30	23	76.67%	07	23.33%
VAM	30	10	3333%	20	66.67%

Table 5.3.2 Effectiveness of VAM, TOCM-VAM and ASM Methods on UTPs

Method	No. of Problems Tested	No. of Problems produced Optimal Solution directly	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
ASM	20	14	70%	06	30%
TOCM-VAM	20	04	20%	16	80%
VAM	20	01	05%	19	95%

Table 5.3.3 Effectiveness of VAM, TOCM-VAM and ASM Methods on TPs

Method	No. of Problems Tested	No. of Problems produced Optimal Solution directly	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
ASM	50	40	80%	10	20%
TOCM-VAM	50	27	54%	23	46%
VAM	50	11	22%	39	78%

6. Conclusion

In this paper, we have tried to expose that the ASM method is the best one for finding an IBFS for any transportation problem. To verify the performance of the method, 30 classical benchmark instances of balanced kind and 20 of unbalanced kind from the literature have been tested. Simulation results on BTPs substantiate that the ASM method produces optimal solution directly to 26 (i.e. 86.67% of) BTPs whereas TOCM-VAM method produces optimal solution straight to only 23 (i.e. 76.67%) BTPs. Another simulation results on UTPs authenticate that the ASM method produces optimal solution directly to 14 (i.e. 70% of) UTPs whereas TOCM-VAM method produces optimal solution straight to only 04 (i.e. 20%) UTPs. Hence, out of 50 total TPs tested the ASM method produces optimal solution directly to 40 (i.e. 80% of) TPs whereas TOCM-VAM method produces optimal solution straight to only 27 (i.e. 54%) TPs. Therefore, it is established and recognized that the ASM method is the best one and TOCM-VAM method is a better one for finding an IBFS to TPs. As a result, it is wise to apply only the ASM method to find IBFS for TPs. Further, the most attractive feature of this method is that it requires only simple arithmetical and logical calculations and hence anyone can easily understand and apply it far better than any other method. Also, this method will be more cost-effective for those decision makers who are trading with logistics and supply chain problems.

7. Tables

Table 4.1: Classical Benchmark Balanced TPs

Problem No.,(Author(s), Year, [Ref. No.])	Problem No.,(Author(s), Year, [Ref. No.])
Problem 1(Ramadan et al., 2012, [31]) $[C_{ij}]$ 3× 3= [32 40 120; 60 68 104; 200 80 60] $[S_i]$ 3× 1= [20, 30, 45] $[D_j]$ 1× 3= [30, 35, 30]	Problem 16 (Opera Jude et al., 2017, [30]) $[C_{ij}]$ 4× 4= [45 52 63 57;58 48 56 54;52 55 62 58;65 48 44 54] $[S_i]$ 4× 1= [15500, 12000, 14400, 11600] $[D_j]$ 1× 4= [12600, 12500, 13000, 15400]

Problem 2(Srinivasan et al., 1977, [40]) $[C_{ij}]$ 3× 4 = [3 6 3 4; 6 5 11 15; 1 3 10 5] $[S_i]$ 3× 1 = [80, 90, 55] $[D_j]$ 1× 4= [70, 60, 35, 60]	Problem 17 (Opera Jude et al., 2017, [30]) $[C_{ij}]$ 4× 4= [2 5 6 3;9 6 2 1;5 2 3 6;7 7 2 4] $[S_i]$ 4× 1= [6, 9, 7, 12] $[D_j]$ 1× 4= [10, 4, 6, 14]
Problem 3(Schrenket al., 2011, [36]) $[C_{ij}]$ 3× 4= [3 6 1 5; 7 9 2 7; 2 4 2 1] $[S_i]$ 3× 1= [6, 6, 6] $[D_j]$ 1× 4= [4, 5, 4, 5]	Problem 18 (Opera Jude et al., 2017, [30]) $[C_{ij}]$ 3× 3= [4 3 5; 6 5 4; 8 10 7] $[S_i]$ 3× 1= [90, 80, 100] $[D_j]$ 1× 3= [70, 120.80]
Problem 4 (Samuel, 2012, [35]) $[C_{ij}]$ 3× 4= [1 2 3 4;4 3 2 0; 0 2 2 1] $[S_i]$ 3× 1= [6, 8, 10] $[D_j]$ 1× 4= [4, 6, 8, 6]	Problem 19 (Babu et al., 2013, [7]) $[C_{ij}]$ 3× 4= [19 30 50 12; 70 30 40 60; 40 10 60 20] $[S_i]$ 3× 1= [7 10 18][Dj] 1× 4= [5, 8, 7, 15]
Problem 5(Imam et al. , 2009, [15]) $[C_{ij}]$ 3× 4= [10 2 20 11;12 7 9 20; 4 14 16 18] $[S_i]$ 3× 1= 15, 25, 10] $[D_j]$ 1× 4= [5, 15, 15, 15]	Problem 20 (Babu et al., 2014, [8]) $[C_{ij}]$ 4× 4= [5 3 6 10;6 8 10 7;3 1 6 7;8 2 10 12] $[S_i]$ 4× 1= [30, 10, 20, 10] $[D_j]$ 1× 4= [20, 25, 15, 10]
Problem 6(Ahmed M.M., et al., 2014, [5]) $[C_{ij}]$ 4× 3= [2 7 4; 3 3 1; 5 4 7; 1 6 2] $[S_i]$ 4× 1= [5, 8, 7, 14] $[D_j]$ 1× 3= [7, 9, 18]	Problem 21 (Mhlanga A, 2014, [24]) $[C_{ij}]$ 4× 5= [4 9 8 10 12;6 10 3 2 3;3 2 7 10 3; 3 5 5 4 8] $[S_i]$ 4× 1= [24, 18, 20, 16] $[D_j]$ 1× 5= [10, 20, 10, 18, 20]
Problem 7(Mollah M A. et al. 2016, , [26]) $[C_{ij}]$ 4× 4= [7 5 9 11;4 3 8 6;3 8 10 5;2 6 7 3] $[S_i]$ 4× 1= [30, 25, 20, 15] $[D_j]$ 1× 4= [30, 30, 20, 10]	Problem 22 (Juman M.S., and Hoque M.A., 2015, [16]) $[C_{ij}]$ 4× 5= [25, 14, 34, 46, 45 10, 47, 14, 20, 41 22, 42, 38, 21, 46 36, 20, 41, 38, 44] $[S_i]$ 4× 1= [27, 35, 37, 45] $[D_j]$ 1× 5= [22, 27, 28, 33, 34]
Problem 8(Juman et al., 2015, [16]) $[C_{ij}]$ 3× 4= [19 30 50 12;70 30 40 60; 40 10 60 20] $[S_i]$ 3× 1= [7, 10, 18] $[D_j]$ 1× 4= [5, 7, 8, 15]	Problem 23 (Deshmukh N.M„ 2012, [12]) $[C_{ij}]$ 3× 4= [19 30 50 10;70 30 40 60; 40 8 70 20] $[S_i]$ 3× 1= [7, 9, 18] $[D_j]$ 1× 4= [5, 8, 7, 14]

Problem 9 (Juman et al., 2015, [16]) $[C_{ij}]$ 3× 4= [13 18 30 8;55 20 25 40;30 6 50 10] $[S_i]$ 3× 1= [8, 10, 11] $[D_j]$ 1× 4= [4, 6, 7, 12]	Problem 24 (Deshmukh N.M,, 2012, [12]) $[C_{ij}]$ 4× 6= [9 12 9 6 9 10;7 3 7 7 5 5; 6 5 9 11 3 11;6 8 11 2 2 10] $[S_i]$ 4× 1= [5, 6, 2, 9] $[D_j]$ 1× 6= [4, 4, 6, 2, 4, 2]
Problem 10 (Aminur R. Khan, 2012, [6]) $[C_{ij}]$ 3× 4= [6 1 9 3;11 5 2 8;10 12 4 7] $[S_i]$ 3× 1= [70, 55, 90] $[D_j]$ 1× 4= [85, 35, 50, 45]	Problem 25 (Russell E.J., 1969, [34]) $[C_{ij}]$ 5× 5= [73 40 9 79 20; 62 93 96 8 13; 96 65 80 5065; 57 58 29 12 87; 56 23 87 18 12] $[S_i]$ 5× 1= [8, 7, 9, 3, 5] $[D_j]$ 1× 5= [6, 8, 10, 4, 4]
Problem 11 (Aminur R. Khan, 2012, [6]) $[C_{ij}]$ 4× 6= [7 10 7 4 7 8;5 1 5 5 3 3;4 3 7 9 1 9; 4 6 9 0 0 8] $[S_i]$ 4× 1= [5, 6, 2, 9] $[D_j]$ 1× 6= [4, 4, 6, 2, 4, 2]	Problem 26 (Shweta Sing et al., 2012, [39]) $[C_{ij}]$ 5× 5= [68 35 4 74 15; 57 88 91 3 8; 91 60 75 45 60; 52 53 24 7 82; 51 18 82 13 7] $[S_i]$ 5× 1= [18, 17, 19, 13, 15] $[D_j]$ 1× 5= [16, 18, 20, 14, 14]
Problem 12 (Adlakha et al., 2009, [4]) $[C_{ij}]$ 4× 5= [2 1 3 2 2; 3 2 1 1 1; 5 4 2 1 3; 7 5 5 3 1] $[S_i]$ 4× 1= [20, 70, 30, 60] $[D_j]$ 1× 5= [50, 30, 30, 50, 20]	Problem 27 (WagenerU.A., 1965, [42]) $[C_{ij}]$ 5× 6= [5 3 7 3 8 5; 5 6 12 5 7 11; 2 8 3 4 8 2; 9 6 10 5 10 9; 5 3 7 3 8 5] $[S_i]$ 5× 1= [3, 4, 2, 8, 3] $[D_j]$ 1× 6= [3, 4, 6, 2, 1, 4]
Problem 13 (Abdul Hakim et al., 2018, [1]) $[C_{ij}]$ 3× 4= [5 3 6 2 ; 4 7 9 1; 3 4 7 5] $[S_i]$ 3× 1= [19, 37, 34] $[D_j]$ 1× 4= [16, 18, 31, 25]	Problem 28 (Das et al., 2014, [10]) $[C_{ij}]$ 4× 5= [10 8 9 5 13; 7 9 8 10 4; 9 3 7 10 6; 11 4 8 3 9] $[S_i]$ 4× 1 = [100, 80, 70, 90] $[D_j]$1× 5= [60, 40, 100, 50, 90]
Problem 14 (Abdul Hakim et al., 2018, [1]) $[C_{ij}]$ 4× 4= [4 6 5 2;6 4 1 4;5 2 3 1;4 6 7 8] $[S_i]$ 4× 1= [6, 10, 12, 14] $[D_j]$ 1× 4= [9, 16, 10, 7]	Problem 29 (Das et al., 2014, [11]) $[C_{ij}]$ 5× 7 = [12 7 3 8 10 6 6;6 9 7 12 8 12 4;10 12 8 4 99 3; 8 5 11 6 7 9 3;7 6 8 11 9 5 6] $[S_i]$ 5× 1 = [60, 80, 70, 100, 90] $[D_j]$15× 7 = [20, 30, 40, 70, 60, 80, 100]

Problem 15 (Ray and Hossain, 2007, [33]) $[C_{ij}]$ 4× 3= [4 3 4;10 7 5;8 8 3;5 6 6] $[S_i]$ 4× 1= [11, 12, 10, 7] $[D_j]$ 1× 3= [16, 10, 14]	Problem 30 (Khan A.R. et al., 2015, [18]) $[C_{ij}]$ 6× 6= [12 4 13 18 9 2; 9 16 10 7 15 11; 4 9 10 8 9 7; 9 3 12 6 4 5;7 11 5 18 2 7; 16 8 4 5 1 10] $[S_i]$ 6× 1= [120, 80, 50, 90, 100, 60] $[D_j]$ 1× 6= [75, 85, 140, 40, 95, 65]

Table 4.2: Classical Benchmark Unbalanced TPs

Problem No.,(Author(s), Year, [Ref. No.])	Problem No.,(Author(s), Year, [Ref. No.])
Problem 1 (Sen et al., 2010, [37]) $[C_{ij}]$ 5× 4 = [60 120 75 180; 58 100 60 165; 62 110 65 170; 65 115 80 175; 70 135 85 195] $[S_i]$ 5× 1= [8000, 9200, 6250, 4900, 6100] $[D_j]$ 1× 4= [5000, 2000, 10000, 6000]	Problem 11 (www.engineeingnotes.com. Balanced and Unbalanced TP, [43]) $[C_{ij}]$ 4× 4= [2, 4, 6, 11; 10, 8, 7, 5; 13, 3, 9, 12; 4, 6, 8, 3] $[S_i]$ 4× 1= [50, 70, 30, 50] $[D_j]$ 1× 4= [25, 35, 105, 20]
Problem 2 (Kulkarni and et al., 2010, [21]) $[C_{ij}]$ 4× 3= [3 4 6; 7 3 8; 6 4 5; 7 5 2] $[S_i]$ 4× 1= [100, 80, 90, 120] $[D_j]$ 1× 3= [110, 110, 60]	Problem 12 (Ahmed M.M.et al., 2014, [5]) $[C_{ij}]$ 3× 4= [10 8 4 3; 12 14 20 2;6 9 23 25] $[S_i]$ 3× 1= [500, 400, 300] $[D_j]$ 1× 4= [250, 350, 600, 150]
Example 3 (Deshmukh, 2012, [12]) $[C_{ij}]$ 3× 4= [19 30 50 10; 70 30 40 60; 40 8 70 20] $[S_i]$ 3× 1= [7, 9, 18][Dj] 1× 4= [40, 8, 7, 14]	Example 13 (MBA, Distance mode, 2014, Anna University, Chennai [23]) $[C_{ij}]$3× 4=[12 7 10 10; 10 9 12 10; 14 12 9 12] $[S_i]$ 3× 1= [40, 30, 20] $[D_j]$ 1× 4= [30, 25, 15, 10]
Example 4 (Geetha and et al., 2015, [13]) $[C_{ij}]$ 3× 4= [6 1 9 3; 11 5 2 8; 10 12 4 7] $[S_i]$ 3× 1= [70, 55, 70] $[D_j]$ 1× 4= [85, 35, 50, 45]	Example 14 (Ahmed M.M.et al., 2014, [5]) $[C_{ij}]$ 3× 5= [5 8 6 6 3; 4 7 7 6 5; 8 4 6 6 4;] $[S_i]$ 3× 1= [800, 500, 900] $[D_j]$ 1× 5= [400, 400, 500, 400, 800]

Example 5 (Geetha and et al., 2015, [13]) $[C_{ij}]$ 4× 3= [5 6 9; 3 5 10; 6 7 6; 6 4 10] $[S_i]$ 4× 1= [100, 75, 50, 75] $[D_j]$ 1× 3= [70, 80, 120]	Example 15 (Nagaraj Balakishnan, 1990, [29]) $[C_{ij}]$ 3× 3= [6, 10, 14 12, 19, 21 15, 14, 17] $[S_i]$ 3× 1= [50, 50, 50] $[D_j]$ 1× 3= [30, 40, 55,]
Example 6 (Geetha and et al., 2015, [13]) $[C_{ij}]$ 3× 4= [10 15 12 12; 8 10 11 9; 11 12 13 10] $[S_i]$ 3× 1= [200, 150, 120] $[D_j]$ 1× 4= [140, 120, 80, 220]	Example 16 (MBA, Distance mode, 2014, Anna University, Chennai [23]) $[C_{ij}]$ 3× 5= [10 8 12 9 3; 4 4 6 6 7; 15 7 11 13 8] $[S_i]$ 3× 1= [15, 12, 16] $[D_j]$ 1× 5= [8, 8, 4, 7,6]
Example 7 (Geetha and et al., 2015, [13]) $[C_{ij}]$ 3× 4= [7 8 11 10; 10 12 5 4; 6 11 10 9] $[S_i]$ 3× 1= [30, 45, 35] $[D_j]$ 1× 4= [20, 28, 19, 33]	Example 17 (Ray and G.C., 2007[33]) $[C_{ij}]$ 3× 4= [25 17 25 14; 15 10 18 24; 16 20 8 13] $[S_i]$ 3× 1= [300, 500, 600] $[D_j]$ 1× 4= [300, 300, 500, 500]
Example 8 (Abdul Quddoos et al., 2016, [2]) $[C_{ij}]$ 4× 3= [2 7 14; 3 3 1; 5 4 7; 1 6 2] $[S_i]$ 4× 1= [5, 8, 7, 15] $[D_j]$ 1× 3= [7, 9, 18]	Example 18 (Ray and G.C., 2007[33]) $[C_{ij}]$ 3× 5= [5 4 8 6 5; 4 5 4 3 2; 3 6 5 8 4] $[S_i]$ 3× 1= [600, 400, 1000] $[D_j]$ 1× 5= [450, 400, 200, 250, 300]
Example 9 (Abdul Quddoos et al., 2016, [2]) $[C_{ij}]$ 4× 4= [4 6 8 13; 13 11 10 8; 14 4 10 13; 9 11 13 8] $[S_i]$ 4× 1= [50, 70, 30, 50] $[D_j]$ 1× 3= [25, 35, 105, 20]	Example 19 (Pannerselvam, 2010, [32]) $[C_{ij}]$ 4× 5= [10 2 16 14 10; 6 18 12 13 16; 8 4 14 12 10; 14 22 20 8 18] $[S_i]$ 4× 1= [300, 500, 825, 375] $[D_j]$ 1× 5= [350, 400, 250, 150, 400]
Example 10 (Abdul Quddoos et al., 2016, [2]) $[C_{ij}]$ 3× 3= [4 8 8; 13 24 16; 8 16 24] $[S_i]$ 3× 1= [76, 82, 77] $[D_j]$ 1× 1= [72, 102,41]	Example 20 (Kanti Swarup et al., 1995 [17]) $[C_{ij}]$ 3× 4= [42 48 38 37; 40 49 52 51; 39 38 40 43] $[S_i]$ 3× 1= [160, 150, 190] $[D_j]$ 1× 4= [80, 90, 110, 160]

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