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MINIMIZATION OF TOTAL WAITING TIME OF JOBS IN SPECIALLY STRUCTURED TWO STAGE FLOWSHOP SCHEDULING INCLUDING DISJOINT JOB BLOCK CRITERIA AND PROBABILITY ASSOCIATED WITH JOBS

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ABSTRACT. The present paper is aimed to provide algorithm for minimizing the total waiting time of jobs for specially structured two stage flowshop scheduling. The model includes disjoint job block criteria and probability associated with jobs. The algorithm is made clear by numerical illustration.

1. INTRODUCTION

Scheduling is the determination of order of various jobs (tasks) for the set of machines (resources) such that certain performance measures are optimized. Scheduling involves time tabling as well as sequencing information of jobs (tasks). Scheduling is generally considered to be one of the most important issues in the planning and operation of a manufacturing system. Better scheduling system has significant impact on cost reduction, increased productivity, customer satisfaction and overall competitive advantage. Scheduling leads to increase in capacity utilization, improves efficiency and thereby reduces the time required to complete the jobs and consequently increases the profitability of an organization in present competitive environment.

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In today's manufacturing and distribution systems, scheduling have significant role to meet customer requirements as quickly as possible while maximizing the profits. A good scheduling help considerably in reducing operational costs, improving customer service and utilizing the resources optimally. The principle of optimality in the given flow shop scheduling problem is precised as minimization of waiting time of jobs. The waiting time of a job is defined as the subtraction of the completion time of job on the first machine from the starting time of job on second machine. Today's large-scale markets and immediate interactions mean that clients expect high-class goods and services at what time they require them, anywhere they require them. Organizations, whether public or private, have to make available these products and services as effectively and efficiently as possible.

2. Preliminaries

To find optimal sequence of jobs the fundamental study was made by Johnson [1] using heuristic approach for n jobs 2 and in restrictive case 3 machines flow-shop scheduling . Ignall and Schrage [2] developed branch and bound algorithms for the permutation flow-shop problem minimizing make-span . Lockett et.al. [3] studied sequencing problems which involves sequence dependent change over times. Maggu and Das and et. al. [4] introduced the equivalent job concept for job block in scheduling problems. Singh T.P. [5] extended the study by introducing various parameters like transportation time, break down interval, weightage of jobs etc. The work was further extended by Gupta J.N.D. [5], Rajendran C. et. al. [6], Singh T.P. et.al. [7] considering criteria other than make-span. Further Singh T.P., Gupta D. et.al. [8, 9] made an attempt to minimize the rental cost of machines including job block through simple heuristic approach. Gupta D. and Bharat Goyal [10] studied specially structured two stage Flow Shop scheduling models with the objective to optimize the total waiting time of jobs. Heydari [12] studied flow shop scheduling problem with processing of jobs in a string of disjoint job blocks. Singh T.P., Kumar, R. and Gupta, D. [13] studied nflow-shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks.

This paper is an extension of study done by Gupta D. and Bharat Goyal [11] in the sense that a string of disjoint job block criteria is taken in account. The concept of job block is significant in scheduling systems where certain orderings of jobs are prescribed either by technological constraints or by externally imposed policy. The string of disjoint job blocks consists of two disjoint job blocks such that in one block the order of jobs is fixed while in other block the order of jobs is arbitrary. In fact the paper is a combination of the study made by [11–13]. The objective of the study is to obtain an optimal sequence of jobs to minimize the total waiting time of the machines. An algorithm is proposed to solve the problem and is validated with the help of a numerical illustration.

3. Practical Situation

Manufacturing units/industries play a momentous role in the economic progress of a country. Flow shop scheduling occurs in various offices, service stations, banks, airports etc. In our routine working in industrial and manufacturing units, diverse jobs are practiced on a variety of machines. In textile industry different types of fabric is produced using different types of yarn. Here, the maximum equivalent time taken in dying of yarn on first machine is always less than or equal to the minimum equivalent time taken in weaving of yarn on the second machine. Flow-shop scheduling occurs in various offices, service stations, airports etc. Routine working in industries and factories have diverse jobs which are to be processed on various machines. Sometimes the manufacturer has a minimum time contract with the customers to complete their job. This condition leads to enquire about the best way to schedule the task so that waiting times for the jobs are reduced and greater satisfaction is achieved. The idea of minimizing the waiting time may be a reasonable aspect from manager's point of view in factories/ industries perspective when he has to make decision on minimum time bond with a profit-making party to complete the jobs.

4. Notations

A_k: Processing time of k^{th} job an machine A.
B_k: Processing time of k^{th} job an machine B.
$A_{k'}$: Expected processing time of k^{th} job an machine A.
$B_{k'}$: Expected processing time of k^{th} job an machine B.

p_k: Probability associated with A_k.
q_k: Probability associated with B_k.
α: Fixed order job block.

Equivalent Job Block Theorem: In processing a schedule s=(1,2,3,...,p) of p jobs on two machines M and N in the order MN with no passing allowed. A job i (i=1,2,3,...,p) has processing time M_i and N_i on each machine respectively. The job block(k,m) is equivalent to the single job α. Now the processing times of job α on the machine M and N are denoted respectively by M_α, N_α are given by

$M_\alpha = M_k - M_m - min(M_m, N_k)$
$N_\alpha = N_k - N_m - min(M_m, N_k)$

The proof of the theorem is given by Maggu P.L. and Das G.

5. Problem Formulation

Let n jobs are to be processed through two machines A and B in order AB. Let A_k and B_k denotes the processing time associated with probabilities for k^{th} job on these machines. Let two job blocks be α and β such that block α consists of i jobs out of n jobs in which the order of jobs is fixed and β consists of r jobs out of n in which order of jobs is arbitrary such that $i+r=n$. Let the two job blocks α and β form a disjoint set in the sense that the two job blocks have no job in common. Also, we consider the structural relationship i.e.$MaxA'_K \leq MinB'_k$ holds good.

Table 1. Matrix form of the problem

Jobs	Machine A		Machine B	
J	A_k	p_k	B_k	q_k
1	A_1	p_1	B_1	q_1
2	A_2	p_2	B_2	q_2
3	A_3	p_3	B_3	q_3
⋮	⋮	⋮	⋮	⋮
n	A_n	p_n	B_n	q_n

Our aim is to find a best possible sequence Si of jobs with minimum total waiting time.

6. ALGORITHM

Step 1: Calculate expected processing times, A'_k and B'_k on machines A and B defined as follows:

$A'_k = A_k \times p_k$, $B'_k = B_k \times q_k$.

Step 2: Define the fictitious machines A and B with processing times A'_k and B'_k with the expected times and verify the structural relationship:

$MaxA'_K \leq MinB'_k$.

Step 3: Take equivalent job $\alpha = (r, m)$ and calculate the processing times A_{α_1} and B_{α_2} on the guidelines of Maggu and Das as followa:

$A_{\alpha_1} = A_{r_1} + A_{m_1} - min(A_{m_1}, B_{r_2})$.

$A_{\alpha_1} = B_{r_2} + B_{m_2} - min(A_{m_1}, B_{r_2})$.

If a job block has three or more than three jobs then to find the expected flow times we use the property that the equivalent job for job-block is associative i.e. $((i_1, i2), i_3) = (i_1, (i_2, i_3))$.

Step 4: Obtain the new job block from the job block (disjoint from job block) by the proposed algorithm. Obtain the processing times and as defined in step 2.

Step 5: Now, reduce the given problem to a new problem by replacing s-jobs by job block α with the processing times A_{α_1} and B_{α_2} and remaining $r = (n - s)$ jobs by a disjoint job block β_k with processing times and as defined in step 2. Form the table in the following format:

TABLE 2

Jobs (J)	Machine A (A'_k)	Machine B (B'_k)	$x_k = B'_k - A'_k$
α	A_{α_1}	B_{α_2}	x_1
β_k	$A_{\beta_{k1}}$	$B_{\beta_{k2}}$	x_2

Step 6: Arrange the jobs in increasing order of x_k and let the sequence be $(\mu_1, \mu_2, \ldots, \mu_n)$.

Step 7: Find MinA'_k. Now two cases arise:

(a) If $A'_{\mu 1} = Min A'_k$, then schedule according to step 3 is required optimal sequence.
(b) If $A'_{\mu 1} \neq Min A'_k$, then go to next step.

Step 8: Consider the different sequences of jobs $S_1, S_2, ..., S_r$ where S_1 is the sequence obtained in step 3 , sequence $S_k (k = 2, 3, ..., r)$ can be obtained by placing k^{th} job in the sequence S_1 to the first position and rest of the sequence remaining same.

Step 9: Form the table in the following format:

TABLE 3

Jobs (J)	Machine A (A'_k)	Machine A (B'_k)	$z_{k_r} = (n-r)x_k$				
			$x_k = B'_k - A'_k$	$r = 1$	$r = 2$	$...$	$r = (n-1)$
1	A'_1	B'_1	x_1	z_{11}	z_{12}	...	$z_{1(n-1)}$
2	A'_2	B'_2	x_2	z_{21}	z_{22}	...	$z_{2(n-1)}$
3	A'_3	B'_3	x_3	z_{31}	z_{32}	...	$z_{3(n-1)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	A'_n	B'_n	x_n	z_{n_1}	z_{n_2}	...	$z_{n(n-1)}$

Step 10: Calculate the waiting time T_w for all the sequences $S_1, S_2, ..., S_r$ using the formula:

$$T_w = nA'_{\mu_1} + \sum_{r=1}^{n-1} Z_{ar} - \sum_{k=1}^{n} A'_k$$

where

A'_{μ_1} =Equivalent processing time of first job on machine X in sequence S_k.

$Z_{ar} = (n-r)x_{ar}$;$a = \mu_1, \mu_2, ..., \mu_n$

The sequence with minimum waiting time is required optimal sequence.

7. NUMERICAL ILLUSTRATION

Assume 5 jobs 1,2,3,4,5 are to be processed on two machines A and B with processing times A_k, B_k and p_k, q_k are their respective probabilities.

Our propose is to achieve a most favourable schedule, minimizing the total waiting time for the jobs.

As per step 1- Expected processing times A'_k and B'_k on machines A and B are calculated in Table 5.

TABLE 4

Jobs	Machine A		Machine B	
J	A_k	p_k	B_k	q_k
1	6	0.3	12	0.3
2	7	0.3	21	0.2
3	12	0.1	34	0.1
4	11	0.1	22	0.2
5	13	0.2	24	0.2

TABLE 5

Jobs J	Machine A A'_k	Machine B B'_k
1	1.8	3.6
2	2.1	4.2
3	1.2	3.4
4	1.1	4.4
5	2.6	4.8

As per step 2: Max$A'_k = 2.6 \leq$Min$B'_k = 3.4$

As per step 3: Take equivalent job $\alpha = (2, 5)$. Then processing times are defined as follows

$A'_\alpha = A'_2 + A'_5 - Min(A'_5 - B'_2) = 2.1$ and $B'_\alpha = B'_2 + B'_5 - Min(A'_5 - B'_2) = 6.4.$

As per step 4: Taking new job block β= (1,3,4) or (γ,4) where γ = (1,3). Then processing times are defined as described in step 2 and forming a table we receive Table 6..

TABLE 6

Jobs J	Machine A A'_k	Machine B B'_k	$x_k = B'_k - A'_k$
β	1.8	8.1	6.3
α	2.1	6.4	4.3

As per step 5- Arrange the jobs in increasing order of x_k i.e. the sequence found to be α,1,3,4 (see Table 7).

TABLE 7

Jobs (J)	Machine A (A'_k)	Machine B (B'_k)	$x_k = B'_k - A'_k$
α	2.1	6.4	4.3
β	1.8	8.1	6.3

TABLE 8

Jobs	Machine A	Machine B	$z_{kr} = (n-r)x_k$				
(J)	(A_k)	(B_k)	$x_k = B'_k - A'_k$	$r=1$	$r=2$	$r=3$	$r=4$
1	1.8	3.6	1.8	7.2	5.4	3.6	1.8
2	2.1	4.2	2.1	8.4	6.3	4.2	2.1
3	1.2	3.4	2.2	8.8	6.6	4.4	2.2
4	1.1	4.4	3.3	13.2	9.9	6.6	3.3
5	2.6	4.8	2.2	8.8	6.6	4.4	2.2

As per step 6- Min$A'_k = 1.8 \neq 2.1$.
As per step 7- The sequences obtained are
$S_1 = (\alpha, \beta) = (2, 5, 1, 3, 4)$
$S_2 = (\beta, \alpha) = (1, 3, 4, 2, 5)$.
As per step 8- Fill the values in Table 8.
As per step 9- Calculate the total waiting time for the sequences S_1, S_2:
$\sum_{k=1}^{n} A_k = 8.8$.
For the sequence $S_1 = (\alpha, \beta) = (2, 5, 1, 3, 4)$.
Total waiting time $T_w = 5 \times 2.1 + 8.4 + 6.6 + 3.6 + 2.2 - 8.8 = 14.1$.
For the sequence $S_2 = (\beta, \alpha) = (1, 3, 4, 2, 5)$.
Total waiting time $T_w = 5 \times 1.8 + 7.2 + 6.6 + 6.6 + 2.1 - 8.8 = 22.7$.
Hence the sequence $S_1 = (2, 5, 1, 3, 4)$ is the required sequence with minimum total waiting time.

8. CONCLUSION

The present study deals simple flowshop scheduling model with the main idea to optimize the total waiting time of jobs. However, it may increase the other costs like machine idle cost or penalty cost of the jobs, yet the idea of minimizing the waiting time is a matter that cannot be avoided in cases when there is a minimum time contract with the customers. The study can be extended by introducing various parameters like weightage of jobs, setup time of machines, breakdown interval of machines, fuzzy processing time etc.

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