

NOISE VARIANCE ESTIMATION IN MAGNITUDE MAGNETIC RESONANCE IMAGES

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ABSTRACT. Noise corrupted Magnetic Resonance Images (MRI) are modelled with Rician probability distribution. Cancellation of such signal dependent noise in MRI is a challenging task. An efficient and robust MR image reconstruction method is required to efficiently estimate and reduce noise in MR images. In this paper, linear minimum square error (LMMSE) estimation method is presented, which employs the self-similarity property of the MRI to restore the noise free images.

1. INTRODUCTION

Magnetic Resonance Images (MRI) is a non-invasive imaging modality and preferred over other ones. MRI provides important information such as internal anatomical details of patient body, soft-tissue, bone structures and internal organ structural details non-invasively. MRI data is usually corrupted by random noise occurred during image acquisition by some environmental operational factors and degrade image quality. This noise corrupted data harm the useful information required for automatic computerised analysis and diagnostic applications in clinical environment [1].

In order to provide useful information for its intended used in clinical practices, these are required to make noise free. An robust noise estimation and

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removal method is required for computerised image reconstruction and for its further automated analysis [2]. Noisy magnitude MR images follows Rician distribution. The zero mean uncorrelated probability of Gaussian noise corrupt the complex image data and assumed to be zero mean and equal variance in both the real and imaginary component of k -space data. The real and imaginary parts of the complex raw data are corrupted with white additive Gaussian noise. The spatial visual MR image is constructed taking the magnitude of the complex image data. After taking magnitude, the MR image data is modelled into Rician distribution. Rician probability also introduced a bias in reconstructed image [2, 3].

The impact of the noise level present in the image can be minimized by taking the average of multiple scans of the desired area of interest during image acquisition of data. However, in real clinical application it is not a relevant practical solution due to some operational practical constraints such as patient comfort due to long data acquisition time [4]. Noisy magnitude MRI contains Gaussian additive noise, both in the real and imaginary channels of the data.

2. MRI NOISE MODELING

The k -space data is raw complex valued data with same mean value at each sample is corrupted by additive white Gaussian noise with square variance. The magnitude MR image is created by taking the root of the sum squared of both real and imaginary components by a non-linear operation. The probability distribution in the constructed magnitude MR image becomes Rician distributed and is model as in [5]:

$$p_M(M|A, \sigma_n) = \frac{M}{\sigma_n^2} e^{-\left(M^2 + \frac{A^2}{2\sigma_n^2}\right)} I_0\left(\frac{AM}{2\sigma_n^2}\right) H(M),$$

where I_0 is the 0^{th} order modified Bessel function of its first kind and M denotes the Rician distribution random variable, $A = \sqrt{(A_R^2 + A_I^2)}$ is the real complex MR signal without any noise and $H(\cdot)$ represents the Heaviside step function. The shape of the Rician distribution depends on the signal to noise ratio that is defined as $\frac{A}{\sigma_n}$. At high SNR when the $\frac{A}{\sigma_n}$, the Rician distribution of image data approaches a Gaussian distribution and is give as in [5]:

$$p_M(M, \sigma_n) = \frac{1}{\sqrt{2\pi}\sigma_n^2} e^{-\left(\frac{(M-A)^2}{2\sigma_n^2}\right)} H(M).$$

The background MR regions has no signal value equal 0, hence the probability here follows Rayleigh distribution that can be defined as in [6]:

$$p_M(M, \sigma_n) = p_M(M|A=0, \sigma_n) = \frac{M}{\sigma_n^2} e^{-\left(\frac{M^2}{2\sigma_n^2}\right)} HM.$$

3. SIGNAL AND NOISE PARAMETER ESTIMATION IN MRI

Similar to Gaussian smoothing and Wiener filtering, a noise free image data A is approximated from the noisy magnitude MRI M . A magnitude MRI M is created from the squared sum of squares of the real and imaginary parts of the complex k -space data.

A very simplified and reliable approach to predict the noise level in the magnitude images is acquired the same image twice and taking their average value. Various noise level estimation methods are found in the literature such as given in [7–10]. For a given noisy magnitude MRI and its components, a few dominant noise variance estimation methods are given here as follows:

(A) Conventional Estimator

The relationship between the second order magnitude data moments and noise contents in a Rician distributed data MRI image, noise free signal is estimated as:

$$A_c = \sqrt{\max(\langle M^2 \rangle - 2\sigma_n^2, 0)},$$

where $\langle M^2 \rangle$ is the simple second order moment and $\langle . \rangle$ is defined as:

$$\langle I \rangle = \frac{1}{|\eta|} \sum_{p \in \eta} Ip,$$

where η is square neighborhood mask window. This is called conventional variance estimator.

(B) Maximum Likelihood Estimator

Maximum likelihood estimator estimates the noise free signal by maximizing the likelihood function as:

$$A_{ML} = \arg \max_A (\log L),$$

where $\log L$ is defined as:

$$\log L = \sum_{i=1}^N \log \left(I_0 \left(\frac{AM_i}{\sigma_n^2} \right) \right) - \frac{NA^2}{2\sigma_n^2} - \sum_{i=1}^N \frac{M_i^2}{2\sigma_n^2}.$$

(C) Expectation Maximization

Expected minimization is a recursive method to estimate noise variance and noise free data simultaneously by maximizing the expected likelihood log is defined below as:

$$A_{K+1} = \frac{1}{N} \sum_{i=1}^N \frac{l_1 \frac{(AM_i)}{\sigma_n^2}}{I_0 \frac{(A_K M_i)}{\sigma_n^2}} M_i$$

and

$$\sigma_{K+1}^2 = \max \left[\frac{1}{N} \sum_{i=1}^N M_i^2 - \frac{A_K^2}{\sigma}, 0 \right],$$

where N is again the number of samples and the initial values of the data signal are computed as:

$$A_0 = \left(2 \left(\frac{1}{N} \sum_{i=1}^N M_i^2 \right)^2 - \frac{1}{N} \sum_{i=1}^N M_i^4 \right)^{1/4},$$

where σ_n^2 in the above equation is defined as given below:

$$\sigma_n^2 = \frac{1}{2} \left(\frac{1}{N} \sum_{i=1}^N M_i^2 - A_0 \right).$$

Commonly, even moments of the Rician distributed data in question are exploited for parameter estimation. The v^{th} moments of the Rician probability density of the MRI data can be given as:

$$E[M^v] = \int_0^\infty \frac{M_{v+1}}{\sigma^2} e^{-\left(\frac{M^2+A^2}{2\sigma^2}\right)} I_0 \left(\frac{AM}{\sigma^2} \right) HM,$$

where $E[.]$ is the expectation operator and $E[M^2]$ is computed from a simple local spatial average and as is defined in [10–13] is given below:

$$E[M^2] = \langle M^2 \rangle = \frac{1}{N} \sum_{i=1}^N M_i^2.$$

This is an unbiased estimator since $E[M^2] = \langle M^2 \rangle$. Therefore, an unbiased variance parameter of magnitude of the squared MR image data is given by $A_c = \langle M^2 \rangle$ and taking its square root that will give the conventional estimator as given in [10–13] and the same is also reproduced here as given below:

$$A_c^2 = \sqrt{\langle M^2 - 2\sigma^2 \rangle}.$$

4. LINEAR MINIMUM MEAN SQUARE ERROR NOISE ESTIMATION

To estimate the true signal from the noisy magnitude MR images, statistical linear minimum mean square error estimation (LMMSE) method appeared an effective technique. This technique provides a closed-form solution from the related functional random data variables where as maximum likelihood and expectation maximization methods provide solution in an iterative manner. The LMMSE method emerges more efficient method than other optimization-based estimation methods. In this approach *sigma*, the noise variance is observed from the cross-covariance vector and the covariance matrix of the actual data and can be defined as:

$$\sigma = E\{\sigma\} + C_{\sigma A} C_{AA}^{-1} (A - E\{A\}).$$

In the above equation, $C_{\sigma A}$ is a cross-covariance vector and C_{AA}^{-1} is the covariance matrix of the image data. In this closed-form solution, A^2 is used instead of A because the even order moments of the Rician distribution are simple polynomials and therefore easier to calculate. For a 2D Rician distributed MRI data, the above model can also be represented as:

$$A_{ij}^2 = E\{A_{ij}^2\} + C_{A_{ij}^2 M_{ij}^2} C_{M_{ij}^2 M_{ij}^2}^{-1} (M_{ij}^2 - E\{M_{ij}^2\}).$$

The co-variance matrices in this case are just scalar values for each pixel location in the image matrix. For the Rician distributed image data, the square linear minimum mean square error estimation for a 2D MR image is given below:

$$A_{ij}^2 = E\{A_{ij}^2\} \frac{E\{A_{ij}^2\} + 2E\{A_{ij}^2\}\sigma_n^2 - E\{A_{ij}^2\}E\{M_{ij}^2\}}{E\{A_{ij}^4\} - E\{A_{ij}^2\}^2} \times (M_{ij}^2 - E\{M_{ij}^2\}).$$

By replacing the expectations by the sample estimator $\langle . \rangle$, the above expression can be represented as:

$$A_{ij}^2 = \langle M_{ij}^2 \rangle - 2\sigma_n^2 + K_{ij}(M_{ij}^2 - \langle M_{ij}^2 \rangle).$$

In this equation, K_{ij} is taken from the expression given below:

$$K_{i,j} = 1 - \frac{4\sigma_n^2(\langle M_{i,j}^2 \rangle - \sigma_n^2)}{\langle M_{i,j}^4 \rangle - \langle M_{i,j}^2 \rangle^2}.$$

5. EXPERIMENTATION RESULTS

In experiments, single slice noise free magnitude MRIs are used from Brian-Web MRI database. Forwarded and inverse Fourier transform methods applied to split transform the image data. To create a noisy magnitude image, it is squared taking sum of squared real and imaginary data with different noise levels. Some related stochastic Rician distributed based model such as given in [10, 14–17] are applied.

The statistics of the noisy MRIs is calculated using local neighborhood sliding window of size 5×5 around each pixel. This method is compared with the Adaptive Wiener filter and Wavelet domain filtering for medical image processing as given in [18, 19]. The neighborhood size can be set as $r = cd^2$. A trade-off exists between the computational accuracy and computation time taken to compute the statistically significant of the outcome results. This method works well with noise variance σ^2 with 5×5 neighborhood window and automatically estimate the noise variance using $\hat{\sigma}_n = \sqrt{\frac{2}{N} bmod(\hat{\mu}1_{i,j})}$. In the noisy MRI background regions, the maximum mean shifted towards a value of noise standard deviation σ_n . The mean of a Rayleigh distribution is defined as:

$$\hat{\mu}1 = \sigma_n \sqrt{\frac{\pi}{2}}.$$

Based on above equation, a new noise estimator can be defined as:

$$\hat{\sigma}_n = \sqrt{\frac{2}{\pi} \arg \max \{p(\hat{\mu}1)\}}.$$

The maximum mean $\hat{\sigma}_n$ of the noise can be calculated from above as the mode of the noise distribution as:

$$\hat{\sigma}_n = \sqrt{\frac{2}{\pi} bmod(\hat{\mu}1_{i,j})}.$$

When the background pixels are properly isolated, the performance of the estimator is largely same for all the estimation around the distribution of noise variance. In [20] histogram mode based noise estimator robustly estimate the noise properly when the background pixels wrongly assigned to foreground pixels.

In Table 1, the estimated noise variance result against the applied pseudo-noise variance on the different images of different sizes is depicted. Figure 1 represents the same. Only gray-scale images with 256 intensity levels and

TABLE 1. Experimental observation of estimated noise variance

	Noise Levels Applied vs Estimated Variance											
Images	3	5	7	10	15	20	25	30	35	40	45	50
Brain-T1	3.40	5.33	7.72	10.19	15.10	19.99	24.68	29.46	34.58	40.09	45.05	50.14
Brain-T2	3.35	5.38	7.24	10.18	14.78	19.87	24.76	30.17	34.84	40.38	44.98	59.64
Saggital	4.21	5.50	7.55	10.63	15.27	20.42	25.17	30.42	34.70	40.16	45.48	50.63
Lena	3.32	5.17	7.10	10.12	14.85	19.62	24.57	29.30	34.55	39.99	44.99	49.01
Barbara	5.88	7.13	8.61	11.03	15.71	20.46	25.45	30.56	34.94	40.53	45.26	50.40

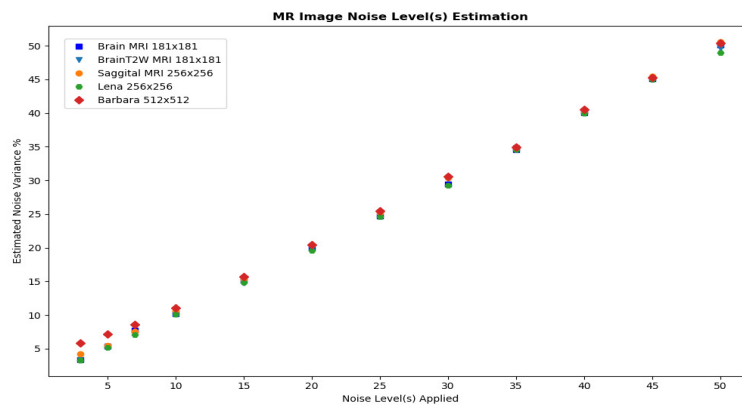


FIGURE 1. Plot of applied vs estimated noise variance

different sizes are used. For example input MRI images are of 181×181 , 256×256 and Lena and Barbara images are used of 256×256 and 512×512 sizes and 10 to 50 iterations are applied using 5×5 neighborhood kernel for noise estimation.

6. CONCLUSION

The accuracy of this method is best in small size medical images, but not perform well in all type of noisy images with other than Rician probability distribution. For mixed type noise levels, this method not perform well and required to tune depending upon the type and distribution of noise. There is a lot of scope to improve the estimation methods to provide more robust estimation criteria in all type of noise corrupted MR images.

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