ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.6, 3289–3294
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.6.8 Spec. Issue on RDESTM-2020

A STUDY ON 0-CONDITIONS IN THE SUBGROUP LATTICES OF 2×2 MATRICES OVER Z_{11}

R. SEETHALAKSHMI[1], V. DURAIMURUGAN, AND P. NAMASIVAYAM

ABSTRACT. In this paper we verify the lattice theoretic properties like 0-modularity, 0-semi modularity, 0-super modularity, 0-distributivity, super 0-distributivity, pseudo 0-distributivity in the subgroup lattice of the group of 2×2 matrices over Z_{11}.

1. INTRODUCTION

Let L(G) be the Lattice of Subgroups of G, where G is a group of 2x2 matrices over Z_p having determinant value 1 under matrix multiplication modulo p, where p is a prime number. Let G = $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \varepsilon Z_p, ad - bc \neq 0 \right\}$ Then G is a group under matrix multiplication modulo p. Let

$$G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \varepsilon G : ad - bc = 0 \right\}.$$

Then G is a subgroup of G. We have,

$$o(G) = p(p2-1)(p-1) and, o(G) = p(p2-1) \tag{1.1}$$

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 06B10.
Key words and phrases. Subgroups, Lattice, Lattice Identities.

For ready reference we give the split up of the lattice of subgroups of G whenp=11 [4]. In this paper we are going to study about all the above properties in L(G) .

2. Preliminaries

Definition 2.1. *(Lattice) [3] Poset (P, $\leq$) is called a lattice if every pair x, y elements of P has a supre mum and an infimum, which are denoted by x $\vee$ y and x $\wedge$ y respectively.*

Definition 2.2. *(0-Modular Lattice) [2] A lattice L is said to be 0-modular if whenever x $\leq$ y and y$\wedge$ z = 0, then x = (x $\vee$ z) $\wedge$ y, for all x,y,z ε L.*

Definition 2.3. *(0-semi modular) [5] A Lattice L is said to be 0-semi modular if whenever a is an atom of L and x ε L such that a $\wedge$ x = 0, then x$\vee$a covers x.*

Definition 2.4. *(0-upermodular) [2] A lattice L is said to be 0-supermodular if for all a, b, c, d ε L, with b$\wedge$ c = c $\wedge$ d = b $\wedge$ d = 0,we have*

Definition 2.5. *(0-Distributive lattice) [2] A Lattice L is said to be 0-distributive if for all x. y, zεL whenever x$\wedge$y = 0 and x$\wedge$z = 0 then x$\wedge$ (y$\vee$z) = 0.*

Definition 2.6. *(super 0-distributive) [2] A Lattice L is said to be super 0- distributive if for all x, y, z ε L , x$\wedge$y = 0 implies that (x$\vee$y)$\wedge$z = (x$\wedge$z)$\vee$(y$\wedge$z).*

Definition 2.7. *(pseudo 0-Distributive) [2] A Lattice L is said to be pseudo 0-Distributive if for all x. y, z ε L with x$\wedge$y = 0, x$\wedge$z = 0 we have (x$\vee$y)$\wedge$z = (y$\wedge$z).*

We give on fig. 1 the diagram of L(G) when P=11.

$$f(x,y) = \{[(f_R(x,y) f_G(x,y) f_B(x,y))]^T : \\ V(x,y)\varepsilon\{0,1,2,\ldots,m-1\} \times \{0,1,2,\ldots,n-1\}\}.$$

Row I: (Left to right) $L_1$to L_{12}
Row II: (Left to right) J_1 to J_{55} and I_1 to I_{12}
Row III: (Left to right) F_1 to $F_5$5 and H_1 to H_{66}
Row IV: (Left to right) C_1 to $C_5$5 and E_1 to $E_5$5
Row V: (Left to right) A_1, $B_1$to $B_5$5 and D_1 to $D_6$6

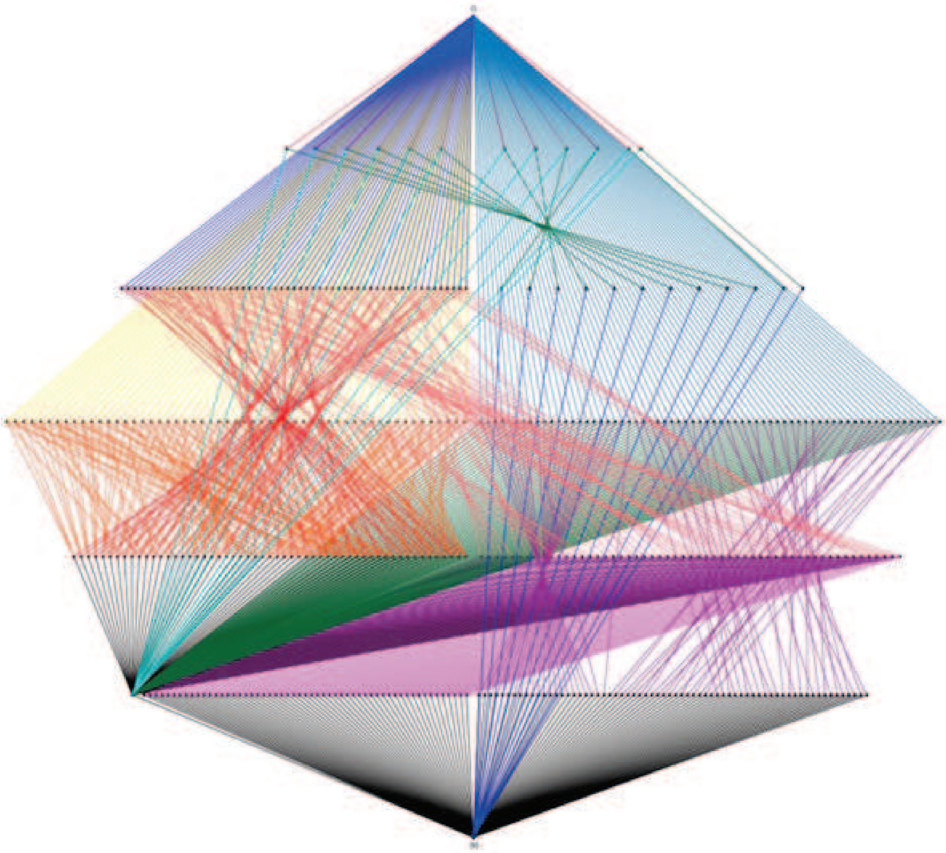

FIGURE 1. L(G) when p = 11

3. LATTICE IDENTITIES IN THE SUBGROUP LATTICE OF THE GROUP OF 2X2 MATRICES OVER Z_{11}

Lemma 3.1. *L(G) is not 0-modular if p = 11.*

Proof. From fig.1, we take three subgroups C_54, J_1, I_2, ε L(G). Let $C_{54} \subset J_1$ and $J_1 \wedge I_2$= e. But, $(C_{54} \vee I_2) \wedge J_1 = G \wedge J_1 = J_1 \neg C_{54}$. Therefore, $(C_{54} \vee I_2) \wedge J_1 \neg C_{54}$. Hence, L(G) is not 0-modular when p = 11. □

Lemma 3.2. *L(G) is not 0-semi modular if p = 11.*

Proof. From fig. 1, we take two subgroups $I_1, C_{19} \varepsilon L(G)$. We know that, I1 is an atom of L(G) and $C_{19} \varepsilon L(G)$ such that $I_1 \wedge C_{19}$=0. Then $I_1 \vee C_{19}$=G which does not cover C19. Hence, L(G) is not 0-semi modular when p = 11. □

Lemma 3.3. *L(G) is not 0-super modular if p = 11.*

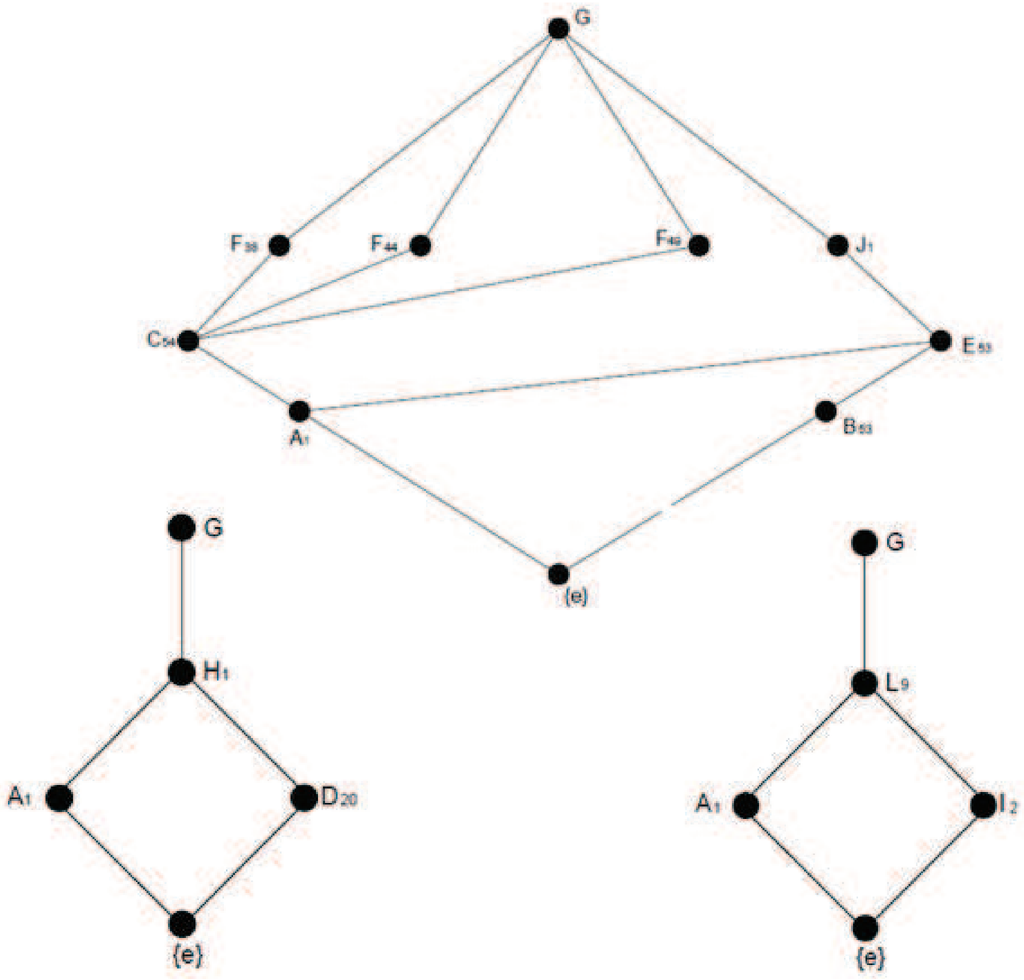

FIGURE 2

Proof. From fig. 2, we choose four subgroups $B_{53}, H_1, F_{38}, I_2 \ \varepsilon$ L(G). Now, Let $H_1 \wedge F_{38} = F_{38} \wedge I_2 = H_1 \wedge I_2$=0. Then, $(B_{53} \vee H_1) \wedge (B_{53} \vee F_{38}) \wedge (B_{53} \vee I_2) = G \wedge G \wedge G = G \neq B_{53}$. Therefore, $(B_{53} \vee H_1) \wedge (B_{53} \vee F_{38}) \wedge (B_{53} \vee I_2) \neq B_{53}$. Hence, L(G) is not 0-super modular when p = 11. □

Lemma 3.4. *L(G) is not 0-distributive if p = 11.*

Proof. From fig. 1, we take three subgroups $C_{19}, I_1, B_1 \varepsilon L$ (G). Let $C_{19} \wedge I_1 = eandC_{19} \wedge B_1$ = e. But, $C_{19} \wedge (I_1 \vee B_1) = C_{19} \wedge G = C_{19} \neq e$. Therefore, $C_{19} \wedge (I_1 \vee B_1) \neq e$. Hence, L(G) is not 0-distributive when p = 11. □

Lemma 3.5. *L(G) is not super 0-distributive if p = 11.*

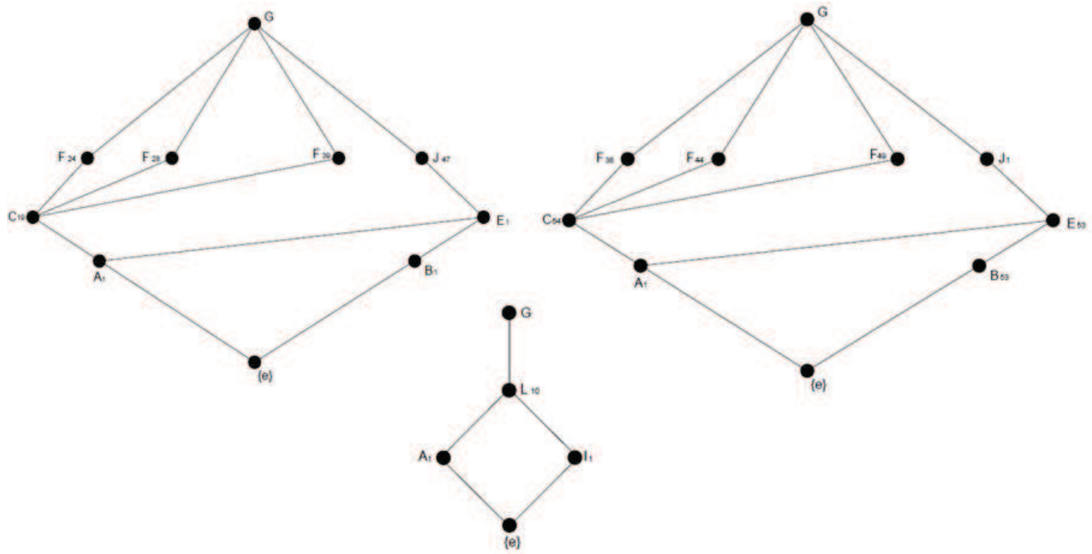

FIGURE 3

Proof. From fig. 3, we take three subgroups $C_{54}, I_2, B_{53} \varepsilon L(G)$. Let $C_{54} \wedge I_2$=0. Then, $(C_{54} \vee 3I_2) \wedge B_{53} = G \wedge B_{53} = B_{53}$. But, $(C_{54} \wedge B_{53}) \vee (I_2 \wedge B_{53}) = e \vee e = e \neq B_{53}$. Therefore, $(C_{54} \vee I_2) \wedge B_{53} \neq (C_{54} \wedge B_{53}) \vee (I_2 \wedge B_{53})$. Hence L(G) is not super 0-distributive when p = 11. □

Lemma 3.6. *L(G) is not pseudo 0-distributive if p = 11.*

Proof. From fig. 1, we take three subgroups $B_1, D_{20}, I_1 \varepsilon L(G)$.. Now, Let $B_1 \wedge D_{20} = 0 and B_1 \wedge I_1 = 0$. Then, $(B_1 \vee D_{20}) \wedge I_1 = G \wedge I_1 = I_1$. But, $(D_{20} \wedge I_1) = e \neq I_1$. Therefore, $(B_1 \vee D_{20}) \wedge I_1 \neq D_{20} \wedge I_1$. Hence, L(G) is not pseudo 0-distributive. □

4. CONCLUSION

In this paper we proved that the 0-modularity, 0- semi modularity, 0-super modularity, 0-distributivity,super 0-distributivity, pseudo 0-distributivity in the subgroup lattice of the group of 2x2 matrices over Z_{11}.

REFERENCES

[1] D. JEBARAJ THIRAVIAM: *A Study on some special types of lattices*, Ph.D thesis, Manonmaniam Sundaranar University, 2015.

[2] I. CHAJDA, S. RADELECZKI: *O-conditions and Tolerance schemes*, Acta Math. Univ. Comenianae, **70**(11,2) (2003), 177–184.

[3] A. ROSENFELD: *Fuzzy groups*, J. Math Anal. and App., **35** (1971), 512–517.

[4] R. SEETHALAKSHMI, V. DURAI MURUGAN, R. MURUGESAN: *On the lattice of subgroups of 2x2 matrices over Z11*, Malaya journal of Matematik, **5**(1) (2020), 451–456.

[5] A. VETHAMANICKAM: *Topics in universal Algebra*, Ph.D thesis, Madurai Kamaraj University, 1994.

DEPARTMENT OF MATHEMATICS
MANONMANIAMSUNDARANAR UNIVERSITY ABISHEKAPATTI
TIRUNELVELI 627 012, TAMILNADU, INDIA
E-mail address: trseethalakshmi@gmail.com

DEPARTMENT OF MATHEMATICS
ST. JOSEPH COLLEGE OF ARTS AND SCIENCE
TENKASI, TAMILNADU, INDIA
E-mail address: vvndurai@gmail.com

DEPARTMENT OF MATHEMATICS
THE MDT HINDU COLLEGE
TIRUNELVELI-627010 INDIA
E-mail address: vasuhe2010@gmail.com