

OPERATIONS ON SOME STAR RELATED PERFECT MEAN CORDIAL GRAPHS

A. ANNIE LYDIA¹ AND M. K. ANGEL JEBITHA

ABSTRACT. A vertex labeling $h : V(G) \rightarrow \{0, 1, 2, 3\}$ is said to be perfect mean cordial labeling of a graph G if it induces an edge labeling h^* defined as follows:

$$h^*(wz) = \begin{cases} 1 & \text{if } 2|(h(w) + h(z)) \\ 0 & \text{otherwise} \end{cases},$$

with the condition that $|e_h(0) - e_h(1)| \leq 1$ and $|v_h(\alpha) - v_h(\beta)| \leq 1$ for all $\alpha, \beta \in \{0, 1, 2, 3\}$, where $e_h(\delta)$ is number of edges label with δ ($\delta = 0, 1$) and $v_h(\lambda)$ denote the number of vertices labeled with λ ($\lambda = 0, 1, 2, 3$). A graph G is said to be perfect mean cordial graph if it admits a perfect mean cordial labeling. In this paper, we prove that operations on some star related perfect mean cordial graphs are perfect mean cordial graphs.

1. INTRODUCTION

In the present era, graph theory has become a highly challenging and interesting area for the study of numerous mathematicians and computer experts. Since it has many applications and scope for various researches, it has attracted the attention of the erudite scholars who have the overwhelming desire for updating the field of mathematics. Particularly graph labeling has become a widely popular and area of concern, since it offers wide range of applications. A graph labeling is an assignment of integers to the nodes or the links, or both, subject to certain conditions.

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In 1987, Cahit introduced the concept of cordial labeling as a weaker version of graceful and harmonious labeling. In perfect mean cordial graph was introduced and proved that some standard graphs are perfect mean cordial graphs. A complete bipartite graph in which one partite set has r vertices and another partite set has s vertices is denoted by $K_{r,s}$. A complete bipartite graph $K_{1,k}$ is said to be a star and is denoted by S_k . A vertex is said to be support if it adjacent to a pendant vertex.

Definition 1.1. *If the vertices of the graph are assigned values subject to certain conditions is known as graph labeling.*

Definition 1.2. *Let $G = (V, E)$ be a graph. A mapping $h : V(G) \rightarrow \{0, 1\}$ is called binary vertex labeling of G and $h(v)$ is called label of the vertex v of G under h . For an edge $e = wz$, the induced edge labeling $h^* : E(G) \rightarrow \{0, 1\}$ is given by $h^*(e) = |h(w) - h(z)|$. Let us denote the number of vertices of G having labels 0, 1 by $v_h(0), v_h(1)$ respectively under h and the number of edges of G having labels $e_h(0), e_h(1)$ respectively under h^* .*

Definition 1.3. *A binary vertex labeling of a graph G is called cordial labeling if $|v_h(0) - v_h(1)| \leq 1$ and $|e_h(0) - e_h(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.*

2. DEFINITIONS

Definition 2.1. *A vertex labeling $h : V(G) \rightarrow \{0, 1, 2, 3\}$ with induced edge labeling $h^* : E(G) \rightarrow \{0, 1\}$ defined by*

$$h^*(wz) = \begin{cases} 1 & \text{if } 2 \mid (h(w) + h(z)) \\ 0 & \text{otherwise} \end{cases}$$

is called perfect mean cordial labeling of a graph G if $|e_h(0) - e_h(1)| \leq 1$ and $|v_h(\alpha) - v_h(\beta)| \leq 1$ for all $\alpha, \beta \in \{0, 1, 2, 3\}$, where $v_h(\lambda)$ is the number of vertices labeled with λ ($\lambda = 0, 1, 2, 3$) and $e_h(\delta)$ is number of edges label with δ ($\delta = 0, 1$). A graph G is said to be perfect mean cordial graph if it admits a perfect mean cordial labeling.

Example 1. *The graph G which is shown in Figure 2.1 is a perfect mean cordial labeling.*

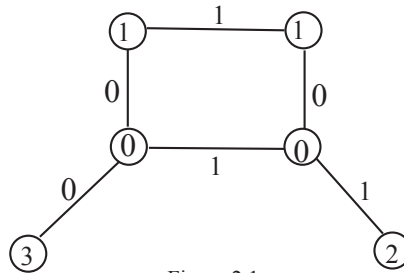


Figure 2.1

Definition 2.2. An edge $wz \in E(G)$ is subdivided if the edge wz is deleted, a new vertex s (called the subdivision vertex) is added, along the new edge ws and sz . A subdivision graph $S(G)$ of a graph G is obtained from G by subdividing all the edges exactly once.

Definition 2.3. Let G be a graph with apex vertex. Consider two copies of a graph G namely G_1 and G_2 , then the graph $G' = \langle G_1 : G_2 \rangle$ is the graph obtained by joining apex vertex of G_1 and G_2 to a new vertex s .

Definition 2.4. Let G be a graph with apex vertex. Consider two copies of a graph G namely G_1 and G_2 , then the graph $G' = \langle G_1 \blacktriangle G_2 \rangle$ is the graph obtained by joining apex vertices of G_1 and G_2 to a new vertex s as well as joining apex vertex of G_1 and G_2 by a edge.

Terms not defined are used in the sense of [2]. In this paper, we prove that two operations on graphs, which are having apex vertex are perfect mean cordial graphs.

3. PRIME CONCLUSIONS

In this session, we utilise two operations on star graphs and subdivided star graphs. In [1] we proved star and subdivided star are perfect mean cordial graphs. Here we prove that two operations on two isomorphic star graphs are perfect mean cordial graphs and two isomorphic subdivided star graphs with the same two operations are perfect mean cordial graphs. In [3–5] authors used these operations in different types of labeling. Here we used this operations on perfect mean cordial labeling of some graphs.

Theorem 3.1. Graph $\langle K_{1,k}^{(1)} : K_{1,k}^{(2)} \rangle$ is perfect mean cordial graph.

Proof. The pendant vertices of $K_{1,k}^{(1)}$ are $w_1^{(1)}, w_2^{(1)}, w_3^{(1)}, \dots, w_k^{(1)}$, the pendant vertices of $K_{1,k}^{(2)}$ are $w_1^{(2)}, w_2^{(2)}, w_3^{(2)}, \dots, w_k^{(2)}$. The apex vertices of $K_{1,k}^{(1)}$ and $K_{1,k}^{(2)}$ be a_1, a_2 and they are adjacent to a new vertex s .

Let $G = \langle K_{1,k}^{(1)} : K_{1,k}^{(2)} \rangle$. We construct vertex labeling $h : V(G) \rightarrow \{0, 1, 2, 3\}$ as proceeds below.

Case 1: k is odd.

$$h(w_j^{(1)}) = \begin{cases} 0 & j \equiv 0 \pmod{4} \\ 3 & j \equiv 1 \pmod{4} \\ 2 & j \equiv 2 \pmod{4} \\ 1 & j \equiv 3 \pmod{4} \end{cases} \quad 1 \leq i \leq k$$

$$h(w_j^{(2)}) = \begin{cases} 0 & j \equiv 0 \pmod{4} \\ 1 & j \equiv 1 \pmod{4} \\ 2 & j \equiv 2 \pmod{4} \\ 3 & j \equiv 3 \pmod{4} \end{cases} \quad 1 \leq i \leq k$$

$h(a_1) = 0; h(a_2) = 0; h(s) = 2$. The induced edge labeling is,

$$h^*(wz) = \begin{cases} 1 & \text{if } 2(h(w) + h(z)) \text{ for all } wz \in E(G) \\ 0 & \text{otherwise} \end{cases}.$$

Examination of vertex, edge demands are illustrated below.

k is odd	vertex values	edge values
$k \equiv 1 \pmod{4}$	$v_h(0) = \frac{k-1}{2} + 2, v_h(1) = v_h(2) = v_h(3) = \frac{k-1}{2} + 1$	$e_h(0) = e_h(1) = k + 1$
$k \equiv 3 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = \frac{k-3}{2} + 2, v_h(2) = \frac{k-3}{2} + 3$	$e_h(0) = e_h(1) = k + 1$

Case 2: k is even.

$$h(w_j^{(1)}) = \begin{cases} 0 & j \equiv 0 \pmod{4} \\ 1 & j \equiv 1 \pmod{4} \\ 2 & j \equiv 2 \pmod{4} \\ 3 & j \equiv 3 \pmod{4} \end{cases} \quad 1 \leq i \leq k$$

$$h(w_j^{(2)}) = \begin{cases} 1 & j \equiv 0 \pmod{4} \\ 2 & j \equiv 1 \pmod{4} \\ 3 & j \equiv 2 \pmod{4} \\ 0 & j \equiv 3 \pmod{4} \end{cases} \quad 1 \leq i \leq k$$

$h(a_1) = 0; h(a_2) = 1; h(s) = 3$. The induced edge labeling is,

$$h^*(wz) = \begin{cases} 1 & \text{if } 2|(f(w) + f(z)) \text{ for all } wz \in E(G) \\ 0 & \text{otherwise} \end{cases}$$

Examination of vertex and edge demands are illustrated below.

k is even	vertex values	edge values
$k \equiv 0 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = \frac{k}{2} + 1,$ $v_h(2) = \frac{k}{2}$	$e_h(0) = e_h(1) = k + 1$
$k \equiv 2 \pmod{4}$	$v_h(0) = \frac{k-2}{2} + 1, v_h(1) = v_h(2) =$ $v_h(3) = \frac{k-2}{2} + 2$	$e_h(0) = e_h(1) = k + 1$

Consequently, the graph G fulfills the demands $|e_h(0) - e_h(1)| \leq 1$ and $|v_h(\alpha) - v_h(\beta)| \leq 1$ for all $\alpha, \beta \in \{0, 1, 2, 3\}$. Accordingly, $\langle K_{1,k}^{(1)} : K_{1,k}^{(2)} \rangle$ is perfect mean cordial graph.

□

Example 2. Illustration of the perfect mean cordial graph $\langle K_{1,5}^{(1)} : K_{1,5}^{(2)} \rangle$ is shown in the figure 3.1

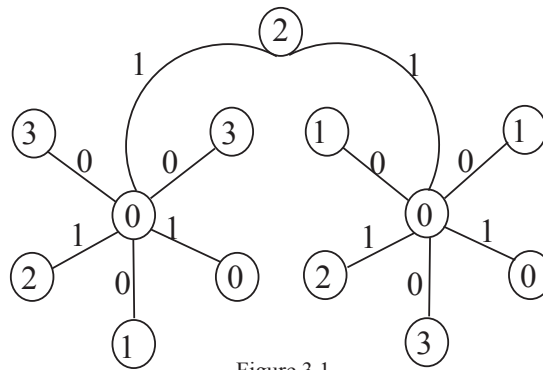


Figure 3.1

Theorem 3.2. Graph $\langle K_{1,k}^{(1)} \blacktriangle K_{1,k}^{(2)} \rangle$ is perfect mean cordial graph.

Proof. The pendant vertices of $K_{1,k}^{(1)}$ are $w_1^{(1)}, w_2^{(1)}, w_3^{(1)}, \dots, w_k^{(1)}$ and the pendant vertices of $K_{1,k}^{(2)}$ are $w_1^{(2)}, w_2^{(2)}, w_3^{(2)}, \dots, w_k^{(2)}$. The apex vertices of $K_{1,k}^{(1)}$ and $K_{1,k}^{(2)}$ be a_1 and a_2 and they are adjacent to a new vertex w .

Let $\langle K_{1,k}^{(1)} \blacktriangle K_{1,k}^{(2)} \rangle$. We construct vertex labeling $h : V(G) \rightarrow \{0, 1, 2, 3\}$ as proceeds below.

Case 1: k is odd.

$$h(w_j^{(1)}) = \begin{cases} 0 & j \equiv 0 \pmod{4} \\ 3 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 2 & j \equiv 2 \pmod{4} \\ 1 & j \equiv 3 \pmod{4} \end{cases}$$

$$h(w_j^{(2)}) = \begin{cases} 0 & j \equiv 0 \pmod{4} \\ 1 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 2 & j \equiv 2 \pmod{4} \\ 3 & j \equiv 3 \pmod{4} \end{cases}$$

$f(a_1) = 0; f(a_2) = 0; f(s) = 2$. The induced edge labeling is,

$$f^*(wz) = \begin{cases} 1 & \text{if } 2 \mid (f(w) + f(z)) \text{ for all } wz \in E(G) \\ 0 & \text{otherwise} \end{cases}$$

Examination of vertices, edges demands are illustrated below.

k is odd	vertex values	edge values
$k \equiv 1 \pmod{4}$	$v_h(0) = \frac{k-1}{2} + 2, v_h(1) = v_h(2) = \frac{k-1}{2} + 1$	$e_h(0) = k + 1, e_h(1) = k + 2$
$k \equiv 3 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = \frac{k-3}{2} + 2, v_h(2) = \frac{k-3}{2} + 3$	$e_h(0) = k + 1, e_h(1) = k + 2$

Case 2: k is even.

$$f(w_j^{(1)}) = \begin{cases} 0 & j \equiv 0 \pmod{4} \\ 1 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 2 & j \equiv 2 \pmod{4} \\ 3 & j \equiv 3 \pmod{4} \end{cases}$$

$$f(w_j^{(2)}) = \begin{cases} 1 & j \equiv 0 \pmod{4} \\ 2 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 3 & j \equiv 2 \pmod{4} \\ 0 & j \equiv 3 \pmod{4} \end{cases}$$

$h(a_1) = 0; h(a_2) = 1; h(s) = 3$. The induced edge labeling is,

$$h^*(wz) = \begin{cases} 1 & \text{if } 2 \mid (f(w) + f(z)) \text{ for all } wz \in E(G) \\ 0 & \text{otherwise} \end{cases}$$

Examination of vertex and edge demands are illustrated below.

k is even	vertex values	edge values
$k \equiv 0 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = \frac{k}{2} + 1, v_h(2) = \frac{k}{2}$	$e_h(0) = k + 2, e_h(1) = k + 1$
$k \equiv 2 \pmod{4}$	$v_h(0) = \frac{k-2}{2} + 1, v_h(1) = v_h(2) = v_h(3) = \frac{k-2}{2} + 2$	$e_h(0) = k + 2, e_h(1) = k + 1$

Consequently, the graph G fulfills the demands $|e_h(0) - e_h(1)| \leq 1$ and $|v_h(\alpha) - v_h(\beta)| \leq 1$ for all $\alpha, \beta \in \{0, 1, 2, 3\}$.

Accordingly, $\langle K_{1,k}^{(1)} \blacktriangle K_{1,k}^{(2)} \rangle$ is a perfect mean cordial graph.

□

Example 3. Illustration , the perfect mean cordial graph $\langle K_{1,5}^{(1)} \blacktriangle K_{1,5}^{(2)} \rangle$ is display in the figure 3.2

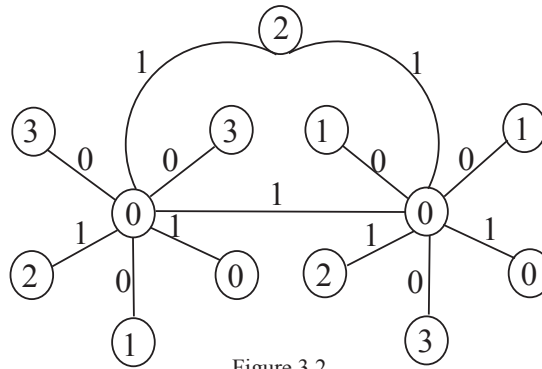


Figure 3.2

Theorem 3.3. Graph $\langle S(K_{1,k}^{(1)}) : S(K_{1,k}^{(2)}) \rangle$ is perfect mean cordial graph.

Proof. Let $w_1^{(1)}, w_2^{(1)}, w_3^{(1)}, \dots, w_k^{(1)}$ be support vertices of $S(K_{1,k}^{(1)})$, $z_1^{(1)}, z_2^{(1)}, z_3^{(1)}, \dots, z_k^{(1)}$ be pendant vertices of $S(K_{1,k}^{(1)})$ and $w_1^{(2)}, w_2^{(2)}, w_3^{(2)}, \dots, w_k^{(2)}$ be support vertices of $S(K_{1,k}^{(2)})$, $z_1^{(2)}, z_2^{(2)}, z_3^{(2)}, \dots, z_k^{(2)}$ be pendant vertices of $S(K_{1,k}^{(2)})$. Let a_1 and a_2 be the apex vertices of $S(K_{1,k}^{(1)})$ and $S(K_{1,k}^{(2)})$ respectively.

Let $G = \langle S(K_{1,k}^{(1)}) : S(K_{1,k}^{(2)}) \rangle$. We construct vertex labeling $h : V(G) \rightarrow \{0, 1, 2, 3\}$ as proceeds below.

$$h(w_j^{(1)}) = \begin{cases} 3 & j \equiv 0 \pmod{4} \\ 1 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 0 & j \equiv 2 \pmod{4} \\ 2 & j \equiv 3 \pmod{4} \end{cases}$$

$$h(z_j^{(1)}) = \begin{cases} 3 & j \equiv 0 \pmod{4} \\ 0 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 2 & j \equiv 2 \pmod{4} \\ 1 & j \equiv 3 \pmod{4} \end{cases}$$

$$h(w_j^{(2)}) = \begin{cases} 1 & j \equiv 0 \pmod{4} \\ 0 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 3 & j \equiv 2 \pmod{4} \\ 2 & j \equiv 3 \pmod{4} \end{cases}$$

$$h(z_j^{(2)}) = \begin{cases} 2 & j \equiv 0 \pmod{4} \\ 3 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 1 & j \equiv 2 \pmod{4} \\ 0 & j \equiv 3 \pmod{4} \end{cases}$$

$h(a_1) = 3 ; h(a_2) = 2 ; h(s) = 1$. The induced edge labeling is,

$$h^*(wz) = \begin{cases} 1 & \text{if } 2|(h(w) + h(z)) \text{ for all } wz \in E(G) \\ 0 & \text{otherwise} \end{cases}$$

Examination of vertex and edge demands are illustrated below

	Vertex values	Edge values
$k \equiv 0 \pmod{4}$	$v_h(0) = k, v_h(1) = v_h(2) =$ $v_h(3) = k + 1$	$e_h(0) = e_h(1) = 2k + 1$
$k \equiv 1 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = k + 1,$ $v_h(2) = k$	$e_h(0) = e_h(1) = 2k + 1$
$k \equiv 2 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = k +$ $1, v_h(2) = k$	$e_h(0) = e_h(1) = 2k + 1$
$k \equiv 3 \pmod{4}$	$v_h(0) = v_h(1) = v_h(2) = k +$ $1, v_h(3) = k$	$e_h(0) = e_h(1) = 2k + 1$

Consequently, the graph G fulfills the demands $|e_h(0) - e_h(1)| \leq 1$ and $|v_h(\alpha) - v_h(\beta)| \leq 1$ for all $\alpha, \beta \in N$.

Accordingly, $\langle S(K_{1,k}^{(1)}) : S(K_{1,k}^{(2)}) \rangle$ is perfect mean cordial graph. $\square$

Example 4. Illustration of the perfect mean cordial graph $\langle S(K_{1,5}^{(1)}) : S(K_{1,5}^{(2)}) \rangle$ is display in the figure 3.3

Theorem 3.4. A graph $\langle S(K_{1,k}^{(1)}) \blacktriangle S(K_{1,k}^{(2)}) \rangle$ is perfect mean cordial graph.

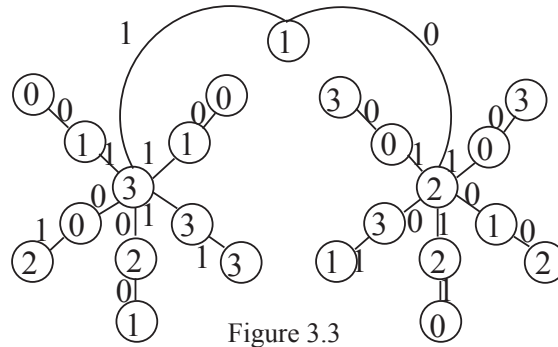


Figure 3.3

Proof. Let $w_1^{(1)}, w_2^{(1)}, w_3^{(1)}, \dots, w_k^{(1)}$ be support vertices of $S(K_{1,k}^{(1)})$, $z_1^{(1)}, z_2^{(1)}, z_3^{(1)}, \dots, z_k^{(1)}$ be pendant vertices of $S(K_{1,k}^{(1)})$ and $w_1^{(2)}, w_2^{(2)}, w_3^{(2)}, \dots, w_k^{(2)}$ be support vertices of $S(K_{1,k}^{(2)})$, $z_1^{(2)}, z_2^{(2)}, z_3^{(2)}, \dots, z_k^{(2)}$ be pendant vertices of $S(K_{1,k}^{(2)})$.

Let a_1 and a_2 be the apex vertices of $S(K_{1,k}^{(1)})$ and $S(K_{1,k}^{(2)})$ respectively and they are adjacent to a new vertex s . Let $G = \langle S(K_{1,k}^{(1)}) \blacktriangle S(K_{1,k}^{(2)}) \rangle$. We construct vertex labeling $h : V(G) \rightarrow \{0, 1, 2, 3\}$ as proceeds below.

$$h(w_j^{(1)}) = \begin{cases} 3 & j \equiv 0 \pmod{4} \\ 1 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 0 & j \equiv 2 \pmod{4} \\ 2 & j \equiv 3 \pmod{4} \end{cases}$$

$$h(z_j^{(1)}) = \begin{cases} 3 & j \equiv 0 \pmod{4} \\ 0 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 2 & j \equiv 2 \pmod{4} \\ 1 & j \equiv 3 \pmod{4} \end{cases}$$

$$h(w_j^{(2)}) = \begin{cases} 1 & j \equiv 0 \pmod{4} \\ 0 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 3 & j \equiv 2 \pmod{4} \\ 2 & j \equiv 3 \pmod{4} \end{cases}$$

$$h(z_j^{(2)}) = \begin{cases} 2 & j \equiv 0 \pmod{4} \\ 3 & j \equiv 1 \pmod{4}, 1 \leq i \leq k \\ 1 & j \equiv 2 \pmod{4} \\ 0 & j \equiv 3 \pmod{4} \end{cases}$$

$f(a_1) = 3; f(a_2) = 2; f(s) = 1$. The induced edge labeling is,

$$h^*(wz) = \begin{cases} 1 & \text{if } 2|(h(w) + h(z)) \text{ for all } wz \in E(G) \\ 0 & \text{otherwise} \end{cases}.$$

Examination of vertex and edge demands are illustrated below.

	vertex values	edge values
$k \equiv 0 \pmod{4}$	$v_h(0) = k, v_h(1) = v_h(2) = v_h(3) = k + 1$	$e_h(0) = 2k + 2, e_h(1) = 2k + 1$
$k \equiv 1 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = k + 1, v_h(2) = k$	$e_h(0) = 2k + 2, e_h(1) = 2k + 1$
$k \equiv 2 \pmod{4}$	$v_h(0) = v_h(1) = v_h(3) = k + 1, v_h(2) = k$	$e_h(0) = 2k + 2, e_h(1) = 2k + 1$
$k \equiv 3 \pmod{4}$	$v_h(0) = k, v_h(1) = v_h(2) = v_h(3) = k + 1$	$e_h(0) = 2k + 2, e_h(1) = 2k + 1$

Consequently, the graph G fulfills the demands $|e_h(0) - e_h(1)| \leq 1$ and $|v_h(\alpha) - v_h(\beta)| \leq 1$ for all $\alpha, \beta \in N$.

Accordingly, $\langle S(K_{1,k}^{(1)}) \blacktriangle S(K_{1,k}^{(2)}) \rangle$ is perfect mean cordial graph. $\square$

Example 5. Illustration of the perfect mean cordial graph $\langle S(K_{1,5}^{(1)}) \blacktriangle S(K_{1,5}^{(2)}) \rangle$ is display in the figure 3.4

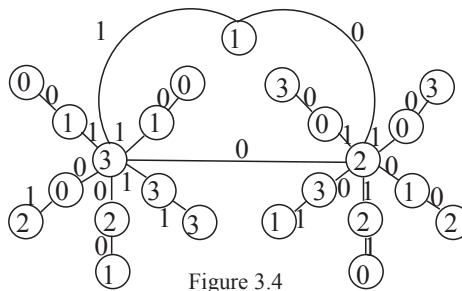


Figure 3.4

REFERENCES

- [1] A. ANNIE LYDIA, M. K. ANGEL JEBITHA: *Star Related Perfect Mean Cordial Graphs*, Recent Trends in Mathematics, February 2019.
- [2] F.HARARY: *Graph Theory*, Addison - Wesley, Reading Mass, 197).

- [3] G. V. GHODASARA, S. G. SONCHHATRA: *Cordial Labeling of Fan Related Graphs*, International Journal of Scientific and Engineering Research, **5** (2013), 470–476.
- [4] S. K. VAIDYA, C. M. BARASARA: *Product Cordial Graphs in the Context of Some Graph Operations*, International Journal of Mathematics and Scientific Computing, **1**(2) (2011), 1–6.
- [5] S. K. VAIDYA, N. A. DANI, K. K. KANNANI, P. L. VIHOL: *Cordial and 3 - Equitable Labeling for Some Shell Related Graphs*, Journal of Scientific Research, **I**(3) (2009), 438–449.

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