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DATA COMPRESSION OF ECG SIGNALS USING CATMULL-ROM SPLINE CURVES

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ABSTRACT. Data compression is a process of removing unnecessary complexity or redundancy. Electro cardio graphic signals may be recorded on a long timescale and the produced ECG recordings amount to huge data size that quickly fills up available storage space and hence data compression is an essential operation.

In this paper we attempt to approximate ECG signals by using Catmull-Rom Spline curves and the residual errors are quantized and encoded with Huffman coding. We developed a program in MATLAB and evaluate the performance with real data set. The result shows that this compression algorithm performs well for the ECG data.

1. INTRODUCTION

Electro Cardiogram (ECG) signals usually sampled at $200-500$ samples with 8-12 bits resolution. Electro Cardiogram data compression helps to reduce storage requirements to develop a more efficient tele cardiology system for cardiac analysis and diagnosis. Splines are special functions consist of piece wise polynomial functions on sub intervals joined together with certain continuity conditions. Splines offer quite flexible and elegant solutions for interpolation or approximation of the irregularly distributed data.

Considering long monitoring periods, compression is required to handle such vast amount of data. It can increase the capacity of database where hundreds of thousands of ECG signals are stored for subsequent monitoring and evaluation.

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Also ECG compression includes other applications such as telephone or mobile radio to ECG center for further processing.

Definition 1.1. *A spline function of degree k having knots $x_0, x_1, \ldots x_n$ is a function such that*

1) *On each interval$[x_{i-1}, x_i]$, S is a polynomial of degree $\leq k$,*
2) *S has a continuous $(k-1)^{th}$ derivative on $[x_0, x_n]$.*

Definition 1.2. *Catmull-Rom spline [2] is a family of the C^1 continuous cubic interpolating splines. Tangent at each point $p_i$is calculated using the previous and the next point on the curve, that is $p_i{}' = \tau(p_{i+1} - p_{i-1})$.*

Definition 1.3. *Quantization [4] is the process of mapping input values from a large set (continuous set) to output values in a (countable) smaller set often with a finite number of elements. Rounding and truncation are typical example of quantization.*

Definition 1.4. *Huffman coding [1] is a famous algorithm which is used for the lossless compression of data. The idea behind Huffman coding is to find a way to compress the storage of data using variable length codes.*

Definition 1.5. *For $A \in K^{m\times n}$, a pseudo inverse of A is defined as a matrix $A^+ \in K^{n\times m}$ satisfying all of the following four criteria*

(1) $AA^+A = A$,
(2) $A^+AA^+ = A^+$,
(3) $(AA^+)^T = AA^+$,
(4) $(A^+A)^T = A^+A$.

Remark 1.1. *A^+exists for any matrix A, when A has full rank:*

When A has linearly independent column $A^+ = (A^TA)^{-1}A^T$ and in this case $A^TA = I$;

When A has linearly independent rows, $A^+ = A^T(A^TA)^{-1}$and in this case $AA^T = I$.

Definition 1.6. *Electro Cardiogram (ECG) [6] is one of the simplest and oldest cardiac investigations available, yet it can provide a wealth of useful information and remains an essential part of the assessment of cardiac patients. It is a recording-a graph of voltage verses time-of the electrical activity of the heart using electrodes placed on the skin. There are 3 main components to an ECG:*

(1) *The P wave, which represents the depolarization of the atria.*
(2) *The QRS complex which represents the depolarization of the ventricles.*
(3) *T wave, which represent the repolarization of the ventricles.*

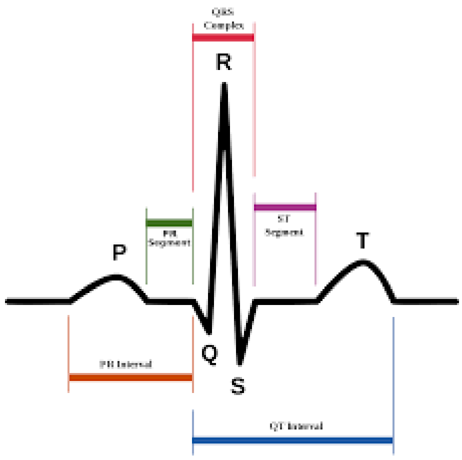

FIGURE 1. ECG

2. RESEARCH METHOD

In [2] Danyang Li etal introduced a compression algorithm for the compression of sensor array data of a hydrologic monitoring system using Catmull Rom Spline. Here we use the Catmull Rom spline to approximate the Ecg signals.

2.1. **Spline based Compression.** Let $p_1, p_2, \ldots p_M$ represent the control points. Each control point p_i represents a two dimensional vector (p_i^x, p_i^y) where $1 \leq i \leq M$ Tangent at each point p_i is calculated using the previous and the next point on the curve that is p_i '=$\tau(p_{i+1} - p_{i-1})$ Parameter τ affects the bend of the curve at control points and set $\tau = 2$.

The i^{th} segment of the curve between p_i and p_{i+1} is determined by four control points $p_{i-1}, p_i, p_{i+1}, p_{i+2}$ and the i^{th} segment can be expressed by a polynomial form :

$$p(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 \tag{2.1}$$

equation (2.1) is restricted by

$$\begin{aligned} p(0) = p_i \quad p(1) = p_{i+1}\,, \\ p\,'(0) = \frac{1}{2}(p_{i+1} - p_{i-1}) \quad p\,'(1) = \frac{1}{2}(p_{i+2} - p_i)\,. \end{aligned} \tag{2.2}$$

By combining equations (2.1) and (2.2)

$$p(t) = 1/2 \begin{pmatrix} -t+2t^2-t^3 \\ 2-5t^2+3t^3 \\ t+4t^2-3t^3 \\ -t^2+t^3 \end{pmatrix}^T \begin{pmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{pmatrix}. \tag{2.3}$$

2.2. **Representing ECG Data using Catmull-Rom Spline.** Consider that there are N signals $S_1, S_2, \ldots S_N$ in the storage space of ECG. Assume that signals are at a distance of $\dfrac{x_N - x_1}{N-1}$ and consider $x_i = i$ for $1 \le i \le N$. Let the value of signalS_i at a moment be denoted as y_i.

To represent the data of signal, we define a $N \times 1$ matrix Y as follows:

$$Y = \begin{pmatrix} y_1 & y_2 & y_3 & \cdots & y_n \end{pmatrix}^T.$$

Let us represent data Y approximately with a small number of spline parameters based on the Catmull-Rom curve. Let M denote the number of control points of the spline and p_i denote the i^{th} control point. Let the control points be equally spaced along the x-axis and

$$p_i^x = 1 + \frac{(i-1)(N-1)}{M-1}.$$

Clearly there are $M-1$ segments in the Catmull-Rom spline curve. The x coordinate of the point in the i^{th} segment satisfies $p_i^x \le x \le p_{i+1}^x i^{\text{th}}$. Let B denote a $M \times 1$ matrix o $\{p_i^y\}$. Then

$$\begin{aligned} B &= \begin{pmatrix} b_1 & b_2 & b_3 & \cdots & b_m \end{pmatrix}^T \\ &= \begin{pmatrix} p_1^y & p_2^y & p_3^y & \cdots & p_M^y \end{pmatrix}^T \end{aligned}$$

Also we place two additional virtual control points for the first and last segments of the spline and they satisfy

$$\begin{aligned} p_0 &= (1 - \frac{N-1}{M-1}, b_1) \\ p_M &= (N + \frac{N-1}{M-1}, b_M). \end{aligned}$$

If the j^{th} signal belongs to the k_j^{th} segment then we have

$$k_j = \lceil \frac{(j-1)(M-1)}{N-1} \rceil$$

where $\lceil x \rceil = \begin{cases} x & x \in Z \\ \text{smallest integer that is greater than } x & \text{otherwise} \end{cases}$

Since the signal S_j belongs to the k_j^{th} segment of the spline , the interpolation parameter t_j satisfies the equation

$$t_j = \frac{x_j - p_(k_j)^x}{p_(k_(j+1))^x - p_(k_j)^x} = \frac{j - p_(k_j)^x}{(N-1)/(M-1)}\,.$$

From equation (2.3) $\widehat{y_j}$, the interpolated value of x_j can be written as

$$\widehat{y_j} = \frac{1}{2}\begin{pmatrix} -t_j + 2t_j^2 - t_j^3 \\ 2 - 5t_j^2 + 3t_j^3 \\ t_j + 4t_j^2 - 3t_j^3 \\ -t_j^2 + t_j^3 \end{pmatrix}^T \begin{pmatrix} b_{kj-1} \\ b_{kj} \\ b_{k_{j+1}} \\ b_{k_{j+2}} \end{pmatrix} \tag{2.4}$$

Thus equation (2.4) can be written as

$$\widehat{y_j} = \begin{pmatrix} a_{j,1} \\ a_{j,2} \\ \vdots \\ a_{j,i-1} \\ a_{j,i} \\ a_{j,i+1} \\ a_{j,i+2} \\ \vdots \\ a_{j,M} \end{pmatrix}^T \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{i-1} \\ b_i \\ b_{i+1} \\ b_{i+2} \\ \vdots \\ b_M \end{pmatrix}, \tag{2.5}$$

where

$$a_{j,i} = \begin{cases} \dfrac{-t_j + 2t_j^2 - t_j^3}{2} & i = k_{j-1} \\ \dfrac{2 - 5t_j^2 + 3t_j^3}{2} & i = k_j \\ \dfrac{t_j + 4t_j^2 - 3t_j^3}{2} & i = k_{j+1} \\ \dfrac{-t_j^2 + t_j^3}{2} & i = k_{j+2} \\ 0 & \text{o}therwise\,. \end{cases}$$

Now equation (2.5) will be of the form

$$\widehat{Y} = AB\,,$$

where $\widehat{Y} = \begin{pmatrix} \widehat{y_1} & \widehat{y_2} & \cdots & \widehat{y_n} \end{pmatrix}^T$ and $A = [a_{ij}]_{N\times M}$. The optimal parameter $\tilde{B}$ is given by

$$\tilde{B} \text{ arg } \min_B \|Y - \widehat{Y}\| = \text{ arg } \min_B \|Y - AB\|\,, \tag{2.6}$$

where

$$\|X_{n\times m}\| = \sum_{i=1}^{n} \left(\sum_{j=1}^{m} |\mathbf{X_{ij}}|^2 \right)^{1/2} .$$

Since the psuedo inverse gives the minimum norm we can write

$$\tilde{B} = A^{+} Y\,. \tag{2.7}$$

The signals generate data Y in each sample period. The optimal spline parameter $\tilde{B}$can be calculated using equation (2.6). The approximate value of the signal data by the spline parameter $\tilde{B}$ is given by [2].

$$\widehat{Y} = A\tilde{B}\,. \tag{2.8}$$

The approximate value captures the major part of the real value. The error between real value and the approximate value is given by [2].

$$\triangle = Y - \widehat{Y}\,.$$

Now quantization can be applied to $\triangle$ and the quantized result can be encoded to E with the Huffman coding. The receiver reconstructs the approximate value $\widehat{Y}$, using the equation (2.8) then decodes E to quantized error $\triangle\,'$.

The final result can be written as

$$Y\,' = \widehat{Y} + \triangle\,' \tag{2.9}$$

2.3. **Steps for Compression Algorithm.**

(1) Evaluate the optimal spline parameter$\tilde{B}$ using equation (2.7).
(2) Find the approximate value $\widehat{Y}$ using equation (2.8).
(3) Quantize $\triangle$ and then encode to E with Huffman coding.
(4) Return $\tilde{B}$ and E .

2.4. **Steps for Decompression Algorithm.**

1 Find $\triangle$ from E with Huffman coding.
2 Generate the approximate value $\widehat{Y}$ using equation (2.8) .
3 Evaluate the reconstructed data $Y\,'$ from equation (2.9) .
4 Return Y

3. RESULTS AND ANALYSIS

The algorithm for spline approximation first selects the the minimum number of control points to construct the spline [5]. To measure the data reduction performance of the compressed algorithm, here we use the formula for compression ratio as

$$r_c = (1 - \frac{s_c}{s_o}) \times 100\,\% \tag{3.1}$$

where s_c is the compressed size that is the data size after running compression algorithm and s_o is the is the original data size. Both are measured with number of bits. Here the original data generated by the ECG signal for the first data set 21374 bytes. After running the program the compressed data signal reduced the size by 6596 bytes with compression ratio 69%.

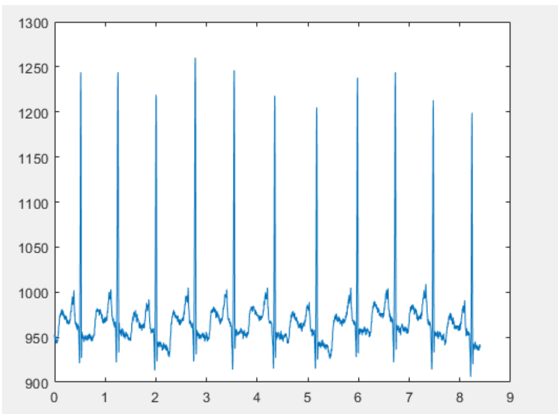

FIGURE 2. ECG signals in data base I before data compression

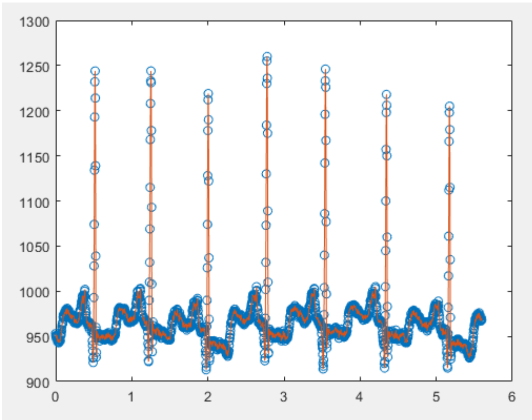

FIGURE 3. ECG signals in data base I after approximated with Catmull Rom Spline

We consider 10 data sets each with 3600 signals and compile the program in matlab with these data sets. The compression ratio r_c using the formula (3.1) is as shown below.

TABLE 1. compression ratio of ECG signals in different data bases

Sr.No.	Data	Original size in bytes	Compressed size in bytes	compression ratio (r_c)
1	Data Set1	21374	6596	69.1401
2	Data set2	21456	6596	69.2580
3	Data set3	21491	6596	69.3081
4	Data set4	21352	6596	69.1083
5	Data set5	20032	6596	67.0727
6	Data set6	21453	6596	69.2537
7	Data set7	21375	6596	69.1415
8	Data set8	21561	6596	69.4077
9	Data set9	21572	6596	69.4233
10	Data set10	21522	6596	69.3523

As per [3]

TABLE 2. comparison of CR with different compression techniques

Sr. No.	Different Compression Techniques	CR
1	Run length coding compression (RLE)	1.4725
2	Amplitude Zone Time Epoch Coding(AZTEC)	1.0314
3	Quadratic Spline Wavelet Transform(Spline)	0.9996
4	Discrete Cosine Transform(DCT)	1.0524
5	Discrete Sine Transform(DST)	1.0685
6	Fast Fourier Transform(FFT)	1.0129
7	Discrete Cosine Transform-II(DCT2)	0.9714
8	Scan-Along Polygonal Approximation Technique(SAPAFAN)	1.6969

Here CR is calculated by the formula

$$CR = \frac{B_o}{B_c},$$

where B_o and $B_c$are the number of bytes of the original and compressed data respectively. In our data base I, $CR = \frac{21374}{6596} = 3.24$ which is higher than the CR values in table 2. Hence our compression method is better.

4. CONCLUSION

We developed a program in matlab for an error bounded data compression algorithm, in which the major part of ECG signal data are represented by Catmull Rom spline and the residual errors are quantized and encoded with Huffman coding. The algorithm is of low complexity and can be implemented easily. Also it is not time consuming. We have evaluated the performance of our program in matlab. The compression ratio shows that this algorithm performs well for the given data. It can increase the capacity of database and hence hundreds of thousands of ECG signals can be stored for subsequent monitoring and evaluation.

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