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SOME STRONG FORMS OF CONTINUOUS FUNCTIONS IN NANO TOPOLOGY

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ABSTRACT. The objective of this paper is to introduce new stronger forms of continuous functions called strongly nano $\alpha\hat{g}$-continuous function and perfectly nano $\alpha\hat{g}$- continuous function. Further, some properties and characterizations of these functions are obtained. Also their relationships with existing continuous functions are investigated.

1. INTRODUCTION

Njastad [8] and Levine [7] have introduced α-open sets and generalized closed sets respectively. Lellis Thivagar [4] introduced Nano Topological space with respect to a subset of X of an universe which is defined interms of lower and upper approximations of X. Davamani Christober and Vinith Mala [2] have defined $\alpha\hat{g}$-closed sets and $\alpha\hat{g}$-continuous function in Nano Topological space.

In this paper, we investigate some stronger forms of continuous functions namely strongly nano $\alpha\hat{g}$-continuous function and perfectly nano $\alpha\hat{g}$- continuous function. In addition, some properties and characterizations of these functions are examined as well as their relationships with existing continuous functions are explored.

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2. Preliminaries

Definition 2.1. *[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.*

(i) *The lower approximation of X with respect to R is denoted by $L_R(X)$ and is defined as $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, R(x) denotes the equivalence class determined by x.*
(ii) *The upper approximation of X with respect to R is denoted by $U_R(X)$ and is defined as $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \bigcap X \neq \emptyset\}$.*
(iii) *The boundary region of X with respect to R is denoted by $B_R(X)$ and is defined as $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. *[4] Let U be an universe. R be an equivalence relation on U and $\tau_R(X) = \{U,\ \emptyset,\ L_R(X),\ U_R(X),\ B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:*

(i) *U and $\emptyset\ \in \tau_R(X)$*
(ii) *The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$*
(iii) *The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$*

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Definition 2.3. *[4] Let $(U, \tau_R(X))$ be a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and is denoted by $Nint(A)$. The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$.*

Definition 2.4. *[2] A subset A of a space (U,$\tau_R(X)$) is called nano $\alpha\hat{g}$-closed if $N_\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano $\hat{g}$-open in $(U, \tau_R(X))$. The complement of nano $\alpha\hat{g}$-closed set is nano $\alpha\hat{g}$-open set.*

Definition 2.5. *A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be*

(i) *Nano continuous [5] on U if the inverse image of every nano open set in V is nano open in U.*

(ii) *Nano α-continuous [5] on U if the inverse image of every nano open set in V is nano α-open in U.*

(iii) *Nano g-continuous[1] on U if the inverse image of every nano open set in V is nano g-open in U.*

(iv) *Nano α-generalized continuous[9] on U if the inverse image of every nano open set in V is nano α- generalized open in U.*

(v) *Nano $\hat{g}$-continuous [3] on U if the inverse image of every nano open set in V is nano $\hat{g}$-open in U.*

(vi) *Nano $\alpha\hat{g}$-continuous [2] if the inverse image of every nano closed set in V is nano $\alpha\hat{g}$-closed in U.*

(vii) *Nano $\alpha\hat{g}$-irresolute[2] if the inverse image of every nano $\alpha\hat{g}$-closed set in V is nano $\alpha\hat{g}$-closed in U.*

(viii) *Nano strongly continuous [6] if $f^{-1}(A)$ is nano clopen in U for every subset A in V.*

(ix) *Nano Perfectly continuous [6] if $f^{-1}(A)$ is nano clopen in U for every nano open set A in V.*

3. Strongly Nano $\alpha\hat{g}$-continuous functions

Definition 3.1. *A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be strongly nano $\alpha\hat{g}$-continuous if $f^{-1}(B)$ is nano closed in U for every nano $\alpha\hat{g}$-closed set B in V.*

Example 1. *Let $U = \{p, q, r, s\}$, $U/R = \{\{p\}, \{q, r\}, \{s\}\}$ and $X = \{q, s\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{s\}, \{q, r\}, \{q, r, s\}\}$. Also let $V = \{a, b, c, d\}, V/R' = \{\{a, b\}, \{c\}, \{d\}\}$, $Y = \{a, c\} \subseteq V$. Then nano $\alpha\hat{g}$-closed sets of V are $\{V, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Define a function $f : U \rightarrow V$ by $f(p) = d$, $f(q) = a$, $f(r) = a$ and $f(s) = a$. Then inverse image of every nano $\alpha\hat{g}$-closed sets in V is nano closed in U. Hence f is strongly nano $\alpha\hat{g}$-continuous.*

Theorem 3.1. *A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is strongly nano $\alpha\hat{g}$-continuous if and only if $f^{-1}(B)$ is nano open in $(U, \tau_R(X))$ for every nano $\alpha\hat{g}$-open set B in $(V, \tau_{R'}(Y))$.*

Proof. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a strongly nano $\alpha\hat{g}$- continuous function and B be any nano $\alpha\hat{g}$-open set in V. Then B^c is nano $\alpha\hat{g}$-closed in V.

Therefore $f^{-1}(B^c)$ is nano closed in U. But $f^{-1}(B^c) = [f^{-1}(B)]^c$ and hence $f^{-1}(B)$ is nano open in U. Converse is obvious. $\square$

Theorem 3.2. *Every strongly nano $\alpha\hat{g}$-continuous function is*

(i) *Nano continuous and hence nano generalized continuous and nano α-continuous.*
(ii) *Nano $\alpha\hat{g}$-continuous and hence nano αg- continuous and nano gs-continuous*
(iii) *Nano $\hat{g}$-continuous.*

Proof. Follows from the definitions. $\square$

Remark 3.1. *The following example shows that the implications in theorem 3.2 are not reversible in general.*

Example 2. *Let $U = \{a, b, c, d\}$, $X = \{a, c\}$ and $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$. Also $V = \{p, q, r, s\}$, $Y = \{q, s\}$ and $\tau_{R'}(Y) = \{V, \emptyset, \{s\}, \{q, r\}, \{q, r, s\}\}$. Define $f : U \to V$ by $f(a) = q$, $f(b) = r$, $f(c) = s$ and $f(d) = p$. Here f is nano continuous, nano $\alpha\hat{g}$-continuous, nano α-continuous, nano g-continuous, nano $\hat{g}$-continuous, nano gs-continuous and nano αg-continuous but not strongly nano $\alpha\hat{g}$-continuous.*

Theorem 3.3. *Every strongly nano continuous function is strongly nano $\alpha\hat{g}$ - continuous function.*

Proof. Follows from the definitions. $\square$

Remark 3.2. *The following example shows that the converse of theorem 3.3 need not be true.*

Example 3. *Let $U = \{p, q, r, s\}$, $U/R = \{\{p\}, \{q, r\}, \{s\}\}$ and $X = \{q, s\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{s\}, \{q, r\}, \{q, r, s\}\}$. Also let $V = \{a, b, c, d\}, V/R' = \{\{a, b\}, \{c\}, \{d\}\}$, $Y = \{a, c\} \subseteq V$. Then nano $\alpha\hat{g}$-closed sets of V are $\{V, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Define a function $f : U \to V$ by $f(p) = d$, $f(q) = a$, $f(r) = a$ and $f(s) = a$. Here f is strongly nano $\alpha\hat{g}$-continuous but not strongly nano continuous.*

Theorem 3.4. *Every strongly nano $\alpha\hat{g}$- continuous function is nano $\alpha\hat{g}$-irresolute.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a strongly nano $\alpha\hat{g}$-continuous function and B be a nano $\alpha\hat{g}$-closed set in V. Then $f^{-1}(B)$ is nano closed in U. That is $f^{-1}(B)$ is nano $\alpha\hat{g}$- closed in U. Hence f is nano $\alpha\hat{g}$-irresolute. $\square$

Corollary 3.1. *Every strongly nano continuous function is nano $\alpha\hat{g}$-irresolute.*

Proof. Follows from theorem 3.3 and theorem 3.4. □

Remark 3.3. *The converse of the theorem 3.4 need not be true as the following example shows.*

Example 4. $U = \{a, b, c, d\}$ *with* $X = \{a, c\} \subseteq U$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$. *Then* $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$. *Also let* $V = \{p, q, r, s\}, V/R' = \{\{q, r\}, \{p\}, \{s\}\}$, $Y = \{q, s\} \subseteq V$. *Then nano $\alpha\hat{g}$-closed sets of V are* $\{V, \emptyset, \{p\}, \{p, q\}, \{p, r\}, \{p, s\}, \{p, q, r\}, [p, q, s\}, \{p, r, s\}$. *Define* $f : U \to V$ *by* $f(a) = s,\ f(b) = r,\ f(c) = q,\ f(d) = p$. *Here f is nano $\alpha\hat{g}$-irresolute but not strongly nano $\alpha\hat{g}$-continuous.*

Theorem 3.5. *The composition of two strongly nano $\alpha\hat{g}$-continuous functions is strongly nano $\alpha\hat{g}$-continuous.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ be two strongly nano $\alpha\hat{g}$-continuous functions. Let B be any nano $\alpha\hat{g}$-closed set in W. since g is strongly nano $\alpha\hat{g}$-continuous, $g^{-1}(B)$ is nano closed in V. Then $g^{-1}(B)$ is nano $\alpha\hat{g}$-closed in V. Since f is strongly nano $\alpha\hat{g}$–continuous, $f^{-1}(g^{-1}(B))$ is nano closed in U. That is $(g \circ f)^{-1}(B)$ is nano closed in U. Hence $g \circ f$ is strongly nano $\alpha\hat{g}$-continuous. □

Theorem 3.6. *Let* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y)$ *and* $g : (V, \tau_{R'}(Y) \to (W, \tau_{R''}(Z))$ *be any two functions such that* $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ *then*

(i) $g \circ f$ *is strongly nano $\alpha\hat{g}$-continuous if g is strongly nano $\alpha\hat{g}$-continuous and f is nano continuous.*
(ii) $g \circ f$ *is nano $\alpha\hat{g}$-irresolute if g is strongly nano $\alpha\hat{g}$-continuous and f is nano $\alpha\hat{g}$-continuous(or f is nano $\alpha\hat{g}$-irresolute)*
(iii) $g \circ f$ *is nano continuous if g is nano continuous and f is strongly nano $\alpha\hat{g}$-continuous.*

Proof. (i) Let B be a nano $\alpha\hat{g}$-closed set in W. Since g is strongly nano $\alpha\hat{g}$-continuous, $g^{-1}(B)$ is nano closed in V. Since f is nano continuous, $f^{-1}(g^{-1}(B))$ is nano closed in U. That is $(g \circ f)^{-1}(B)$ is nano closed in U. Hence $g \circ f$ is strongly nano $\alpha\hat{g}$-continuous.
(ii) Let B be a nano $\alpha\hat{g}$-closed set in W. Since g is strongly nano $\alpha\hat{g}$-continuous, $g^{-1}(B)$ is nano closed in V. Since f is nano $\alpha\hat{g}$-continuous, $f^{-1}(g^{-1}(B))$ is nano

$\alpha\hat{g}$-closed in U. That is $(g \circ f)^{-1}(B)$ is nano $\alpha\hat{g}$-closed in U. Hence $g \circ f$ is nano $\alpha\hat{g}$-irresolute.

(iii) Let B be a nano closed set in W. Since g is nano continuous, $g^{-1}(B)$ is nano closed in V. Hence $g^{-1}(B)$ is nano $\alpha\hat{g}$-closed in V. Since f is strongly nano $\alpha\hat{g}$-continuous, $f^{-1}(g^{-1}(B))$ is nano closed in U. That is $(g \circ f)^{-1}(B)$ is nano closed in U. Hence $g \circ f$ is nano continuous. □

4. Perfectly Nano $\alpha\hat{g}$-continuous functions

Definition 4.1. *A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be perfectly nano $\alpha\hat{g}$-continuous if $f^{-1}(B)$ is nano clopen in U for every nano $\alpha\hat{g}$-closed set B in V.*

Example 5. *Let $U = \{a, b, c, d\}$ with $X = \{a, b, c\} \subseteq U$, $U/R = \{\{a, c\}, \{b, d\}\}$. Then $\tau_R(X) = \{U, \emptyset, \{a, c\}, \{b, d\}\}$. Also let $V = \{a, b, c, d\}, V/R' = \{\{a, b\}, \{c, d\}\}$, $Y = \{a, b, c\} \subseteq V$. $\tau_{R'}(Y) = \{V, \emptyset, \{a, b\}, \{c, d\}\}$. Define $f : U \to V$ by $f(a) = c$, $f(b) = a$, $f(c) = d, f(d) = b$. Here inverse image of every nano $\alpha\hat{g}$-closed sets in V is nano clopen in U. Hence f is perfectly nano $\alpha\hat{g}$-continuous.*

Theorem 4.1. *A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is perfectly nano $\alpha\hat{g}$-continuous if and only if $f^{-1}(B)$ is nano clopen in $(U, \tau_R(X))$ for every nano $\alpha\hat{g}$-open set B in $(V, \tau_{R'}(Y))$.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a perfectly nano $\alpha\hat{g}$- continuous function and B b a nano $\alpha\hat{g}$-open set in V. Then B^c is nano $\alpha\hat{g}$ -closed in V. Therefore $f^{-1}(B^c)$ is nano clopen in U. But $f^{-1}(B^c) = [f^{-1}(B)]^c$ and hence $f^{-1}(B)$ is nano clopen in U. Converse is obvious. □

Theorem 4.2. *Every perfectly nano $\alpha\hat{g}$-continuous function is strongly nano $\alpha\hat{g}$-continuous.*

Proof. Follows from the definitions. □

Remark 4.1. *The converse of the theorem 4.2 need not be true.*

Example 6. *In example 1, f is strongly nano $\alpha\hat{g}$-continuous but not perfectly nano $\alpha\hat{g}$-continuous.*

Theorem 4.3. *If $(U, \tau_R(X))$ is extremely disconnected then every strongly nano $\alpha\hat{g}$-continuous is perfectly nano $\alpha\hat{g}$-continuous.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a strongly nano $\alpha\hat{g}$-continuous function. Then for every nano $\alpha\hat{g}$-closed set in V, $f^{-1}(B)$ is nano closed in U. Since U is extremely disconnected, $f^{-1}(B)$ is nano clopen in U. Hence f is perfectly nano $\alpha\hat{g}$-continuous. □

Theorem 4.4. *Let $(U, \tau_R(X))$ be a indiscrete nano topological space, $(V, \tau_{R'}(Y))$ be a nano topological space and $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be any function, then the following statements are equivalent.*

(i) *f is perfectly nano $\alpha\hat{g}$-continuous.*
(ii) *f is strongly nano $\alpha\hat{g}$–continuous.*

Proof. (i)⇒(ii) Follows from theorem 4.2.
(ii) ⇒(i) Let B be a nano $\alpha\hat{g}$-closed set in V. By hypothesis, $f^{-1}(B)$ is nano closed in U. Since $(U, \tau_R(X))$ is a indiscrete nano topological space, $f^{-1}(B)$ is nano clopen in U. Hence f is perfectly nano $\alpha\hat{g}$-continuous. □

Theorem 4.5. *Every strongly nano continuous function is perfectly nano $\alpha\hat{g}$-continuous.*

Proof. Let f be a strongly nano continuous function. Let B be a nano $\alpha\hat{g}$-closed set in $(V, \tau_{R'}(Y))$, then $f^{-1}(B)$ is nano clopen in $(U, \tau_R(X))$. Hence f is perfectly nano $\alpha\hat{g}$-continuous. □

Remark 4.2. *The following example shows that the converse of the above theorem need not be true.*

Example 7. *In example 5, f is perfectly nano $\alpha\hat{g}$-continuous, but not strongly nano continuous.*

Theorem 4.6. *The composition of two perfectly nano $\alpha\hat{g}$-continuous functions is perfectly nano $\alpha\hat{g}$-continuous.*

Proof. If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ be two perfectly nano $\alpha\hat{g}$-continuous functions. Let B be a nano $\alpha\hat{g}$-closed set in $(W, \tau_{R''}(Z))$, since g is perfectly nano $\alpha\hat{g}$-continuous, $g^{-1}(B)$ is nano clopen in $(V, \tau_{R'}(Y))$. Since every nano closed set is nano $\alpha\hat{g}$-closed, $g^{-1}(B)$ is nano

$\alpha\hat{g}$-closed in V. Since f is perfectly nano $\alpha\hat{g}$-continuous, $f^{-1}(g^{-1}(B))$ is nano clopen in $(U, \tau_R(X))$. That is $(g \circ f)^{-1}(B)$ is nano clopen in U. Hence $g \circ f$ is perfectly nano $\alpha\hat{g}$-continuous. $\square$

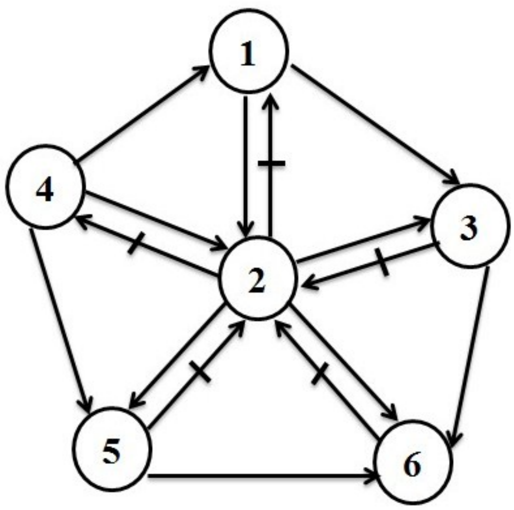
Theorem 4.7. *Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ be any two functions such that $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ then,*

(i) *$g \circ f$is perfectly nano $\alpha\hat{g}$-continuous if g is strongly nano $\alpha\hat{g}$-continuous and f is perfectly nano $\alpha\hat{g}$-continuous.*

(ii) *$g \circ f$ is strongly nano $\alpha\hat{g}$-continuous if g is perfectly nano $\alpha\hat{g}$-continuous and f is strongly nano $\alpha\hat{g}$-continuous(or f is nano continuous).*

(iii) *$g \circ f$ is nano $\alpha\hat{g}$- irresolute if g is perfectly nano $\alpha\hat{g}$-continuous and f is nano $\alpha\hat{g}$-continuous(or f is nano $\alpha\hat{g}$-irresolute).*

Remark 4.3. *From the above theorems and examples we have the following diagram.*



1. Perfectly nano $\alpha\hat{g}$-continuous
2. Strogly nano $\alpha\hat{g}$-continuous
3. Nano α-continuous
4. Strongly nano continuous
5. Nano $\alpha\hat{g}$-irresolute
6. Nano $\alpha\hat{g}$-continuous

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