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DETERMINATION OF BEST INITIAL BASIC FEASIBLE SOLUTION OF A TRANSPORTATION PROBLEM: A TOCM-ASM APPROACH

R. MURUGESAN[1] AND T. ESAKKIAMMAL

ABSTRACT. Reinfeld and Vogel (1958) developed a method known as Vogel's Approximation Method (VAM), which is the most efficient solution procedure for more than five decades, for obtaining an Initial Basic Feasible Solution (IBFS) for the transportation problems (TPs) as it provides a very good IBFS. Maharajan and Meenakshi (2004) extended the Total Opportunity Cost Matrix (TOCM) of Kirca and Satir (1990) by using VAM procedure on the TOCM, called TOCM-VAM method. It yields a very efficient initial solution for TPs. Abdul Quddooset al. developed a new method called ASM method (July 2012) and Revised Version of ASM method (June 2016) for obtaining the best IBFS for TPs with minimum effort of mathematical calculations. In this paper, we have applied the ASM procedure on the TOCM (called the TOCM-ASM approach) which yields best IBFS for TPs in the sense that which is either optimal or very close to the optimal solution. To verify the performance of the proposed approach, 50 classical benchmark instances (30 of balanced category and 20 of unbalanced category) from the literature have been tested. Simulation results validate that the proposed TOCM-ASM approach has produced optimal solution directly to 40 TPs. Further, the most attractive feature of this approach is that it requires only uncomplicated arithmetical and logical calculations and hence any one can easily understand and apply it far better than any other method. Also, this approach will be more cost-effective for those decision makers who are trading with logistics and supply chain problems.

[1]*corresponding author*

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1. Introduction

Transportation problems have been broadly studied in Operations Research and Computer Science. They play a vital role in logistics and supply-chain management for reducing the distribution cost and improving the service. In 1941 Hitchcock [5] developed the basic transportation problem along with the constructive method of solution and later in 1949 Koopmans [8] discussed the problem in detail. Again in 1951 Dantzig [4] formulated the transportation problem as linear programming problem and also provided the solution method. During 1960s, quite few methods such as North West Corner (NWC) Method, Least Cost Method (LCM) and Vogel's Approximation Method (VAM) [6, 10, 21, 22, 23] have been established for finding the IBFS of TPs.

In the recent years several methods have been projected by several researchers to find the optimal solution for TPs. directly. But no method is attaining optimal solution directly to all TPs. Among them, in July 2012, Abdul Quddoos et al. [2] proposed a new method, named ASM method, based on making allocations to zero entry cell of reduced cost matrix, for finding an optimal solution directly for a wide range of TPs. In October 2012, Mohammad Kamrul Hasan [11] proposed that direct methods (including ASM method) for finding optimal solution of a TP do not reflect optimal solution continuously. Murugesan [12] confessed and recognized the statement of Mohammad Kamrul Hasan by testing the ASM method for various benchmark problems. Meanwhile by doing further research, Abdul Quddoos et al.[1] encountered a few problems in which ASM method does not directly provide optimal solution to each and every problem, but provides a best IBFS, which is very close to optimal solution. One basic problem encountered was the unbalanced TP (UTP) in which an IBFS, not optimal but very close to optimal, was obtained. To overcome this problem, in July 2016, Abdul Quddoos et al. [1] presented a Revised Version of the ASM method, which provides optimal solution directly for most of the problems, and if not, it provides best IBFS. Murugesan and Esakkiammal [13 to 20] established Abdul Quddoos et al.'s claim by testing 30 benchmark instances of balanced category and 20 of unbalanced category. Again by our further research we have observed that Kirca and Satir (1990) [7] first introduced the concept of Total Opportunity Cost Matrix (TOCM) and applied the Least Cost Method with some tie-breaking policies on the TOCM to determine the feasible solution of the TP. Mathirajan and Meenakshi (2004) [9] extended TOCM of Kirca and Satir by using VAM procedure on the TOCM (called the VAM-TOC, also same as the TOCM-VAM).

According to the authors, this approach yielded the optimal solution and about 80% of the time it yielded a solution very close to the optimal (0.5% loss of optimality). Md. Amirul Islam et al. (2012) [3] extended TOCM of Kirca Satir by applying Extremum Difference Method (EDM) techniques on the TOCM (called the TOCM-EDM approach) to determine the IBFS of TP. Md. Amirul Islam et al. (2012) [3] again applied Highest Cost Difference Method (HCDM) on TOCM (called the TOCM-HCDM approach) to find the IBFS of TP. Aminur Rahman Khan et al. (2015) [3] calculates the pointer cost as the sum all entries in the respective row or column of the TOCM (called the TOCM-SUM approach) to find the IBFS of TP. Aminur Rahman Khan et al. (2015) [3] also applied Russel's Appoximation Method (RAM) on TOCM (called the TOCM-RAM approach) to compute the IBFS of TP. Murugesan and Esakkiammal (2020) [20] compared TOCM-VAM method with the ASM method by testing 50 benchmark TPs and established that the ASM method has produced optimal solution directly to 40 TPs, whereas TOCM-VAM method has produced optimal solution directly to only 27 TPs. Though the ASM method produces optimal solution to a good number of TPs, Murugesan and Esakkiammal (2020) [19] have identified some challenging TPs for which the ASM method produces only near optimal solution.

For getting IBFS to a given TP, various modern methods such as EDM, HCDM, SUM and RAM (2015) [3] have been applied on the TOCM of the given TP. But so far, the more efficient ASM method has not been applied on the TOCM of a TP. Hence in this paper, we have proposed the new approach called TOCM-ASM which applies the ASM method on the TOCM of the given TP. The performance of this approach is compared only with the TOCM-VAM method. Murugesan and Esakkiammal [20] have showed that the TOCM-VAM method produces better IBFS to good number of TPs next to the ASM method. That is why the TOCM-VAM method only is considered for comparison with the proposed TOCM-ASM approach.

The paper is organized as follows: Following the brief introduction in Section 1, in Section 2.1 and 2.2 step-by-step algorithms of TOCM-VAM and ASM are presented. In Section 2.3, the proposed algorithm for the TOCM-ASM approach for determining the best IBFS to the BTP and UTP are presented. In Section 3, one benchmark problem from balanced type and another from unbalanced type have been illustrated by the methods of TOCM-VAM method as well as by the proposed TOCM-ASM approach. Classical benchmark TPs from balanced category and unbalanced category of different sizes from some reputed journals published by several authors are shown in Section 4. Section 5 demonstrates the comparison of the results of TOCM-ASM approach with TOCM-VAM method. Section 6 shows the advantages of the proposed TOCM-ASM approach over the TOCM-VAM method. Finally, in Section 7 conclusions are drawn.

1.1. **Row Opportunity Cost Matrix (ROCM).** For each row of the given balanced TP, the smallest cost of that row is subtracted from each element of the same row. The resulting matrix is called the ROCM.

1.2. **Column Opportunity Cost Matrix (COCM).** For each column of the given balanced TP, the smallest cost of that column is subtracted from each element of the same column. The resulting matrix is called the COCM.

1.3. **Total Opportunity Cost Matrix (TOCM).** The TOCM is obtained by adding the ROCM with the COCM.

2. METHODOLOGY

In this section we present the algorithm for the proposed TOCM-ASM approach as well as that of for the TOCM-VAM and ASM methods.

2.1. **Algorithm of TOCM-VAM Method.** A systematic procedure for TOCM-VAM due to Mathiraj et al. [9] proceeds as follows:

(1) Balance the given transportation problem if either (total supply > total demand) OR (total supply < total demand).
(2) Obtain the Total Opportunity Cost Matrix (TOCM).
(3) Apply VAM on TOCM and obtain feasible allocations.
(4) Compute the total transportation cost for the feasible allocations obtained in Step3 using the original balanced-transportation cost matrix.

2.2. **Algorithm of the ASM method.** The stepwise procedure of ASM method by Abdul Quddoos et al. [1] can also be seen in Murugesan and Esakkiammal (2020) [19, 20].

2.3. **Algorithm for the Proposed TOCM-ASM Approach.** In this section, separate algorithm for the proposed TOCM-ASM approach for determining the IBFS of balanced and unbalanced TPs has been proposed.

2.3.1. **Algorithm for Balanced Case.** The proposed algorithm for determining the IBFS for a BTP consists of the following steps:

(1) Obtain the Row Opportunity Cost Matrix (ROCM) from the given original BTP.
(2) Obtain the Column Opportunity Cost Matrix (COCM) from the given original BTP.
(3) Obtain the Total Opportunity Cost Matrix (TOCM) just by adding ROCM and COCM.
(4) Apply the ASM method on the obtained TOCM and get the feasible allocations.
(5) Compute the total transportation cost for the feasible allocations obtained in Step4 using the unit transportation costs of the original BTP.

2.3.2. **Algorithm for Unbalanced Case.** The proposed algorithm for determining the IBFS for an UTP consists of the following steps:

(1) Convert the given UTP into a BTP by adding either a dummy row or a dummy column with zero unit transportation cost in the cells.
(2) Obtain the Row Opportunity Cost Matrix (ROCM) from the obtained BTP in Step 1.
(3) Obtain the Column Opportunity Cost Matrix (COCM) from the obtained BTP in Step 1.
(4) Obtain the Total Opportunity Cost Matrix (TOCM) just by adding ROCM and COCM.
(5) Set the unit transportation cost for each of the cells of the dummy row or the dummy column of the TOCM as M, where $M > 0$ is a very large but finite positive quantity.
(6) Apply the ASM method on the obtained new TOCM in Step 5 and get the feasible allocations.
(7) Compute the total transportation cost for the feasible allocations obtained in Step 6 using the unit transportation costs of the original BTP in Step 1.

3. NUMERICAL ILLUSTRATION

The above said algorithms for finding an IBFS of TPs are illustrated by the following benchmark problem from the literature due to Murugesan and Esakkiammal (2020) [20].

3.1. **Illustration 1.** Consider the following cost minimizing BTP with four sources and four destinations given in Table 1.:

TABLE 1. The given BTP

Sources	D1	D2	D3	D4	D5	Supply
S1	10	8	9	5	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	

3.1.1. *Solution by the ASM Method.* First, the given BTP is solved using the algorithm of ASM method. The obtained IBFS is obtained as shown in Table 2.

TABLE 2. Allocation table due to ASM method

Sources	D1	D2	D3	D4	D5	Supply
S1	10	8	9	5	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	340

Writing the Allocation Values. X11 = 60, X13 = 40, X25= 80, X32 = 40, X33 = 20, X35 = 10, X43 = 40, X44 = 50 and all other Xij = 0. Note that the generated solution is a non-degenerate one as it contains exactly eight allocations.
Computing the Total Transportation Cost. $Z = (60 \times 10) + (40 \times 9) + (80 \times 4) + (40 \times 3) + (20 \times 7) + (10 \times 6) + (40 \times 8) + (50 \times 3) = 600 + 360 + 320 + 120 + 140 + 60 + 320 + 150 = \$2070.$

Optimality Checking. By applying the MODI method, this solution has been checked for optimality and we have found that the obtained solution is an optimal one only.

3.1.2. *Solution by the TOCM-ASM Approach.* Next, the given BTP is solved using the proposed approach of TOCM-ASM. The IBFS is obtained also as shown in Table , which is also optimal.

Important Observation. It is noted that for a given TP (whether it is balanced or unbalanced), the ASM method as well as the TOCM-ASM approach produces the same identical allocations in the same sequence. This statement is true for any TP. This is because the first and subsequent reduced cost matrices effected from the given TP (in the ASM method) and the first and subsequent reduced cost matrices effected from the TOCM (in the TOCM-ASM approach) produce the zero entry cells at the same positions. Also, it is well-known that, in the ASM method, the allocations are made only at the appropriate zero entry cells of the first and subsequent reduced cost matrices.

3.1.3. *Solution by the TOCM-VAM Method.* Next, the given BTP is solved using the TOCM-VAM method. The IBFS is obtained as shown in Table 3.

TABLE 3. Allocation table due to TOCM-VAM method

Sources	D1	D2	D3	D4	D5	Supply
S1	[illegible] 10	8	[illegible] 9	5	13	100
S2	7	9	8	10	[illegible] 4	80
S3	9	[illegible] 3	[illegible] 7	10	6	70
S4	11	4	[illegible] 8	[illegible] 3	[illegible] 9	90
Demand	60	40	100	50	90	340

Writing the Allocation Values. X11 = 60, X13 = 40, X25= 80, X32 = 40, X33 = 30, X43 = 30, X44 = 50, X45 = 10 and all other Xij = 0. Note that the generated solution is a non-degenerate one as it contains exactly eight allocations.

Computing the Total Transportation Cost. $Z = (60 \times 10) + (40 \times 9) + (80 \times 4) + (40 \times 3) + (30 \times 7) + (30 \times 8) + (50 \times 3) + (10 \times 9) = 600 + 360 + 320 + 120 + 210 + 240 + 150 + 90 = \$2090.$

Optimality Checking. It is observed that for the illustrated problem, the ASM method produced the optimal solution of Z = $2070 directly, where as the TOCM-VAM method produced a near optimal solution of Z = $2090 only. By applying the MODI method, this solution has been improved towards optimality with Z = $2070 in a single iteration. The optimal solution due to the MODI method is shown in Table 2.

3.2. **Illustration 2.** Consider the following cost minimizing UTP with three sources and five destinations given in Table 4:

TABLE 4. The given BTP

Sources	D1	D2	D3	D4	D5	Supply
S1	10	8	12	9	3	15
S2	4	4	6	6	7	12
S3	15	7	11	13	8	16
Demand	8	8	4	7	6	

3.2.1. *Solution by the TOCM-ASM Approach.* First, the given UTP is solved using the approach of TOCM-ASM. The IBFS is obtained as shown in Table 5.

TABLE 5. Allocation table due to TOCM-ASM approach

Sources	D1	D2	D3	D4	D5	D6	Supply
S1	10	[2] 8	12	[7] 9	[6] 3	0	15
S2	[8] 4	4	[4] 6	6	7	0	12
S3	15	[6] 7	11	13	8	[illegible] 0	16
Demand	8	8	4	7	6	10	43

Writing the Allocation Values. X12= 2, X14 = 7, X15 = 6, X21 = 8, X23 = 4, X32 = 8, X36 = 10 and all other Xij = 0. Note that the generated solution is a degenerate one as it contains only seven allocations (instead of eight allocations).

Computing the Total Transportation Cost. $Z = (2 \times 8) + (7 \times 9) + (6 \times 3) + (8 \times 4) + (4 \times 6) + (6 \times 7) + (10 \times 0) = 16 + 63 + 18 + 32 + 24 + 42 + 0 = \$195.$

Optimality Checking. By checking the condition for optimality by MODI method, it is found that the generated solution by TOCM-ASM is not an optimal one. By applying the MODI method, this solution has been improved towards optimality with Z = $193 in a single iteration. The optimal solution due to the MODI method is shown in Table 3.2.2.

Writing the Optimal Allocation Values. X14 = 7, X15 = 6, X16= 2, X21 = 8, X23 = 4, X32 = 8, X36 = 8 and all other Xij = 0. Note that the generated solution is a degenerate one as it contains only seven allocations (instead of eight allocations).

Computing the Total Minimum Transportation Cost. $Z = (7 \times 9) + (6 \times 3) + (2 \times 0) + (8 \times 4) + (4 \times 6) + (8 \times 7) + (8 \times 0) = 63 + 18 + 0 + 32 + 24 + 56 + 0 = \$193.$

TABLE 6. Optimal allocation table due to MODI method

Sources	D1	D2	D3	D4	D5	D6	Supply
S1	10	8	12	[7] 9	[6] 3	[2] 0	15
S2	[8] 4	4	[4] 6	6	7	0	12
S3	15	[8] 7	11	13	8	[8] 0	16
Demand	8	8	4	7	6	10	43

3.2.2. *Solution by the TOCM-VAM Method.* Next, the given UTP is solved using the TOCM-ASM method. The IBFS is obtained as shown in Table 5, which is not optimal.

4. NUMERICAL EXAMPLES

To justify the efficiency of the proposed approach we have solved a good number of classical benchmark problems (Murugesan and Esakkiammal (2020) [20] from balanced and unbalanced categories in different sizes, from various literature and books, which are listed in Table 4.1 and Table 4.2 respectively. Please refer the article for the problems listed.

5. Result Analysis

For evaluating the performance of the TOCM-VAM and TOCM-ASM methods, simulation experiments were carried out on balanced and unbalanced categories of TPs. The main purpose of the experiment was to evaluate the effectiveness of the IBFSs obtained by TOCM-VAM and TOCM-ASM methods by comparing them with optimal solutions. Effectiveness indicates closeness level which is the least iteration number between IBFS and the optimal solution.

5.1. **Analysis for Balanced Case.**

The assessment of the results for 30 classical benchmark problems of balanced case (Refer Table 5.1) has been studied in this research to measure the effectiveness of the proposed TOCM-ASM approach over the TOCM-VAM method. This assessment is shown in following Table 5.1.

Table 7. Comparison of results obtained by focused methods for BTPs

Problem No. #	VAM	TOCM-VAM	ASM	Proposed TOCM-ASM	Opt. Soln.
1.	5600	5600	5600	5600	5600
2.	955^	880	880	880	880
3.	59	59	59	59	59
4.	28	28	28	28	28
5.	475 ^	435	435	435	435
6.	80 ^	76	76	76	76
7.	470 ^	410	410	410	410
8.	859 ^	809	809	809	809
9.	476 ^	417	417	417	417
10.	1220 ^	1165^	1160	1160	1160
11.	68	68	68	68	68

Note. The near optimal solutions due to VAM, TOCM-VAM, ASM and TOCM-ASM methods are denoted by the symbol $\wedge$.

From Table 7, we discover that TOCM-VAM has produced optimal solution directly to 23 BTPs, whereas TOCM-ASM has produced optimal solution directly to 26 BTPs. Among the identified four challenging problems (Problem Nos. 21, 22, 25 and 26) to the proposed TOCM-ASM approach, two problems have the same near optimal solution by the TOCM-VAM method and the other two (Problem No. 25, 26) has better near optimal solution by the proposed TOCM-ASM approach than the TOCM-VAM method.

5.2. **Analysis for Unbalanced Case.**

The evaluation of the results for 20 classical benchmark problems of unbalanced case (Refer Table 5.2) has been studied in this research to measure the effectiveness of the TOCM-ASM approach over TOCM-VAM method. This comparison is shown in following Table 5.2.

TABLE 8. Comparison of results obtained by focused methods for UTPs

Pr. No. #	VAM	TOCM-VAM	ASM	TOCM-ASM	Opt. Soln.
1.	2164000	2164000	2164000	2164000	2146750
2.	880	840 ∨	840∨	840∨	840
3.	779	813	779	779	743
4.	1010	965	960∨	960∨	960
5.	1555	1465 ∨	1465∨	1465∨	1465
6.	5020	4780	4720∨	4720∨	4720
7.	620	630	606∨	606∨	606
8.	76	76	79	79	75
9.	1510	1540	1305∨	1305∨	1305
10.	2566	2361 ∨	2361∨	2361∨	2361
11.	965	965	950∨	950∨	950
12.	8150	8350	7750∨	7750∨	7750
13.	740	740	710∨	710∨	710

Note. The optimal solutions due to VAM, TOCM-VAM and ASM and TOCM-ASM methods are denoted by the symbol $\vee$.

From Table 8, we discover that the proposed TOCM-ASM approach has produced optimal solution directly to 14 UTPs, whereas, the TOCM-VAM method has produced optimal solution directly to only 4 UTPs, Among the identified six challenging problems (Problem Nos. 1, 3, 8, 15, 16 and 20) to the TOCM-ASM approach, four problems, (numbered as 1, 15, 16 and 20) have the same near optimal solution by the TOCM-ASM and TOCM-VAM methods and one problem (numbered with 3) has better near optimal solution by the TOCM-ASM approach than the TOCM-VAM method and one problem (numbered with 8) has better near optimal solution by the TOCM-VAM than the TOCM-ASM approach.

5.3. **Effectiveness of TOCM-ASM over TOC-VAM.** The overall analysis of the results produced by the TOCM-VAM and TOCM-ASM methods reflect their efficiency. The efficiency of the two methods on 30 BTPs is shown in Table 9 and that of on 20 UTPs is shown in Table 5.3.2 and hence that of on 50 TPs is shown in Table 11.

TABLE 9. Effectiveness of TOCM-VAM and TOCM-ASM Methods on BTPs

Method / Approach	No. of Problems Tested	No. of Problems produced Optimal Solution directly	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
TOCM-VAM	30	23	76.67%	07	23.33%
TOCM-ASM	30	26	86.67%	04	13.33%

TABLE 10. Effectiveness of TOCM-VAM and TOCM-ASM Methods on UTPs

Method / Approach	No. of Problems Tested	No. of Problems produced Optimal Solution directly	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
TOCM-VAM	20	04	20%	16	80%
TOCM-ASM	20	14	70%	06	30%

TABLE 11. Effectiveness of TOCM-VAM and TOCM-ASM Methods on Tps

Method / Approach	No. of Problems Tested	No. of Problems produced Optimal Solution directly	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
TOCM-VAM	50	27	54%	23	46%
TOCM-ASM	50	40	80%	10	20%

6. ADVANTAGES OF TOCM-ASM APPROACH

The TOCM-ASM approach is originating to have the following advantages:

(1) It is an excellent method to find the best IBFS, which is either optimal directly or very close to the optimal solution.

(2) It has produced optimal solution to 86.67% of the BTPs and 80% of the UTPs.

(3) It is based on making allocations only to zero entry cells of the reduced cost matrices effected from the TOCM of a TP.
(4) It is very easy to understand and apply.
(5) Mathematical computations involved in this method are very easy, so no expertise in mathematics is required to use this method.
(6) It is more cost-effective.

7. CONCLUSION

In this paper, we have proposed a new approach called TOCM-ASM for finding best IBFS to the TPs. To verify the performance of the proposed approach, 30 classical benchmark instances of balanced kind and 20 of unbalanced kind from the literature have been tested. Simulation results on BTPs substantiate that the TOCM-ASM approach produces optimal solution directly to 26 (i.e. 86.67% of) BTPs. Another simulation results on UTPs authenticate that the TOCM-ASM approach produces optimal solution directly to 14 (i.e. 70% of) UTPs. Hence, out of 50 total TPs tested the TOCM-ASM approach produces optimal solution directly to 40 (i.e. 80% of) TPs. Therefore, it is established and recognized that the TOCM-ASM approach is the best one for finding the best IBFS to TPs. As a result, it is wise to apply the proposed TOCM-ASM approach also to find IBFS for TPs. Further, the most attractive feature of this method is that it requires only simple arithmetical and logical calculations and hence anyone can easily understand and apply it, like the ASM method, far better than any other method. Also, this method will be more cost-effective for those decision makers who are dealing with logistics and supply chain problems.

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DEPARTMENT OF MATHEMATICS, ST. JOHN'S COLLEGE, PALAYAMKOTTAI,
(AFFILIATED TO MANONMANIAM SUNDARANAR UNIVERSITY, TIRUNELVELI)
TIRUNELVELI DISTRICT, TAMILNADU, INDIA.
Email address: rmurugesa2020@gmail.com

DEPARTMENT OF MATHEMATICS, ST. JOHN'S COLLEGE, PALAYAMKOTTAI,
(AFFILIATED TO MANONMANIAM SUNDARANAR UNIVERSITY, TIRUNELVELI)
TIRUNELVELI DISTRICT, TAMILNADU, INDIA.
Email address: tesakki98@gmail.com