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SPECIAL STRUCTURES IN FLOWSHOP SCHEDULING WITH SEPARATED SET-UP TIMES AND CONCEPT OF JOB BLOCK: MINIMIZATION OF WAITING TIME OF JOBS

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ABSTRACT. In the present paper a Flow shop scheduling model in two stage has been studied where the time taken by machines to set-up is separately considered from the processing time. The probabilities with the processing times as well as with the set-up times are also taken into account. The problem is structured specially for the cases when the minimum of expected processing times on second fictitious machine can never be less than to the maximum of expected processing times on first fictitious machine with the objective of minimizing the total of the waiting time for all the jobs. The two of the jobs has been grouped as a block. The significance of the objective has been made clear by computational experiments in comparison to the existing makespan approaches of Johnson and Palmer.

1. INTRODUCTION

The problem of deciding when to perform given jobs with the purpose of optimizing a function while taking attention of chronological constraints and be located in the limitation resources is known as scheduling. The procedure of sharing the same pre described order of all the machines by the jobs is known as Flow Shop Scheduling. In the present paper we talk about the two stage

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specially structured Flow Shop Scheduling in which it is assumed that minimum of the dealing out times of all the jobs on second fictitious machine can never be less than to the maximum of dealing out times of all the jobs on first fictitious machine. The intention of study is minimization of the total of the waiting time of jobs. When the jobs come for the processing, the waiting time for their turn on the first machine is considered to be zero. But in order to process a job on second machine they may have to wait for their turn for many reasons such as the previous job can take some time for the operation on second machine, machine can take time to set-up, break down in the machine etc. This time which is devoted in waiting for the processing of job on the subsequent machine is known as the waiting time of the job. And the sum of all the waiting time of jobs is known as the total of the waiting time of all the jobs.

2. Preliminaries

The Johnson's [1] algorithm for Flow Shop Scheduling problem in two and three stage to lessen the total elapsed time is popular among the analytical approaches that are used for solving two and three stage scheduling problem. Jackson J.R. [2] generalized Johnson's method [1] for capably solving certain two stage production scheduling problems together with cases in which a number of jobs need only one stage and also the jobs needed two stages may possibly need the machines in both of the possible orders. Gupta J.N.D.[3] with the intention of minimizing the total throughput time in which every jobs finish processing on each and every machine, has developed a number of straight forward algorithms however the processing times are not wholly arbitrary, but stands with a definite relationship to ane another. The n-job, m-machine problem has been studied very wisely by many researchers. Maggu P.L.et.al. [4] made an effort to widen the study by initiating the notion of equivalent job for job block. Bhatnagar V.et.al. [5] investigates the n-job, 2 machine flow shop scheduling models with the intention to optimize the total of the waiting times of all the jobs. Singh T.P.et.al. [6] with the intention to minimize the cost of machines which is consumed on rent studied Flow shop scheduling model in two stage together with the concept of job-block. Further Gupta D.et.al. [7] widened the study by considering separated set-up times from processing times and both allied with probabilities with the same objective as in [6]. Gupta D.et.

al.[8] also studied the two stage flow shop scheduling models which was specially structured with the intention to achieve the schedule which lessens the rental cost of the machines. Palmer [11] applied the heuristic approach for minimizing make-span in n-job m- machine problem. Gupta D.et.al. [9],[11] studied Flow Shop Scheduling models in two stage with the idea to optimize the total of the waiting time of all the jobs where the parameters like job block concept, separated set-up times are well thought of.

3. Practical Situation

Industrialized units play an imperative role in the monetary development of a country. Flow shop scheduling happens in banks, airports, factories etc. Regular working in industries and factories has diverse jobs which are to be practiced on various machines. The idea of lessening the total of the waiting time for all the jobs may be a reasonable aspect from managers of Factory /Industry perspective when he has contract to made the work with less waiting with a viable party to finish the work.

4. Notations

S_j - Schedule of the jobs
m_{1j} - Time taken by first machine to process j-th job.
m_{2j} - Time taken by second machine to process j-th job.
p_{1j} - Probability associated with processing time on first machine to process j-th job.
p_{2j} - Probability associated with processing time on second machine to process j-th job.
s_{1j} - Set up time of first machine after processing j-th job.
s_{2j} - Set up time of second machine after processing j-th job.
q_{1j} - Probability associated with set-up time on first machine to process j-th job.
q_{2j} - Probability associated with the set-up time on second machine to process j-th job.
X'_j - Equivalent processing time taken by machine X to process j-th job.
Y'_j - Equivalent processing time taken by machine Y to process j-th job.
T_{aY} - Completion time of job 'a' on machine Y.

W_μ - Waiting time of job μ.
W - Total of the waiting time of all the jobs.

5. Problem Formulation

The Machine M_1 and M_2 are dealing out n jobs in the sort M_1M_2, m_{1j} and m_{2j} are the processing times of the j-th with probabilities p_{1j} and p_{2j}, on machines M_1 and M_2 correspondingly. s_{ij} and s_{2j} are the set up times with probabilities q_{1j} and q_{2j} of machines M_1 and M_2 correspondingly after processing j-th job such that $\sum_{j=1}^{n} p_{ij} = \sum_{j=1}^{n} q_{ij}$; i=1,2. The formulation of the problem in matrix form as defined by Gupta D.et.al.[7] can be seen in Tab.1. Our goal is to come across a best possible sequence S_j of jobs by considering pair of the jobs 1,m as a job block (1,m) with minimum of the total of the waiting time of all the jobs. The Fictitious machines X and Y with equivalent processing times of j-th job are defined by Gupta D. et.al.[7] are given as:

$$X'_j = m_{1j} \times p_{1j} - s_{2j} \times q_{2j} \qquad Y'j = m_{2j} \times p_{2j} - s_{1j} \times q_{1j}. \tag{5.1}$$

Satisfying processing times structural relationship

$$MaxX'_j \leq MinY'_j. \tag{5.2}$$

TABLE 1. Problem Formulation in Matrix Form

Job	**Machine M_1**				**Machine M_2**			
j.	$\boldsymbol{m_{1j}}$	$\boldsymbol{p_{1j}}$	$\boldsymbol{s_{1j}}$	$\boldsymbol{q_{1j}}$	$\boldsymbol{m_{2j}}$	$\boldsymbol{p_{2j}}$	$\boldsymbol{s_{2j}}$	$\boldsymbol{q_{2j}}$
1.	m_{11}	p_{11}	s_{11}	q_{11}	m_{21}	p_{21}	s_{21}	q_{21}
2.	m_{12}	p_{12}	s_{12}	q_{12}	m_{22}	p_{22}	s_{22}	q_{22}
3.	m_{13}	p_{13}	s_{13}	q_{13}	m_{23}	p_{23}	s_{23}	q_{23}
...	...	...	...	...	...	...	...	...
n.	m_{1n}	p_{1n}	s_{1n}	q_{1n}	m_{2n}	p_{2n}	s_{2n}	q_{2n}

5.1. **Assumption.**

(1) Machines M_1 and M_2 are processing n jobs, the jobs firstly processed on machines M_1 after that on machine M_2 and no passing is permissible.

(2) $\sum_{j=1}^{n} p_{ij} = \sum_{j=1}^{n} q_{ij} = 1$; i=1,2.

(3) At the same time no job will be processed by both of the machines.
(4) The course of action of machines can't be interrupted until a job which is in execution can't be completed.
(5) Time to transport jobs from first machine to second machine, Break down interval of machines is neglible.
(6) It is given two jobs 1,m as a block with priority of processing job 1 over job m in the block(1,m).

TABLE 2. Processing Time Matrix of Fictitious Machines with Job Block

Job	**Machine X**	**Machine Y**	$\boldsymbol{x_j}$
j.	$\boldsymbol{X'_j}$	$\boldsymbol{Y'_j}$	$\boldsymbol{Y'_j - X'_j}$
1.	X'_1	Y'_1	x_1
2.	X'_2	Y'_2	x_2
...	...	...	...
α.	X'_α	Y'_α	x_α
...	...	...	...
n-1.	X'_{n-1}	Y'_{n-1}	x_{n-1}

Lemma 5.1. *Two machines X,Y are handing out n jobs in sort XY among no passing is permissible.* $\{X'_j\}_{j=1}^n$ *and* $\{Y'_j\}_{j=1}^n$ *are the dealing out times of n jobs on machines X and Y correspondingly satisfying processing times structural realtionship defined in equation (5.2) in that case for the n job sequence*

$$\zeta : \mu_1, \mu_2, \ldots, \mu_n, \qquad T_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} + \ldots + Y'_{\mu_n}.$$

Proof. Using principle of Mathematical Induction on number of jobs, consider:

$$S(n) : T_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} + \ldots + Y'_{\mu_n}$$

$$T_{\mu_{1X}} = X'_{\mu_1}$$

$$T_{\mu_{1Y}} = X'_{\mu_1} + Y'_{\mu_1}.$$

S(n) is true for n=1. Assume the result holds for less than n jobs,

$$T_{\mu_n Y} = Max(T_{\mu_n X}, T_{\mu_{n-1Y}}).$$

As $MaxX'_j \leq MinY'_j$. Consequently,

$$T_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} + \ldots + Y'_{\mu_n}.$$

$S(n)$ is true for all n $\in N$ □

Lemma 5.2. *Following the similar notation as used in 5.1 Lemma, for n job sequence* ζ*:* $\mu_1, \mu_2, \ldots, \mu_n$*,*

$$W_{\mu_1} = 0 \tag{5.3}$$

$$W_{\mu_n} = X'_{\mu_1} + \sum_{r=1}^{n-1} x_{\mu_r} + X'_{\mu_n} \tag{5.4}$$

x_{μ_r} *is defined as* $x_{\mu_r} = Y'\mu_r - X'\mu_r$ *,* $\mu_r \in \{1, 2, 3, ..., n\}$.

Proof.

$$W_{\mu_1} = 0$$

$$W_{\mu_n} = Max(T_{\mu_{nX}}, T_{\mu_{n-1Y}}) - T_{\mu_{nX}}$$

$$W_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} + \ldots + Y'_{\mu_{n-1}} - X'_{\mu_1} - X'_{\mu_2} - \ldots - X'_{\mu_n}$$

$$W_{\mu_n} = X'_{\mu_1} + \sum_{r=1}^{n-1} (Y'_{\mu_r} - X'\mu_r) + X'_{\mu_n}$$

$$W_{\mu_n} = X'_{\mu_1} + \sum_{r=1}^{n-1} x_{\mu_r} + X'_{\mu_n}$$

□

Theorem 5.1. *Following the similar notations as used in Lemma 5.1 for the n job sequence* ζ*:* $\mu_1, \mu_2, \ldots, \mu_n$ *the total waiting time (W) is given by*

$$W = nX'\mu_1 + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{r=1}^{n-1} X'_k, \tag{5.5}$$

where

$$y_{\mu_r} = (n-r)x_{\mu_r}; \mu_r \in \{1, 2, \ldots n\}.$$

Proof. Using equation (5.3) and equation (5.4) we have

$$W_{\mu_1} = 0$$

$$W = \sum_{i=1}^{n} W_{\mu_i}$$

$$W = nX'\mu_1 + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{r=1}^{n-1} X'_k,$$

where

$$y_{\mu_r} = (n-r)x_{\mu_r}; \mu_r \in \{1, 2, \dots n\}.$$

□

Theorem 5.2. ***Equivalent Job Block Theorem*** *Assuming the two machines X and Y are processing n jobs in the sort XY. $\{X'_j\}_{j=1}^n$ and $\{Y'_j\}_{j=1}^n$ are the processing time of job i, $1 \leq i \leq n$ on machine X and Y respectively. (1,m) is the group job or job block which can be made equivalent to the one job α (called equivalent job α) job α has processing times X'_α and Y'_α on the machines X and Y and are given by*

(5.6) $X'\alpha = X'_l + X'_m - min(X'_m, Y'_l)$ *and* $Y'\alpha = Y'_l + Y'_m - min(X'_m, Y'_l)$.

The theorem is proved by Maggu et.al. [4].

6. Heuristic Algorithm

Step 1: Calculate the processing times for the fictitious machines X and Y denoted by X'_j and Y'_j defined as in equation (5.1)

$$X'_j = m_{1j} \times p_{1j} - s_{2j} \times q_{2j}$$

$$Y'j = m_{2j} \times p_{2j} - s_{1j} \times q_{1j}.$$

Step 2: Verify the processing time structural relationship Max$X'_j \leq$ MinY'_j as defined is equation (5.2).

Step 3: Take equivalent job α = (1,m) and calculate processing times using equations (5.6) and put back the couple of jobs (1,m) in this order by the single job α.

Step 4: Calculate the values for $x_j = Y'_j - X'_j$ in the Tab. 2.

Step 5: Assemble the jobs in ascending order of x_j.
Assume the schedule thus found be $(\mu_1, \mu_2, \dots, \mu_{n-1})$.

Step 6: Find the order schedules of jobs $S_1, S_2, \dots, S_{n-1}$. Where S_1 is the schedule obtained in 5^{th} step, schedule $S_i, 1 \leq i \leq n-1$ can be obtained by taking i^{th} job in the sequence S_1 to the 1^{st} position and considering respite of the schedule same.

Step 7: Evaluate the total of the waiting time (W) of all the jobs for all the schedules $S_1, S_2, \dots., S_{n-1}$ using the equations (5.6)

$$W = nX'\mu_1 + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{r=1}^{n-1} X'_k,$$

where

$$y_{\mu_r} = (n-r)x_{\mu_r}; \mu_r \in \{1, 2, \ldots n\}.$$

6.1. **Numerical Illustartion.** Assuming two machines M_1 and M_2 are processing 5 jobs in Flow Shop in Tab. 3.

TABLE 3. Problem Formulation in Matrix Form of Illustartion

Job	Machine M_1				Machine M_2			
j.	m_{1j}	p_{1j}	s_{1j}	q_{1j}	m_{2j}	p_{2j}	s_{2j}	q_{2j}
1	13	0.1	5	0.2	8	0.2	4	0.2
2	4	0.2	6	0.2	10	0.2	5	0.1
3	6	0.3	2	0.1	3	0.3	8	0.2
4	5	0.2	2	0.2	13	0.1	2	0.3
5	4	0.2	5	0.3	11	0.2	2.5	0.2

Our intention is to attain most favourable schedule of jobs lessening the total of the waiting time for all the jobs by considering jobs 4,2 in a block (4,2)

Solution

As per step 1-: Evaluate the processing times for the fictitious machines X and Y in Tab. 4 using the equation (5.1).

TABLE 4. Processing Time Matrix of Fictitious Machines of Illustartion

Job	**Machine X**	**Machine Y**
j.	X'_j	Y'_j
1.	0.5	0.6
2.	0.3	0.8
3.	0.2	0.7
4.	0.4	0.9
5.	0.3	0.7

As per step 2-: Max $X'_j = 0.5 \leq$ Min $Y'_j = 0.6$ hence the processing time structural relationship is satisfied.

As per step 3-: Taking (4,2) as a job block denoting this job block by α. The processing times on both of the machines X and Y for single job α are calculated using equation (5.6):

$X'_\alpha = X'_4 + X'_2 - min(X'_2, Y_4) = 0.4$
$Y'_\alpha = Y'_4 + Y'_2 - min(X'_2, Y_4) = 0.4$

As per step 4-: Finding the values for $x_j = Y'_j - X'_j$ in Tab.5.

TABLE 5. Processing Time Matrix of Fictitious Machines with Job Block of Illustration

Job	**Machine X**	**Machine Y**	$\boldsymbol{x_j}$
j.	$\boldsymbol{X'_j}$	$\boldsymbol{Y'_j}$	$\boldsymbol{Y'_j - X'_j}$
1.	0.5	0.6	0.1
3.	0.2	0.7	0.5
5.	0.3	0.7	0.4
α	0.4	1.4	1.0

As per step 5-: Assemble the jobs in ascending order of x_j. The scheduke S_1 thus found be 1,5,3,α

As per step 6-: Consider all the possible schedules S_1:1,5,3,α; S_2:5,1,3,α ; S_3:3,1,5,α; S_4:α,1,5,3.

As per step 7-: Evaluate the total of the waiting time (W) of all the jobs for all the schedules S_1, S_2, S_3, S_4 using equations (5.5)

For this problem $\sum_{j=1}^{5} X'_j = 1.7$

For the schedule S_1:1,5,3,α or S_1:1,5,3,4,2.

W=3.9

For the schedule S_2:5,1,3,α or S_2:5,1,3,4,2.

W=3.2

For the schedule S_3:3,1,5,α or S_3:3,1,5,4,2.

W=2.9

For the schedule S_4:α,1,5,3 or S_4:4,2,1,5,3.

W=4.4

Hence the schedule S_3:3,1,5,4,2 is the requisite schedule with waiting time 2.9 with the consideration of (4,2) as a group job.

7. COMPUTATIONAL EXPERIMENTS

To check the effectiveness of the proposed method, a number of several examples of various groups are randomly considered in which each group varies upon different number of jobs. Here seven groups are generated with job sizes 5, 10, 20, 30, 50, 55, 70 and each group is observed over 10 different arbitrarily generated tribulations. The job 4 and job 2 has been considered as a block in all groups. The mean of the total waiting time of each problem for proposed algorithm is compared with the mean of already existed make-span approaches of Johnson [1] and Palmer [11] shown in Table 6 and are plotted in graph as shown in Fig.1, which reveals that the curve of proposed method is lower than the other two curves whereas Palmer's algorithm curve is high among all.

TABLE 6. Comparison of Computational results

No.of Jobs	**Mean Waiting Time of Jobs (Johnson's Method)**	**Mean waiting Time of Jobs (Palmer's method)**	**Mean waiting Times of Jobs (Proposed method)**
5	89.66	99.44	77.89
10	215.86	246.37	161.37
20	454.27	523.03	350.27
30	690.97	815.68	539.30
50	1173.70	1366.33	900.41
55	1323.54	1495.07	991.60
70	1681.69	1916.85	1265.14

In addition, the percentage of error for each of the problem is also calculated by using the formula

$$e_{rr} = [(W_\delta - W_\theta)/W_\theta] * 100,$$

where W_δ is the total waiting time of existed algorithms and W_θ is the total waiting time of the same job computed by using proposed algorithm. For the sake of measuring the wellness of the proposed algorithm, mean of percentage error is calculated for all job groups and then figured out in the graph below,shown in Fig. 2.

Furthermore it can be seen that Palmer's algorithm produces an error significantly larger than the Johnson's algorithm.

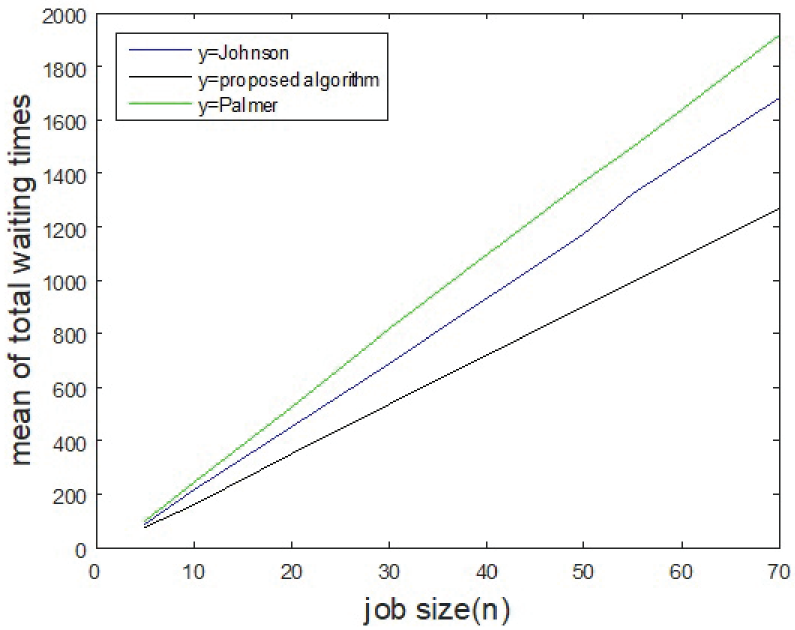

FIGURE 1. Comparison of Computational results

TABLE 7. Mean of percentage errors

N	Mean of percentage error of total waiting times in Johnson's Algorithm	Mean of percentage error of total waiting times in Palmer's Algorithm
5	15.48	27.98
10	34.26	53.62
20	29.82	49.52
30	28.29	51.51
50	30.37	51.85
55	33.53	50.82
70	33.02	51.68

8. CONCLUSION

The present paper deals with Special Structures in Flow Shop scheduling Models with separated set up times incorporating a group of two jobs as a block and proposed heuristic method which provides a near optimal schedule to minimize the total waiting time of jobs. The computational experiments shows that the

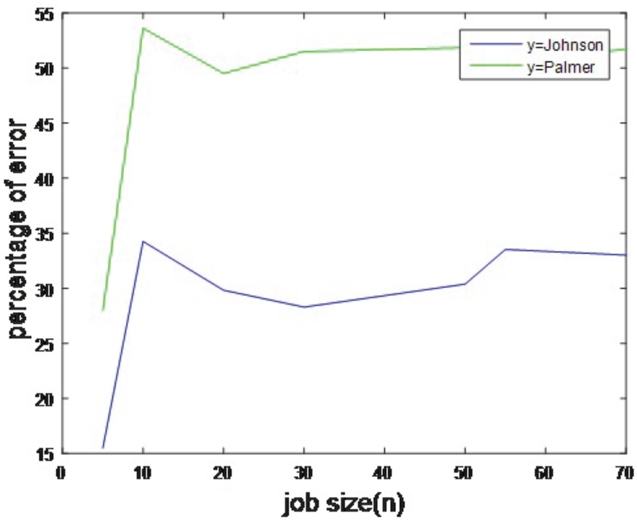

FIGURE 2. Mean of percentage errors

TABLE 8. Average of mean percentage errors

No.of Jobs	Average of mean percentage errors
Palmer's	48.14
Johnson's	29.25

approaches of Johnson [1] and Palmer[11] no doubt minimize the completion time but they however delay the jobs to be processed from first machine to second machine. The proposed algorithm keeps in mind not to make jobs too much wait for processing on second machine when they got free from first machine. The objective of minimizing the waiting time of jobs will be significant to manager's point of view when he has contract with the party to complete their job without making too much wait once the process started

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