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STUDY OF BULK ARRIVAL PARALLEL QUEUE NETWORK MODEL

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ABSTRACT. The present paper emphasis on, developing a queue network model consisting of biserial and parallel queuing subsystems each being connected in series to a single server. The parallel queuing subsystem is characterized with bulk arrival of fixed size. The arrival pattern is poisson and service rate is exponential. Service discipline is first come first served rule (FCFS). The solution and performance measures of the model are obtained by formulating difference equations in steady state form and applying generating function techniques. The model is well illustrated numerically and graphically.

1. INTRODUCTION

Queuing theory has been studied by various researchers with different applications and augmentations. Bulk queue model provides an effectual mechanism for evaluating the performance measures of the queuing system. The Queuing problem with biserial servers and group arrival was studied various investigator including Jackson [1] and Maggu [2]. Shanthikumar [3] approximated the single sever queue. Arumuganathan [4] described the state dependent queue. The analysis was further extended by Gupta D ([5, 6]) with biserial and parallel servers. Kang W.[7] considered many server queue system with impatient customers. Gupta Deepak and others [8] examined the behaviour of heterogeneous servers in steady state. Mittal M. [9] observed the threshold effect on

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queue models under fuzzy environment. Queuing Processes with phase type Service and reneging was investigated by Kaur, M.R. [10]. Feedback queuing processes under stochastic environment and chances of revisiting servers was developed and analysed by Kumar [11]. Whereas Singh T.P. [12] designed the network model for a practical situation occurring in a passport office system and implemented in managing the waiting line problem. Jayarajan [13] studied the priority queue model using energy minimization scheme. In the present manuscript the study made by earlier researchers has been extended by introducing the arrival in batches of fixed size at parallel subsystem instead of individual arrival.

2. Symbols

TABLE 1. Symbols

Symbols	**1st Biserial server S_{11}**	**2nd Biserial server S_{12}**	**1st Parallel server S_{21}**	**2nd Parallel server S_{22}**	**Common server S_3**
No. of customers at any time t	n_1	n_2	n_3	n_4	n_5
Mean Arrival Rate	λ_{11}	λ_{12}	λ_{21}	λ_{22}	
Mean Service Rate	μ_{11}	μ_{12}	μ_{21}	μ_{22}	μ_3
Batch size	β_1	β_2	$-$	$-$	
Transition probabilities	$S_{11} \to S_{12} = \tau_{12}$ $S_{11} \to S_3 = \tau_{15}$ such that $\tau_{12} + \tau_{15} = 1$			$S_{21} \to S_{12} = \tau_{21}$ $S_{21} \to S_3 = \tau_{25}$ such that $\tau_{21} + \tau_{25} = 1$	

3. Problem Description

- The model recommended in the current manuscript includes three service units S_1, S_2 and S_3. The subsystem S_1 incorporates two bi-serial servers S_{11} and S_{12} and constituents of S_2 are two non serial parallel servers S_{21} and S_{22}.
- The server S_3 is allied to each of these two subsystems in series for completion of final phase of services solicited at either S_1 or S_2.
- The customers approaching the system with mean arrival rates λ_{11} and λ_{12} join the queues Q_1 and Q_2 formed in front of the biserial servers S_{11} and S_{12} respectively and those approaching to parallel servers S_21

and S_{22} with mean arrival rates λ_{21} and λ_{22} join the queues Q_3 and Q_4 (formed in front of these servers) in batches of fixed size $\beta_1 \wedge \beta_2$ respectively.

- The customer coming with arrival rate λ_{11}, after availing the service at $\S_{11}$ may either join the server $\S_{12}$ with probability τ_{12} or may directly move to the server $\S_3$ with probability τ_{15} for completion of service such that $\tau_{12} + \tau_{15} = 1$.
- Similarly the customer entering into the system with mean arrival rate λ_{12}, after availing the service at S_{21} either join S_{11} with probability τ_{21} or may directly move to the server S_3 with probability τ_{25} such that $\tau_{21} + \tau_{25} = 1$.
- The customers approaching the with mean arrival rates $\lambda_{21} \wedge \lambda_{21}$ after getting service at parallel servers $S_{21} \wedge S_{21}$ respectively, approach to the server S_3 for completion of the service.
- The problem is also described graphically in the figure 1.

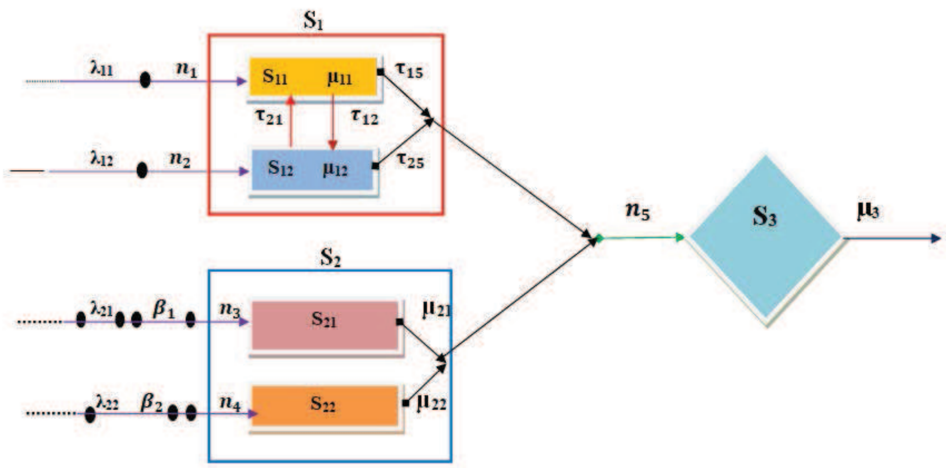

FIGURE 1. Queue Network Model

4. EQUATIONS FORMULATION

$P_{n1,n2,n3,n4,n5}(t)$ denotes the probability that there are $n_1, n_2, n_3, n_4 \wedge n_5$ calling units in the queuing system at any time t, waiting for service in front of servers

$S_1 1, S_1 2, S_2 1, S_2 2 \wedge Server S_3$ respectively. Difference equations in steady-state form for the model are depicted as follows:

For $n_1 > 0, n_2 > 0, n_3 > \beta_1, n_4 > \beta_2 \wedge n_5 > 0$,

(4.1)

$$\begin{gathered}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22} + \mu_{11} + \mu_{12} + \mu_{21} + \mu_{22} + \mu_3)\, P_{n_1,n_2,n_3,n_4,n_5} \\ \lambda_{11} P_{n_1-1,n_2,n_3,n_4 n_5} + \lambda_{12} P_{n_1,n_2} - 1, n_3, n_4, n_3 + \lambda_{21} P_{n_1,n_2,n_3-\beta_1,n_4,n_5} \\ +\lambda_{22} P_{n_1,n_2,n_3,n_4-\beta_2,n_5} + \mu_{11}\tau_{12} P_{n_1+1,n_2-1,n_3,n_4,n_5} + \mu_{11}\tau_{15} P_{n_1+1,n_2,n_3,n_4,n_5-1} \\ +\mu_{12}\tau_{21} P_{n_1-1,n_2+1,n_3,n_4,n_5} + \mu_{12}\tau_{25} P_{n_1,n_2+1,n_3,n_4,n_5-1} + \mu_{21} P_{n_1,n_2,n_3+1,n_4,n_5-1} \\ +\mu_{22} P_{n_1,n_2,n_3,n_4+1,n_5-1} + \mu_5 P_{n_1,n_2,n_3,n_4,n_5+1}\end{gathered}$$

Similarly taking into consideration all the possible conditions on $n_1, n_2, n_3, n_4 \wedge n_5$ and applying them one by one we obtain 71 more steady state difference equations The system of these 72 difference equations in steady state is solved using generating function technique.

4.1. **Solution Process.**

The generating function to solve the system of difference equation is described as:

$$H(x,y,z,s,t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5}\left(x^{n_1} y^{n_2} z^{n_3} s^{n_4} t^{n_5}\right) \tag{4.2}$$

Partial generating function is defined as:

$$\begin{aligned} H_{n_2,n_3,n_4,n_5}(x) &= \sum_{n_1=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5} x^{n_1} \\ H_{n_3,n_4,n_5}(x,y) &= \sum_{n_2=0}^{\infty} H_{n_2,n_3,n_4,n_5}(x) y^{n_2} \\ H_{n_4,n_5}(x,y,z) &= \sum_{n_3=0}^{\infty} H_{n_3,n_4,n_5}(x,y) z^{n_3} \\ H_{n_5}(x,y,z,s) &= \sum_{n_4=0}^{\infty} H_{n_4,n_5}(x,y,z) s^{n_4} \\ H(x,y,z,s,t) &= \sum_{n_5=0}^{\infty} H_{n_5}(x,y,z,s) t^{n_3} \end{aligned} \tag{4.3}$$

Evaluating the system of difference equations applying generating function technique and making use of (4.2) and (4.3), the procured equation is:
(4.4)

$$H(x,y,z,s,t)=\left[\frac{\begin{array}{c}\mu_{11}\left(1-\frac{\tau_{12}y}{x}-\frac{\tau_{15}t}{x}\right)H_1+\mu_{12}\left(1-\frac{\tau_{21}x}{y}-\frac{\tau_{25}t}{y}\right)H_2\\ +\mu_{21}\left(1-\frac{t}{z}\right)H_3+\mu_{22}\left(1-\frac{t}{s}\right)H_4+\mu_3\left(1-\frac{1}{t}\right)H_5\end{array}}{\begin{array}{c}\lambda_{11}(1-x)+\lambda_{12}(1-y)+\lambda_{21}\left(1-z^{\beta_1}\right)+\lambda_{22}\left(1-s^{\beta_2}\right)\\ +\mu_{11}\left(1-\frac{\tau_{12}y}{x}-\frac{\tau_{15}t}{x}\right)+\mu_{12}\left(1-\frac{\tau_{21}x}{y}-\frac{\tau_{25}t}{y}\right)\\ +\mu_{21}\left(1-\frac{t}{z}\right)+\mu_{22}\left(1-\frac{t}{s}\right)+\mu_3\left(1-\frac{1}{t}\right)\end{array}}\right]$$

Here,

$H_1=H_0(y,z,s,t);H_2=H_0(x,z,s,t);H_3=H_0(x,y,s,t);$
$H_4=H_0(x,y,z,t);H_5=H_0(x,y,z,s);$
$H(1,1,1,1,1)=1,n_1,n_2,n_3,n_4,n_5\neq 0\ 0, otherwise$

Simplifying equation (4.4) further applying L'Hospital for limits by differentiating numerator and denominator separately w.r.t. each variable one by keeping other variables as constant under the conditions:

$$\tau_{12}+\tau_{15}=1$$

$$\tau_{21}+\tau_{25}=1$$

The results procured are as follows:

$$\mu_{11}H_1-\mu_{12}H_2\tau_{21}=-\lambda_{11}+\mu_{11}-\mu_{12}\tau_{21} \tag{4.5}$$

$$-\mu_{11}\tau_{12}H_1+\mu_{12}H_2=-\lambda_{12}-\mu_{11}\tau_{12}+\mu_{12} \tag{4.6}$$

$$\mu_{21}H_3=-\beta_1\lambda_{21}+\mu_{21} \tag{4.7}$$

$$\mu_{22}H_4=-\beta_2\lambda_{22}+\mu_{22} \tag{4.8}$$

$$-\mu_{11}\tau_{15}H_1+\mu_{12}\tau_{25}H_2-\mu_{21}H_3-\mu_{22}H_4+\mu_3H_5 \tag{4.9}$$

Eliminating $H_1,H_2,H_3,H_4\wedge H_5$ from equations (4.5) to (4.9), using laws of calculus, we obtain:

$$H_1=1-\frac{\lambda_{11}+\lambda_{12}\tau_{21}}{\mu_{11}\left(1-\tau_{12}\tau_{21}\right)} \tag{4.10}$$

$$H_2=1-\frac{\lambda_{11}\tau_{12}+\lambda_{12}}{\mu_{12}\left(1-\tau_{12}\tau_{21}\right)} \tag{4.11}$$

$$H_3 = 1 - \frac{\beta_1 \lambda_{21}}{\mu_{21}} \tag{4.12}$$

$$H_4 = 1 - \frac{\beta_2 \lambda_{22}}{\mu_{22}} \tag{4.13}$$

$$H_5 = 1 - \left[\frac{\beta_1 \lambda_{21} + \beta_2 \lambda_{22}}{\mu_3} + \frac{\tau_{15}\left(\lambda_{11} + \lambda_{12}\tau_{21}\right) + \tau_{25}\left(\lambda_{11}\tau_{12} + \lambda_{12}\right)}{\mu_3\left(1 - \tau_{12}\tau_{21}\right)}\right] \tag{4.14}$$

4.2. **The Solution.** The acquired solution of the model in steady state form is as follows:

(4.15)

$$P_{n_1,n_2,n_3,n_4,n_5} = \rho_1^{n_1}\rho_2^{n_2}\rho_3^{n_3}\rho_4^{n_4}\rho_5^{n_5}\left(1-\rho_1\right)\left(1-\rho_2\right)\left(1-\rho_3\right)\left(1-\rho_4\right)\left(1-\rho_5\right)$$

$$\begin{gathered} \rho_1 = \frac{\lambda_{11}+\lambda_{12}\tau_{21}}{\mu_{11}(1-\tau_{12}\tau_{21})} \\ \rho_2 = \frac{\lambda_{11}\tau_{12}+\lambda_{12}}{\mu_{12}(1-\tau_{12}\tau_{21})} \\ \rho_3 = \frac{\beta_1\lambda_{21}}{\mu_{21}}, \rho_4 = \frac{\beta_2\lambda_{22}}{\mu_{22}} \\ \rho_5 = \frac{\beta_1\lambda_{21}+\beta_2\lambda_{22}}{\mu_3} + \frac{\tau_{15}(\lambda_{11}+\lambda_{12}\tau_{21})+\tau_{25}(\lambda_{11}\tau_{12}+\lambda_{12}}{\mu_3(1-\tau_{12}\tau_{21})} \end{gathered} \tag{4.16}$$

Here, the solution in steady state exists only if $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5 < 1$

5. Measures of the Model

5.1. **Mean Queue Length.** The average number of units waiting in the system for service is computed as follows:

$$L_s = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\left(n_1+n_2+n_3+n_4+n_5\right)P_{n_1,n_2,n_3,n_4,n_5} \tag{5.1}$$

$$L_s = \begin{cases} \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_1 P_{n_1,n_2,n_3,n_4,n_5} \\ +\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_2 P_{n_1,n_2,n_3,n_4,n_5} \\ +\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_3 P_{n_1,n_2,n_3,n_4,n_5} \\ +\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_4 P_{n_1,n_2,n_3,n_4,n_5} \\ +\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_5 P_{n_1,n_2,n_3,n_4,n_5} \end{cases}$$

$$L_s = L_1 + L_2 + L_3 + L_4 + L_5$$

Now,

$$L_1 = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_1 P_{n_1,n_2,n_3,n_4,n_5}$$
$$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_1 \rho_1^{n_1}\rho_2^{n_2}\rho_3^{n_3}\rho_4^{n_4}$$
$$\rho_5^{n_5}\left(1-\rho_1\right)\left(1-\rho_2\right)\left(1-\rho_3\right)\left(1-\rho_4\right)\left(1-\rho_5\right)$$

Expanding summations, we obtain:

$$L_1 - \frac{\rho_1}{(1-\rho_1)}$$

Similarly,

$$L_2 = \frac{\rho_2}{(1-\rho_2)}, L_3 = \frac{\rho_3}{(1-\rho_3)}, L_4 = \frac{\rho_4}{(1-\rho_4)} L_5 = \frac{\rho_5}{(1-\rho_5)}$$

Thus,

$$L_s = \frac{\rho_1}{(1-\rho_1)} + \frac{\rho_2}{(1-\rho_2)} + \frac{\rho_3}{(1-\rho_3)} + \frac{\rho_4}{(1-\rho_4)} + \frac{\rho_5}{(1-\rho_5)} \tag{5.2}$$

5.2. **Variance of Queue Length.**

$$\operatorname{Var}\left(n_1+n_2+n_3+n_4+n_5\right) = \left\{ \begin{array}{c} \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5} \\ \left(n_1+n_2+n_3+n_4+n_5\right)^2 - L_S^2 \end{array} \right.$$

Expanding the summations and utilizing the values from (4.15), we get:

(5.3)

$$\operatorname{Var}\left(n_1+n_2+n_3+n_4+n_5\right) = \frac{\rho_1}{\left(1-\rho_1\right)^2}+\frac{\rho_2}{\left(1-\rho_2\right)^2}+\frac{\rho_3}{\left(1-\rho_3\right)^2}+\frac{\rho_4}{\left(1-\rho_4\right)^2}+\frac{\rho_5}{\left(1-\rho_5\right)^2}$$

5.3. **Average waiting time.** The average time of customers to wait in a queue:

$$W_s = \frac{L_s}{\lambda} = \frac{1}{\lambda}\left\{\frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} + \frac{\rho_4}{1-\rho_4} + \frac{\rho_5}{1-\rho_5}\right\} \tag{5.4}$$

Here, $\lambda = \lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}$.

6. Statistical Analysis

Considering the numeric values of various parameters of the model and transition probabilities as given in the table 2; compute the mean queue length and other measures of the model.

TABLE 2. Numerical Values of Parameters

	Numeric Values				
Mean Arrival Rate	$\lambda_{11}=4$	$\lambda_{12}=6$	$\lambda_{21}=3$	$\lambda_{22}=2$	$-$
Mean Service Rate	$\mu_{11}=8$	$\mu_{12}=11$	$\mu_{21}=15$	$\mu_{22}=9$	$\mu_3=28$
Batch size	$\beta_1=4$	$\beta_2=3$	$-$	$-$	$-$
Transition probabilities	$\tau_{12}=0.2, \tau_{15}=0.8$ $\tau_{12}+\tau_{15}=1$			$\tau_{21}=0.6, \tau_{25}=0.4$ $\tau_{21}+\tau_{25}=1$	

Utilizing the parametric values from the table in (4.16); we obtain:

$$\rho_1=\frac{\lambda_{11}+\lambda_{12}\tau_{21}}{\mu_{11}\left(1-\tau_{12}\tau_{21}\right)}=0.824<1$$

$$\rho_2=\frac{\lambda_{11}\tau_{12}+\lambda_{12}}{\mu_{12}\left(1-\tau_{12}\tau_{21}\right)}=0.702<1$$

$$\rho_3=\frac{\beta_1\lambda_{21}}{\mu_{21}}=0.8<1$$

$$\rho_4=\frac{\beta_2\lambda_{22}}{\mu_{22}}=0.666<1$$

$$\rho_5=\left[\frac{\beta_1\lambda_{21}+\beta_2\lambda_{22}}{\mu_3}+\frac{\tau_{15}\left(\lambda_{11}+\lambda_{12}\tau_{21}\right)+\tau_{25}\left(\lambda_{11}\tau_{12}+\lambda_{12}\right)}{\mu_3\left(1-\tau_{12}\tau_{21}\right)}\right]=0.94<1$$

Making use of the values obtained above in (5.2), (5.3) and (5.4); we get:

$$L_s=\frac{\rho_1}{(1-\rho_1)}+\frac{\rho_2}{(1-\rho_2)}+\frac{\rho_3}{(1-\rho_3)}+\frac{\rho_4}{(1-\rho_4)}+\frac{\rho_5}{(1-\rho_5)}$$

$$L_s=28.78$$

$$\mathrm{Var}\left(n_1+n_2+n_3+n_4+n_5\right)=\frac{\rho_1}{(1-\rho_1)^2}+\frac{\rho_2}{(1-\rho_2)^2}+\frac{\rho_3}{(1-\rho_3)^2}+\frac{\rho_4}{(1-\rho_4)^2}+\frac{\rho_5}{(1-\rho_5)^2}$$

$$\mathrm{Var}\left(n_1+n_2+n_3+n_4+n_5\right)=321.5$$
$$W_S=\tfrac{L_S}{\lambda}=\tfrac{28.78}{15}=1.91$$

7. GRAPHICAL ANALYSIS

Behaviour of marginal queue length and queue length of the entire system with respect to size of batch at first parallel server by keeping the values of other parameter fixed is depicted in Table 3.

TABLE 3. Mean queue length with respect to batch size

Batch size	$\lambda_{11}=4, \lambda_{12}=6, \lambda_{21}=3, \lambda_{22}=2, \beta_2=3$ $\mu_{11}=8, \mu_{12}=11, \mu_{21}=25, \mu_{22}=9, \mu_3=28$ $\tau_{12}=0.2, \tau_{15}=0.8, \tau_{21}=0.6, \tau_{25}=0.4$					
β_1	L_1	L_1	L_2	L_3	L_4	L_s
1	0.82	0.7	0.1	0.66	0.62	2.9
2	0.824	0.70	0.2	0.66	0.72	3.1
3	0.824	0.70	0.3	0.66	0.83	3.3
4	0.824	0.70	0.5	0.66	0.94	3.5
5	0.824	0.70	0.6	0.66	1.04	3.7

8. REMARKS

If instead of batch arrival at parallel servers individual arrival is taken into consideration $\beta_1 = \beta_2 = 1$; then the results bears resemblance with that of Gupta D [6].

9. RESULTS AND DISCUSSION

In the recommended paper we propound a network model with two types of arrivals, individual and batch in stochastic environment. The time independent solution of the model and various performance measures have been computed using generating function technique. Graphically study made concludes that mean queue length of the system increases gradually as the size of batch increases. Particular case and numerical illustration provides the validity of the model. The model finds its applications in various fields like banking system, administrative setup, supermarkets, handling of games and production management.

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