

APPROACH ON THE STRUCTURE OF NANO TOPOLOGY VIA GRAPH

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ABSTRACT. This paper focus, an alternative formulation of nano topological space can be developed based on graph with respect to the neighborhood of vertices. The aspect of this present paper is to spotlight on using new definitions of lower, upper approximation space and boundary region of a graph, which are the basic concepts of the nano topological space. These definitions follow naturally from a particular property on the approximation in a graph is examined and discussed. We establish the bigraphs G_1, G_2 of nano topology also we discuss the application of the adjacency matrix and social network illuminated in detail.

1. INTRODUCTION

Lellis Thivagar et al [3] introduced a nano topological space for a subset X of a universe. In 1982, Pawlak [6] introduced the concept of rough set theory based on equivalence relation has been extended to the binary relation and by using concept of rough set in decision theoretical approach [8], [9]. In many authors have studied the binary relation based on neighborhoods [1], [4], [7]. In this paper, we initially construct a new approach of nano topology induced by a graph. The nano topology of graph theory will be an important base for the modification of knowledge extraction and processing. Moreover, the property of

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approximations is studied. In addition to this, we investigate the properties between lower, upper approximation are induced graph with nano topology. Then by using the definition of nano topology induced by a graph, we can deal with an application of social network data to represent the graph and adjacency matrix are examined. Moreover, we give several examples for a better understanding of the subject.

2. PRELIMINARIES

In the current section we recollect the basic definitions of nano topology and graph theory.

Definition 2.1. [3] Let \mathcal{U} be a non-empty finite set of objects called the universe, \mathcal{R} be an equivalence relation on \mathcal{U} named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (\mathcal{U}, R) is said to be approximation space. Let $X \subseteq \mathcal{U}$.

- (1) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{ \bigcup_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\} \right\}$, where $R(x)$ denotes the equivalence class determined by x .
- (2) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{ \bigcup_{x \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}$.
- (3) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [3] Let \mathcal{U} be the universe, R be an equivalence relation on \mathcal{U} and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:

- (1) \mathcal{U} and $\emptyset \in \tau_R(X)$
- (2) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- (3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on \mathcal{U} called as the nano topology on \mathcal{U} with respect to X . We call $\{\mathcal{U}, \tau_R(X)\}$ as the nano topological space.

Definition 2.3. [2] A graph G consist of a pair $(V(G), E(G))$, where $V(G)$ is a non empty finite set whose elements are called points (or) vertices and $E(G)$ is a set of unordered pairs of distinct elements of $V(G)$. The elements of $E(G)$ are called lines (or) edges of the graph.

Definition 2.4. [2] A graph G is simple if it has no loop and no two of its links join the same pair of vertices. A graph which has no edge is called a null graph. A graph which has no vertices is called an empty graph.

Definition 2.5. [2] The set of all neighbors of a vertex $v \in V$ of G , denoted by $N(v)$ is called the neighborhood of v . If A is a subset of V , we denote by $N(A)$. The set of all vertices in G that are adjacent to atleast one vertex in A .

Definition 2.6. [2] Let $[a_{ij}]$ be adjacency matrix for graph with n vertices is an $n \times n$ matrix

$$[a_{ij}] = \begin{cases} 1 & \text{if } \{i, j\} \in E \\ 0 & \text{otherwise} \end{cases}$$

3. THE GRAPH BASED ON NANO TOPOLOGICAL SPACE

To view this section is to propose a new method to define the lower, upper approximations and boundary region of a graph with respect to the neighborhood of vertex respectively.

Definition 3.1. Let $G(V, E)$ be a graph and $\mathcal{N}(v)$ be the neighborhood of a vertex $v \in V$, $\tau_G(A) = \{G, \emptyset, L_G(A), U_G(A), B_G(A)\}$ where the lower approximation $L_G(A)$, upper approximation $U_G(A)$ and boundary region $B_G(A)$ of every subset A of a graph $V(G)$ are defined by

- (i) $L_G(A) = \{v \in V(G) : \mathcal{N}_G(v) \subseteq A\}$
- (ii) $U_G(A) = \{v \in V(G) : \mathcal{N}_G(v) \cap A \neq \emptyset\}$
- (iii) $B_G(A) = U_G(A) - L_G(A)$.

That is, $\tau_G(A)$ forms a topology on graph $G(V, E)$ called as the nano topology induced by graph with respect to neighborhood of vertex $\mathcal{N}_G(v)$. We call $\{G, \tau_G(A)\}$ as the nano topological space induced by graph.

Example 1. Let $G(V, E)$ be a graph with $G = V(G) = \{a, b, c, d\}$. Let $X = \{a, d, c\} \subseteq V(G)$. Then $\mathcal{N}(a) = V(G)$, $\mathcal{N}(b) = \{a, b, c\}$, $\mathcal{N}(c) = \{a, b, c, d\}$ and $\mathcal{N}(d) = \{a, c, d\}$.

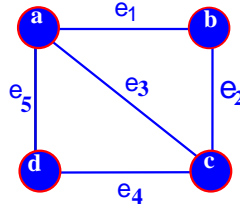


FIGURE 1. Example on Graph

We have $L_G(X) = \{d\}$, $U_G(X) = V(G)$, $B_G(X) = \{a, b, c\}$. Hence $\tau_G(X) = \{G, \emptyset, \{d\}, \{a, b, c\}\}$.

Definition 3.2. Let $G(V, E)$ be a graph and $\mathcal{N}(v)$ be the neighborhood of a vertex $v \in V$. We define the characterization of nano topological space induced by graph as follows as:

Graph Nano Type-1 (\mathcal{GNT}_1):

If $L_G(X) \neq U_G(X)$ or $L_G(X) = U_G(X)$, where $L_G(X) \neq \emptyset$ and $U_G(X) \neq G$, then either $\tau_G(X) = \{G, \emptyset, L_G(X), U_G(X), B_G(X)\}$ is called discrete nano topology induced by graph or $\tau_G(X) = \{G, \emptyset, L_G(X)\}$.

Graph Nano Type-2 (\mathcal{GNT}_2):

If $L_G(X) = \emptyset$ and $U_G(X) \neq G$, then $\tau_G(X) = \{G, \emptyset, U_G(X)\}$.

Graph Nano Type-3 (\mathcal{GNT}_3):

If $L_G(X) \neq \emptyset$ and $U_G(X) = G$, then $\tau_G(X) = \{G, \emptyset, L_G(X), B_G(X)\}$.

Graph Nano Type-4 (\mathcal{GNT}_4):

If $L_G(X) = \emptyset$ and $U_G(X) = G$, then $\tau_G(X) = \{G, \emptyset\}$ is called indiscrete nano topological space induced by graph.

Theorem 3.1. Let $(G, \tau_G(X))$ be nano topological space induced by graph and $V(G)$ be vertex of a graph defined on $G(V, E)$. Let $X, Y \subseteq V(G)$, then the approximation of the following properties hold:

- (i) $L_G(\emptyset) = \emptyset = U_G(\emptyset)$.
- (ii) $L_G(V) = V(G) = U_G(V)$.
- (iii) $L_G(X) \subseteq U_G(X)$.
- (iv) If $X \subseteq Y \Rightarrow L_G(X) \subseteq L_G(Y)$ and $U_G(X) \subseteq U_G(Y)$.

- (v) $L_G(X \cap Y) = L_G(X) \cap L_G(Y)$.
- (vi) $L_G(X \cup Y) \supseteq L_G(X) \cup L_G(Y)$.
- (vii) $U_G(X \cup Y) = U_G(X) \cup U_G(Y)$.
- (viii) $U_G(X \cap Y) \subseteq U_G(X) \cap U_G(Y)$.

Proof. (i) The proof (i) and (ii) is directly from the Definition 3.1.

- (iii) Let $\mathcal{N}(v) \in L_G(A)$. Then $\mathcal{N}(v) \subseteq X$, which implies that $\mathcal{N}(v) \cap A \neq \emptyset$. Therefore we have $\mathcal{N}(v) \in U_G(A)$. Hence $L_G(X) \subseteq U_G(X)$.
- (iv) Let $\mathcal{N}(v) \in L_G(X)$. Then $\mathcal{N}(v) \subseteq X$ and also $X \subseteq Y$, which implies that $\mathcal{N}(v) \subseteq Y$, then we have $\mathcal{N}(v) \in L_G(Y)$. Hence $L_G(X) \subseteq L_G(Y)$. Similarly we have proved that $U_G(X) \subseteq U_G(Y)$.
- (v) Let $\mathcal{N}(v) \in L_G(X) \cap L_G(Y)$. Then $\mathcal{N}(v) \subseteq X$ and $\mathcal{N}(v) \subseteq Y$, which implies that $\mathcal{N}(v) \subseteq X \cap Y$, then we have $\mathcal{N}(v) \in L_G(X \cap Y)$. It can be obtain that $L_G(X \cap Y) = L_G(X) \cap L_G(Y)$.
- (vi) The proof follows from the above (v).
- (vii) Let $\mathcal{N}(v) \in U_G(X) \cup U_G(Y)$. Then $\mathcal{N}(v) \cap X \neq \emptyset$ and $\mathcal{N}(v) \cap Y \neq \emptyset$, which implies that $\mathcal{N}(v) \cap (X \cup Y) \neq \emptyset$, then we have $\mathcal{N}(v) \in U_G(X \cup Y)$. It can be obtain that $U_G(X \cup Y) = U_G(X) \cup U_G(Y)$.
- (viii) The proof follows from the above (vii).

□

Proposition 3.1. Let $(G, \tau_G(X))$ be nano topological space induced by graph and $X \subseteq V(G)$, then the approximation of the following properties are hold:

- (i) $L_G(X) \subseteq L_G(U_G(X))$
- (ii) $L_G(X) \subseteq L_G(L_G(X))$ for all $X \subseteq U$
- (iii) $U_G(U_G(X)) \subseteq U_G(X)$.

Proof. (i) Let $\mathcal{N}(v) \in L_G X$. Then $\mathcal{N}(v) \subseteq X$ Since for all $\mathcal{N}(v) \in \mathcal{N}(u)$, then $\mathcal{N}(v) \subseteq \mathcal{N}(u)$, which implies that $\mathcal{N}(v) \in \mathcal{N}(u) \cap X \neq \emptyset$, then $\mathcal{N}(u) \in \mathcal{N}(v) \cap X \neq \emptyset$. We have $\mathcal{N}(u) \in U_G(X)$, which implies that $\mathcal{N}(v) \in L_G(U_G(X))$, it can be obtain that $L_G(X) \subseteq L_G(U_G(X))$.

(ii) Let $\mathcal{N}(v) \in L_G X$. Then $\mathcal{N}(v) \subseteq X$ and for all $\mathcal{N}(v) \in \mathcal{N}(u)$, which implies that $\mathcal{N}(v) \in \mathcal{N}(u) \subseteq X$, then $\mathcal{N}(u) \in \mathcal{N}(v) \subseteq X$. We have $\mathcal{N}(u) \in L_G(X)$, which implies that $\mathcal{N}(v) \in L_G(L_G(X))$, it can be obtain that $L_G(X) \subseteq L_G(L_G(X))$.

(iii) The proof is follows from above(ii).

□

4. ON BIGRAPH G_1 AND G_2 WITH NANO TOPOLOGY

In this section, we study the union of bigraph $G_1(V_1, G_1)$ and $G_2(V_2, G_2)$ be the approximation of nano topological space with respect to the neighborhood of vertices are established and the properties are studied.

Definition 4.1. Let $G_1(V_1, E_1), G_2(V_2, E_2)$ be a bigraph and $\mathcal{N}_{G_1}(u), \mathcal{N}_{G_2}(v)$ be the neighborhood of a vertex $u \in V_1, v \in V_2$. Thus $\tau_{G'}(A) = \{G', \emptyset, L_{G'}(A), U_{G'}(A), B_{G'}(A)\}$ where $G' = G_1 \cup G_2$ also the lower approximation $L_{G'}(A)$, upper approximation $U_{G'}(A)$ and boundary region $B_{G'}(A)$ of every subset A of a graph V' by $V' = V_1 \cup V_2$ are defined as follows:

- (i) $L_{G'}(A) = \bigcup \{\mathcal{N}_{G_1}(u) \cup \mathcal{N}_{G_2}(v) \mid \{\mathcal{N}_{G_1}(u) \subseteq A\} \cup \{\mathcal{N}_{G_2}(v) \subseteq A\}\}$
- (ii) $U_{G'}(A) = \bigcup \{\mathcal{N}_{G_1}(u) \cap \mathcal{N}_{G_2}(v) \mid \{\mathcal{N}_{G_1}(u) \cap A \neq \emptyset\} \cap \{\mathcal{N}_{G_2}(v) \cap A \neq \emptyset\}\}$
- (iii) $B_{G'}(A) = U_{G'}(A) - L_{G'}(A)$.

That is, $\tau_{G'}(A)$ forms a topology on bigraph G_1, G_2 called as the nano topology induced by bigraph with respect to neighborhood of vertex $\mathcal{N}_{G_1}(u), \mathcal{N}_{G_2}(v)$. We call $\{G', \tau_{G'}(A)\}$ as the nano topological space induced by bigraph.

Example 2. Let G_1, G_2 be bigraph with $V_1(G_1) = \{a, b, c, d\}, V_2(G_2) = \{a, b, c, e\}$ and $V(G') = \{a, b, c, d, e\} = V_1(G_1) \cup V_2(G_2)$.

Let $X = \{a, b, c\} \subseteq V_1(G_1) \cup V_2(G_2)$, then $\mathcal{N}_{v_1}(a) = \{a, b, d\}, \mathcal{N}_{v_1}(b) = \{a, b, c\},$

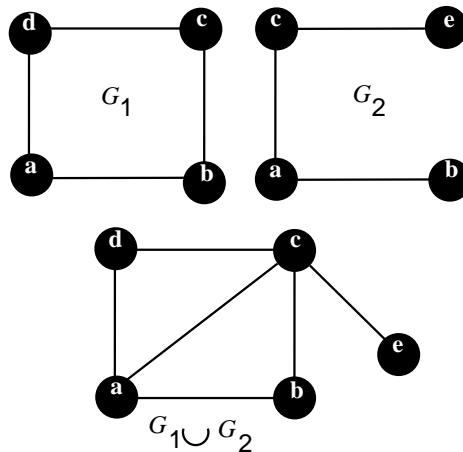


FIGURE 2. Example on Bigraph

$\mathcal{N}_{v_1}(c) = \{b, c, d\}$ and $\mathcal{N}_{v_1}(d) = \{a, c, d\}$ and $\mathcal{N}_{v_2}(a) = \{a, b, c\}, \mathcal{N}_{v_2}(b) = \{a, b\},$

$\mathcal{N}_{v_2}(c) = \{a, c, e\}$ and $\mathcal{N}_{v_2}(e) = \{c, e\}$. We have $L_{G'}(X) = \{a, b, c\}$, $U_{G'}(X) = \{a, b, c\}$, $B_{G'}(X) = \emptyset$. Then $\tau_{G'}(X) = \{G', \emptyset, \{a, b, c\}\}$.

Theorem 4.1. Let $(G', \tau_{G'}(X))$ be nano topological space induced by bigraph where $G' = G_1 \cup G_2$ and $V_1(G_1), V_2(G_2)$ be vertex of a bigraph defined on G_1, G_2 . Let $X \subseteq G'$, then the approximation of the following properties hold:

- (i) $L_{G'}(\emptyset) = \emptyset = U_{G'}(\emptyset)$.
- (ii) $U_{G'}(X) \supseteq (X)$.
- (iii) $L_{G'}(G') = G'$.
- (v) $U_{G'}(G') = G'$.
- (vi) $L_{G_1 \cup G_2}(X) = L_{G_2 \cup G_1}(X)$.
- (vii) $U_{G_1 \cup G_2}(X) = U_{G_2 \cup G_1}(X)$.

Proposition 4.1. Let G_1, G_2 be bigraph and $X \subseteq G'$ where $G' = G_1 \cup G_2$. Then approximation as follows

- (i) $L_{V(G')}(X) = L_{V(G_1)}(X) \cup L_{V(G_2)}(X)$.
- (ii) $U_{V(G')}(X) \subseteq U_{V_1(G_1)}(X) \cup U_{V_2(G_2)}(X)$.

Proof. (i) If for all $X \subseteq V_1 \cup V_2$, then by definition 4.1 we have $L_{G'}(X)$ which is equal to $\bigcup \{u, v \in V_1(G_1) \cup V_2(G_2) \mid N_{G_1}(u) \subseteq X \cup N_{G_2}(v) \subseteq X\}$, which implies that $\bigcup \{u \in V_1(G_1) \mid \{N_{G_1}(u) \subseteq X\}\} \cup \{v \in V_2(G_2) \mid N_{G_2}(v) \subseteq X\}$. Therefore we can obtain that $L_{V_1(G_1)}(X) \cup L_{V_2(G_2)}(X)$. Hence we have $L_{V(G')}(X) = L_{V_1(G_1)}(X) \cup L_{V_2(G_2)}(X)$.

- (ii) If for every $u, v \in V(G')$, $X \subseteq V_1 \cup V_2$. Then we have $U_{G_1 \cup G_2}(X)$ which is equal to $\bigcup \{u, v \in (V_1(G_1)) \cup (V_2(G_2)) \mid \{N_{G_1}(u) \cap X \neq \emptyset\} \text{ and } \{N_{G_2}(v) \cap X \neq \emptyset\}\}$, which implies that $\bigcup \{u \in V_1(G_1) \mid N_{G_1}(u) \cap X \neq \emptyset\} \cap \{v \in V_2(G_2) \mid N_{G_2}(v) \cap X \neq \emptyset\}$. Since we can obtain that $U_{V_1(G_1)}(X) \cup U_{V_2(G_2)}(X)$. Hence $U_{G'}(X) \subseteq U_{V_1(G_1)}(X) \cup U_{V_2(G_2)}(X)$

□

Proposition 4.2. Let V_1 and V_2 be bigraph of G_1 and G_2 . If $V_1(G_1) = V_2(G_2)$ then $L_{G_1}(X) = L_{G_2}(X)$.

Proof. If for all $V_1 \in G_1, V_2 \in G_2$ and $V_1 = V_2$, then by definition 3.1 we have $u \in V_1(G_1), v \in V_2(G_2)$ and $u = v$, which implies that $N(u) \in L_{G_1}(X), N(v) \in L_{G_2}(X)$, therefore we have $N(u) \subseteq X$ and $N(v) \subseteq X$. Since we have $N(v) \in L_{G_1}(X), N(u) \in L_{G_2}(X)$ which is equal to $N(u) = N(v)$ i.e., $V_1(G_1) = V_2(G_2)$. Hence $L_{G_1}(X) = L_{G_2}(X)$. □

Proposition 4.3. *Let V_1 and V_2 be bigraph of G_1 and G_2 . If $V_1(G_1) \subseteq V_2(G_2)$ then $U_{G_1}(X) \subseteq U_{G_2}(X)$.*

5. APPLICATION OF GRAPH WITH REPRESENTATION OF MATRIX

We introduce the application of social networks induced by a graph of nano topological space. Using social network techniques, these data can be used to indicate characteristics of positions held in a network and characteristics of the network structure [5]. Here network theory provides a set of techniques for analyzing graph with nodes in the network and information exchange relationships as connectors between nodes.



FIGURE 3. Social Network

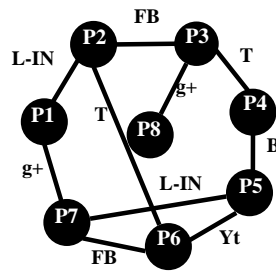


FIGURE 4. Social Network Graph

Let G be a graph of social network with vertex of persons are $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$ and edges of social links are L-In stands for Linked in, FB -stands for facebook, T-stands for twitter, B- stands for Blogger, yt -stands for youtube, g+-stands for google plus. Then the neighborhood of vertex are and the graph is

$\mathbf{v} \in \mathbf{V}$	$\mathbf{N}(v)$
$P1$	$\{P1, P2, P7\}$
$P2$	$\{P1, P2, P3, P6\}$
$P3$	$\{P2, P3, P4, P8\}$
$P4$	$\{P3, P4, P5\}$
$P5$	$\{P4, P5, P6, P7\}$
$P6$	$\{P2, P5, P6, P7\}$
$P7$	$\{P1, P5, P6, P7\}$
$P8$	$\{P3, P8\}$

TABLE 1. Neighborhood of the Vertices

represented by adjacency matrix as follows.

$$[a_{ij}] = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

For any set $X = \{P2, P3, P4, P5, P8\} \subseteq V(G)$. Then $L_G(X) = \{P3, P4, P8\}$ and $U_G(X) = \{V(G)\}$, $B_G(X) = \{P1, P2, P5, P6, P7\}$. Hence $\tau_G(X) = \{G, \emptyset, \{P3, P4, P8\}, \{P1, P2, P5, P6, P7\}\}$ induced the graph G .

Observation: Here we try and ensure whether the most credible used in people of linked in, google plus, Facebook, youtube, and twitter to find the boundary region induced by a graph in nano topology.

6. CONCLUSION

In this paper is to hybridize the graph and nano topology. We define the neighborhood of vertices in nano topological space. Like wise, the application of social network promote nano topological space induced by graph and adjacency matrix are discussed.

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