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NEUTROSOPHIC SET IN INK-ALGEBRA

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ABSTRACT. The notion of neutrosophic INK-Algebra, neutrosophic INK-filter, neutrosophic near INK-filter, neutrosophic ideal and neutrosophic INK-ideal of INK-algebra are introduced, and several properties are investigated. Condition for neutrosophic sets to be neutrosophic INK-filter, neutrosophic near INK-filter, neutrosophic ideal and neutrosophic INK-ideal of INK-algebra are provided. Relation between neutrosophic sub algebra and neutrosophic INK-ideal are considered.

1. INTRODUCTION

In 1965 Zadeh introduced the fuzzy set theory, then so many researchers applied fuzzy set in BCI/BCK-algebras. Also, Atanassov introduced the intuitionistic fuzzy set on the universal set X as generalization of fuzzy set in 1986. Kaviyarasu, Indhira and Chandrasekaran introduced a new algebraic structure called INK-algebra and also, they applied fuzzy set, intuitionistic fuzzy set, Translation and interval-valued concepts in INK-algebras, see [1–11].

In this paper, the notions of neutrosophic INK-subalgebras, neutrosophic near INK-filters, neutrosophic INK-filters, neutrosophic ideals, and neutrosophic INK-ideals of INK-algebras are introduced, and several properties are investigated.

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Conditions for neutrosophic sets to be neutrosophic INK-subalgebras, neutrosophic near INK-filters, neutrosophic INK-filters, neutrosophic ideals, and neutrosophic INK-ideals of INK-algebras are provided.

2. Preliminaries

Before we begin our study, we will give the definition and useful properties of INK-algebras.

Definition 2.1. *An algebra $(X, *, 0)$ is called a INK-algebra if you meet the ensuing conditions for every $x, y, z \in X$.*

INK-1: $((x * y) * (x * z)) * (z * y) = 0$.
INK-2: $((x * z) * (y * z)) * (x * y) = 0$.
INK-3: $x * 0 = x$.
INK-4: $x * y = 0$ *and* $y * x = 0$ *imply* $x = y$.

Definition 2.2. *A non-empty subset S of a INK-algebra $(X, *, 0)$ is said to be a subalgebra of X,if $x * y \in S$, whenever $x, y \in X$.*

Definition 2.3. *Let $(X, *, 0)$ be a INK-algebra. A nonempty subset I of X is called an ideal of X if it satisfies*

(i) $0 \in I$,
(ii) $x * y \in I$ *and* $y \in I$ *imply* $x \in I$ *for all* $x, y \in X$. *Any ideal I has the property that* $y \in I$ *and* $x \leq y$ *imply* $x \in I$.

Definition 2.4. *let I be a non-empty subset of a INK-algebra X. Then I is called a INK-ideal of X, if*

(i) $0 \in I$.
(ii) $((z * x) * (z * y)) \in I$ *and* $y \in I$ *imply* $x \in I$ *for all* $x, y, z \in X$.

Definition 2.5. *A nonempty subset S of a INK-algebra $(X, *, 0)$ is called a near INK-filter of X if*

(i) *The constant* 0 *of* X *is in* S,
(ii) $y \in S \Rightarrow x * y \in S$ *for all* $x, y \in X$.

Definition 2.6. *A nonempty subset S of a INK-algebra $(X, *, 0)$ is called a INK-filter of X if*

(i) *The constant* 0 *of* X *is in* S,

(ii) $x * y \in S, x \in S \Rightarrow y \in S$ *for all* $x, y \in X$.

3. Neutrosophic Set In INK-Algebra

In this section we applied neutrosophic set in INK-algebra.

Definition 3.1. *A neutrosophic set* $\wedge$ *in a nonempty set* X *is a structure of the form* $\wedge = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \, | x \in X\}$, *where* $\lambda_T : X \to [0,1]$, *is a truth membership function* $\lambda_I : X \to [0,1]$ *is a indeterminate membership function and* $\lambda_F : X \to [0,1]$ *is a false membership function.*

Definition 3.2. *A neutrosophic set* $\wedge$ *in* X *is called a neutrosophic INK-subalgebra of* X *if it satisfies the following condition, for all* $x, y, z \in X$

(i) $\lambda_T(x * y) \geq min\,\{\lambda_T(x), \lambda_T(y)\}$

(ii) $\lambda_I(x * y) \leq max\,\{\lambda_I(x), \lambda_I(y)\}$

(iii) $\lambda_F(x * y) \geq min\,\{\lambda_F(x), \lambda_F(y)\}$.

Definition 3.3. *A neutrosophic set* $\wedge$ *in* X *is called a neutrosophic near INK-filter of* X *if it satisfies the following condition, for all* $x, y \in X$.

(i) $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x),$ *and* $\lambda_F(0) \geq \lambda_F(x)$

(ii) $\lambda_T(x * y) \geq \lambda_T(x)$

(iii) $\lambda_I(x * y) \leq \lambda_I(x)$

(iv) $\lambda_F(x * y) \geq \lambda_F(x)$.

Definition 3.4. *A neutrosophic set* $\wedge$ *in* X *is called a neutrosophic INK-filter of* X *if it satisfies the following condition, for all* $x, y \in X$.

(i) $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x),$ *and* $\lambda_F(0) \geq \lambda_F(x)$.

(ii) $\lambda_T(y) \geq min\,\{\lambda_T(x * y), \lambda_T(x)\}$

(iii) $\lambda_I(y) \leq max\,\{\lambda_I(x * y), \lambda_I(x)\}$

(iv) $\lambda_F(y) \geq min\,\{\lambda_F(x * y), \lambda_F(x)\}$.

Definition 3.5. *A neutrosophic set* $\wedge$ *in* X *is called a neutrosophic ideal of* X *if it satisfies the following condition, for all* $x, y \in X$.

(i) $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x),$ *and* $\lambda_F(0) \geq \lambda_F(x)$

(ii) $\lambda_T(x) \geq min\,\{\lambda_T(x * y), \lambda_T(y)\}$

(iii) $\lambda_I(x) \leq max\,\{\lambda_I(x * y), \lambda_I(y)\}$

(iv) $\lambda_F(x) \geq min\{\lambda_F(x * y), \lambda_F(y)\}$.

Definition 3.6. *A neutrosophic set $\wedge$ in X is called a neutrosophic INK-ideal of X if it satisfies the following condition, for all $x, y \in X$.*

(i) $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$, *and* $\lambda_F(0) \geq \lambda_F(x)$
(ii) $\lambda_T(x) \geq min\{\lambda_T((z * x) * (z * y)), \lambda_T(y)\}$
(iii) $\lambda_I(x) \leq max\{\lambda_I((z * x) * (z * y)), \lambda_I(y)\}$
(iv) $\lambda_F(x) \geq min\{\lambda_F((z * x) * (z * y)), \lambda_F(y)\}$.

Example 1. *let $X = \{0, 1, a, b\}$ be a INK-algebra with a fixed element 0 and a binary operation $*$ defined by the following Cayley table*

$*$	0	1	a	b
0	0	0	a	a
1	1	0	a	a
a	a	a	0	0
b	b	a	1	0

We define a neutrosophic $\wedge$ in X as follows

Theorem 3.1. *Every neutrosophic INK-subalgebra of X satisfies the conditions $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$, and $\lambda_F(0) \geq \lambda_F(x)$*

Proof. Assume that $\wedge$ is neutrosophic INK-subalgebra of X. Then for all $x \in X$.
$\lambda_T(0) = \lambda_T(x * y) \geq min\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x)$
$\lambda_I(0) = \lambda_I(x * y) \leq max\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x)$
$\lambda_F(0) = \lambda_F(x * y) \geq min\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x)$. □

Theorem 3.2. *A neutrosophic set $\wedge$ in X is constant if and only if it is a neutrosophic INK-ideal of X.*

Proof. Assume that $\wedge$ is constant for all $x \in X$.
$\lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$. Next for all $x, y, z \in X$.
$\lambda_T(x) = \lambda_T(0) = min\{\lambda_T(0), \lambda_T(0)\} = min\{\lambda_T((z * x) * (z * y)), \lambda_T(y)\}$
$\lambda_I(x) = \lambda_I(0) = max\{\lambda_I(0), \lambda_I(0)\} = max\{\lambda_I((z * x) * (z * y)), \lambda_I(y)\}$
$\lambda_F(x) = \lambda_F(0) = min\{\lambda_F(0), \lambda_F(0)\} = min\{\lambda_F((z * x) * (z * y)), \lambda_F(y)\}$
Hence,$\wedge$ is a neutrosophic INK-ideal of X. conversely, assume that $\wedge$ is a neutrosophic INK-ideal of X. For any $x \in X$ we have,
$\lambda_T(x) \geq min\{\lambda_T((x * x) * (x * 0)), \lambda_T(0)\}$

$\geq min\{\lambda_T(0*x), \lambda_T(y)\} \geq min\{\lambda_T(0), \lambda_T(y)\} \geq \lambda_T(0)$,
$\lambda_I(x) \leq max\{\lambda_I((x*x)*(x*0)), \lambda_I(0)\}$
$\leq max\{\lambda_I(0*x), \lambda_I(y)\} \leq max\{\lambda_I(0), \lambda_I(y)\} \leq \lambda_I(0)$,
$\lambda_F(x) \geq min\{\lambda_F((x*x)*(x*0)), \lambda_F(0)\}$
$\geq min\{\lambda_F(0*x), \lambda_F(y)\} \geq min\{\lambda_F(0), \lambda_F(y)\} \geq \lambda_F(0)$. □

Theorem 3.3. *A neutrosophic set $\wedge$ in X is a neutrosophic INK-ideal if and only if it is a neutrosophic INK-ideal of X.*

Proof. Assume that $\wedge$ is neutrosophic INK-ideal for all X. The $\wedge$ is satisfies the condition $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$ and $\lambda_F(0) \geq \lambda_F(x)$ by the theorem 3.2 we we have $\wedge$ constant, then for all $x \in X$.
$\lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$, thus
$\lambda_T(x) \geq min\{\lambda_T((z*x)*(z*y)), \lambda_T(y)\}$
put $z = 0$ and $0*x = x$
$\geq min\{\lambda_T((0*x)*(0*y)), \lambda_T(y)\}$
$\geq min\{\lambda_T(x*y), \lambda_T(y)\}$,
$\lambda_I(x) \leq max\{\lambda_I((z*x)*(z*y)), \lambda_I(y)\}$
put $z = 0$ and $0*x = x$
$\leq max\{\lambda_I((0*x)*(0*y)), \lambda_I(y)\}$
$\leq max\{\lambda_I(x*y), \lambda_I(y)\}$,
$\lambda_F(x) \geq min\{\lambda_F((z*x)*(z*y)), \lambda_F(y)\}$
put $z = 0$ and $0*x = x$
$\geq min\{\lambda_F((0*x)*(0*y)), \lambda_F(y)\}$
$\geq min\{\lambda_F(x*y), \lambda_F(y)\}$.
Therefore $\wedge$ is a neutrosophic ideal of X .
Conversely, $\wedge$ is a neutrosophic INK-ideal of X . □

Theorem 3.4. *Every neutrosophic INK-ideal of X is a neutrosophic INK-filter, if $0*x = x$.*

Proof. Assume that $\wedge$ is neutrosophic INK-ideal of X . The $\wedge$ is satisfies the condition $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$ and $\lambda_F(0) \geq \lambda_F(x)$. Let $x \in X$.
$\lambda_T(y) \geq min\{\lambda_T((z*x)*(z*y)), \lambda_T(x)\}$
put $z = 0$ and $0*x = x$
$\geq min\{\lambda_T((0*x)*(0*y)), \lambda_T(x)\}$
$\geq min\{\lambda_T(x*y), \lambda_T(x)\}$,

$\lambda_I(y) \leq max\,\{\lambda_I((z*x)*(z*y)), \lambda_I(x)\}$
put $z = 0$ and $0*x = x$
$\leq max\,\{\lambda_I((0*x)*(0*y)), \lambda_I(x)\}$
$\leq max\,\{\lambda_I(x*y), \lambda_I(x)\}$,
$\lambda_F(y) \geq min\,\{\lambda_F((z*x)*(z*y)), \lambda_F(x)\}$
put $z = 0$ and $0*x = x$
$\geq min\,\{\lambda_F((0*x)*(0*y)), \lambda_F(x)\}$
$\geq min\,\{\lambda_F(x*y), \lambda_F(x)\}$.
Hence, $\wedge$ is a neutrosophic INK-filter of X . □

Theorem 3.5. *Every neutrosophic INK-filter of X is a neutrosophic near INK-filter, if $0*x = x$.*

Proof. Assume that $\wedge$ is neutrosophic INK-filter of X . The $\wedge$ is satisfies the condition $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$ and $\lambda_F(0) \geq \lambda_F(x)$. Let $x \in X$.
$\lambda_T(x*y) \geq min\,\{\lambda_T(y*(x*y)), \lambda_T(y)\}$
$= min\,\{\lambda_T(0), \lambda_T(y)\} = \lambda_T(y)$.
$\lambda_I(x*y) \leq max\,\{\lambda_I(y*(x*y)), \lambda_I(y)\}$
$= max\,\{\lambda_I(0), \lambda_I(y)\} = \lambda_I(y)$.
$\lambda_F(x*y) \geq min\,\{\lambda_F(y*(x*y)), \lambda_F(y)\}$
$= min\,\{\lambda_F(0), \lambda_F(y)\} = \lambda_F(y)$.
Hence, $\wedge$ is a neutrosophic near INK-filter of X . □

Theorem 3.6. *Every neutrosophic near INK-filter of X is a neutrosophic near INK-subalgebra.*

Proof. Assume that $\wedge$ is neutrosophic INK-filter of X.
$\lambda_T(x*y) \geq \lambda_T(y) \geq min\,\{\lambda_T(x), \lambda_T(y)\}$
$\lambda_I(x*y) \leq \lambda_I(y) \leq max\,\{\lambda_I(x), \lambda_I(y)\}$
$\lambda_F(x*y) \geq \lambda_F(y) \geq min\,\{\lambda_F(x), \lambda_F(y)\}$
Hence, $\wedge$ a neutrosophic near INK-subalgebra of X. □

Theorem 3.7. *If $\wedge$ is a neutrosophic INK-subalgebra of X satisfies the following condition*
$x*y \neq 0 \Rightarrow (\lambda_T(x) \geq \lambda_T(y), \lambda_I(x) \leq \lambda_I(y), \lambda_F(x) \geq \lambda_F(y))$.
Then $\wedge$ is a neutrosophic near INK-filter of X .

Proof. Assume that $\wedge$ is neutrosophic INK-subalgebra of X (3.7) satisfying the condition by the Theorem 3.2, we have $\wedge$ satisfies the condition $\lambda_T(0) \geq \lambda_T(x)$, $\lambda_I(0) \leq \lambda_I(x)$ and $\lambda_F(0) \geq \lambda_F(x)$. Let $x, y, z \in X$.

Case 1: $x * y = 0$. Then

$$\lambda_T(x * y) = \lambda_T(0) \geq \lambda_T(y),$$
$$\lambda_I(x * y) = \lambda_I(0) \leq \lambda_I(y),$$
$$\lambda_F(x * y) = \lambda_F(0) \geq \lambda_F(y).$$

Case 2: $x * y \neq 0$. Then

$$\lambda_T(x * y) \geq min\{\lambda_T(x), \lambda_T(y)\} = \lambda_T(y),$$
$$\lambda_I(x * y) \leq max\{\lambda_I(x), \lambda_T(y)\} = \lambda_I(y),$$
$$\lambda_F(x * y) \geq min\{\lambda_F(x), \lambda_T(y)\} = \lambda_F(y).$$

Then $\wedge$ is a neutrosophic near INK-filter of X .

□

Theorem 3.8. *If $\wedge$ is a neutrosophic near INK-filter of X satisfies the following condition $\lambda_T = \lambda_I = \lambda_F$. Then $\wedge$ is a neutrosophic near INK-filter of X.*

Proof. Assume that $\wedge$ is neutrosophic near INK-filter of X satisfies the following condition $\lambda_T = \lambda_I = \lambda_F$. Then $\wedge$ satisfies the condition $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$ and $\lambda_F(0) \geq \lambda_F(x)$. Let $x, y \in X$. Then

$$min\{\lambda_T(x * y), \lambda_T(x)\} \geq min\{\lambda_T(y), \lambda_T(x)\}$$
$$= min\{\lambda_T(y), \lambda_T(x)\} \leq \lambda_T(y),$$
$$max\{\lambda_I(x * y), \lambda_I(x)\} \leq max\{\lambda_I(y), \lambda_I(x)\}$$
$$= max\{\lambda_I(y), \lambda_I(x)\} \leq \lambda_I(y),$$
$$min\{\lambda_F(x * y), \lambda_F(x)\} \geq min\{\lambda_F(y), \lambda_F(x)\}$$
$$= min\{\lambda_F(y), \lambda_F(x)\} \leq \lambda_F(y),$$

Hence, $\wedge$ is a neutrosophic near INK-filter of X. □

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