ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.7, 4787–4795
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.7.45

$^*G\hat{\alpha}$-HOMEOMORPHISM IN TOPOLOGICAL SPACES

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ABSTRACT. In this article, we introduce a special type of generalised closed and open maps, briefly $^*g\hat{\alpha}$-closed maps, $^*g\hat{\alpha}$-open maps in topological spaces also analyze some important characterizations of these new type of maps and then we study $^*g\hat{\alpha}$-homeomorphisms. We also obtain strongly $^*g\hat{\alpha}$-homeomorphisms and proved that under the operation $\circ$ the collection of all strongly $^*g\hat{\alpha}$- homeomorphisms form a group.

1. INTRODUCTION

In topological space, R. Malghan [10] introduced and investigated the notion of generalised-closed maps. In 1994, R. Devi [5] introduced the notions of semi-generalized-closed maps and generalized semi-closed maps. In topological space, many authors have been introduced different types of generalized homeomorphisms. In 1972, Crossely and Hildebrand [2] have analyzed semi-homeomorphisms. In 1991, Maki et al [9] have characterized the notion of generalized-homeomorphisms. In our present study,we introduce a special type of generalised closed and open maps, briefly $^*g\hat{\alpha}$-closed maps, $^*g\hat{\alpha}$-open maps in topological spaces also analyze some important characterizations of these new type of maps and then we study $^*g\hat{\alpha}$-homeomorphisms. We also obtain strongly

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2010 *Mathematics Subject Classification.* 54A05, 54D01, 54F65, 54G05.

Key words and phrases. $^*g\hat{\alpha}$-closed maps, $*g\hat{\alpha}$ - open map, $^*g\hat{\alpha}$-homeomorphisms, strongly $^*g\hat{\alpha}$-homeomorphisms.

$^*g\hat{\alpha}$-homeomorphisms and proved that under the operation $\circ$ the collection of all strongly $^*g\hat{\alpha}$-homeomorphisms form a group.

2. Preliminaries

In this section, we give existing definitions and some important results.

Definition 2.1. *Consider a mapping $f:(X,\tau)\to(Y,\sigma)$. Then f is called*

(1) *generalized-continuous [8] if $f^{-1}(A)\in gC(X,\tau)$ for each $A\in\sigma^c$.*
(2) *α-generalized-continuous [4] if $f^{-1}(A)\in \alpha gC(X,\tau)$ for each $A\in\sigma^c$.*
(3) *generalized semi-continuous [3] if $f^{-1}(A)\in gsC(X,\tau)$ for each $A\in\sigma^c$.*
(4) *generalized*-continuous [14]if $f^{-1}(A)\in g^*C(X,\tau)$ for each $A\in\sigma^c$.*
(5) **generalizedα-continuous [11] if $f^{-1}(A)\in {}^*g\alpha C(X,\tau)$ for each $A\in\sigma^c$.*
(6) *generalized pre-continuous [1] if $f^{-1}(A)\in gpC(X,\tau)$ for each $A\in\sigma^c$.*
(7) *generalized semipre-continuous [6] if $f^{-1}(A)\in gspC(X,\tau)$ $\forall A\in\sigma^c$.*
(8) *generalized pre regular-continuous [7] if $f^{-1}(A)\in gprC(X,\tau)$ $\forall A\in\sigma^c$.*
(9) *generalized $alpha$-continuous [4] if $f^{-1}(A)\in g\alpha C(X,\tau)$ for each $A\in\sigma^c$.*

The collection of all generalized-closed sets, αgeneralized-closed sets, generalized semi-closed sets, generalized*-closed sets, *generalizedα-closed sets, generalized pre-closed sets, generalized semi pre-closed sets, generalized pre regular-closed sets and generalizedα-closed sets denoted by $gC(X,\tau)$, $\alpha gC(X,\tau)$, $gsC(X,\tau)$, $g^*C(X,\tau)$, $^*g\alpha C(X,\tau)$, $gpC(X,\tau)$, $gspC(X,\tau)$, $gprC(X,\tau)$ and $g\alpha C(X,\tau)$ respectively.

Definition 2.2. *[12] If A is $^*g\hat{\alpha}$-closed set, then $cl(A)\subseteq O$ for every $^*g\alpha$-open set O which contains A.*

Definition 2.3. *[13] Consider a mapping $f:(X,\tau)\to(Y,\sigma)$. Then a $^*g\hat{\alpha}$-continuous mapping defined by $f^{-1}(A)\in {}^*g\hat{\alpha}C(X,\tau)$ for each $A\in\sigma^c$.*

Definition 2.4. *[13] Consider a mapping $f:(X,\tau)\to(Y,\sigma)$. Then a $^*g\hat{\alpha}$-irresolute mapping defined by $f^{-1}(A)\in {}^*g\hat{\alpha}C(X,\tau)$ for each $A\in {}^*g\hat{\alpha}C(Y,\sigma)$.*

Theorem 2.1. *[13] Assume that $V\subseteq B\subseteq$. And V is a $^*g\hat{\alpha}$-closed set relative to $B\in\tau$ and $B\in {}^*g\hat{\alpha}C(X,\tau)$. Then, we have the set $V\in {}^*g\hat{\alpha}C(X,\tau)$.*

Proposition 2.1. *[13] A $^*g\hat{\alpha}$-irresolute mapping $f:(X,\tau)\to(Y,\sigma)$ implies $^*g\hat{\alpha}$-continuous mapping.*

3. $*g\hat{\alpha}$-CLOSED MAPS AND OPEN MAPS

Definition 3.1. *Consider a mapping $f : (X,\tau) \to (Y,\sigma)$. Then a $^*g\hat{\alpha}$-closed map defined by $f(B)$ is $^*g\hat{\alpha}C(Y,\sigma)$ whenever B is a closed set in (X,τ), where $^*g\hat{\alpha}C(X,\tau)$ denotes the collection of all $^*g\hat{\alpha}$-closed sets and $^*g\hat{\alpha}O(X,\tau)$ denotes collection of all $^*g\hat{\alpha}$-open sets.*

Proposition 3.1. *Consider a mapping $f : (X,\tau) \to (Y,\sigma)$. Then f is $^*g\hat{\alpha}$-closed if and only if $^*g\hat{\alpha} - cl(f(B)) \subseteq f(cl(B))$ for each set B in (X,τ).*

Proof. Assume that f is $^*g\hat{\alpha}$-closed, $B \subseteq X$. Then we have $f(cl(B)) \in {}^*g\hat{\alpha}C(Y,\sigma)$. We have $f(B) \subseteq f(cl(B))$ and $^*g\hat{\alpha}-cl(f(B)) \subseteq {}^*g\hat{\alpha}-cl(f(cl(B))) = f(cl(B))$.
Conversely, choose $B \in \tau^c$. Then $B = cl(B)$, from the statement of the proposition, $f(B) = f(cl(B)) \supseteq {}^*g\hat{\alpha} - cl(f(B))$. We have $f(B) \subseteq {}^*g\hat{\alpha} - cl(f(B))$. Therefore $f(B) =^* g\hat{\alpha} - cl(f(B))$. That is, $f(B) \in {}^*g\hat{\alpha}C(Y,\sigma)$. Therefore, f is $^*g\hat{\alpha}$-closed. □

Now, we give a result for necessary and sufficient condition for $^*g\hat{\alpha}$-closed map

Theorem 3.1. *Let a map $f : (X,\tau) \to (Y,\sigma)$ and f is a $^*g\hat{\alpha}$-closed iff for every $A \in Y$ and for every $f^{-1}(A) \subseteq U \in \tau$, $\exists\ B \in {}^*g\hat{\alpha}O(Y,\sigma)$ where $A \subseteq B$, $f^{-1}(B) \subseteq U$.*

Proof. Assume that f is a $^*g\hat{\alpha}$-closed map. Consider, $C \subseteq Y$, $U \subseteq X$, where $U \in \tau$ such that $f^{-1}(C) \subseteq U$. Now, we have $B = (f(C^c))^c \in {}^*g\hat{\alpha}O(Y,\sigma)$ containing C such that $f^{-1}(B) \subseteq U$.
Conversely, consider $C \in \tau^c$. Then $f^{-1}(f(C)^c) \subseteq C^c$ and $C^c \in \tau$. From the statement of the theorem, $\exists\ B \in {}^*g\hat{\alpha}O(Y,\sigma)$ where $(f(C))^c \subseteq B$, $f^{-1}(B) \subseteq C^c$ and so $C \subseteq (f^{-1}(B))^c$. Hence $B^c \subseteq f(C) \subseteq f((f^{-1}(B))^c) \subseteq B^c$ that implies $f(C) = B^c$. Thus f is $^*g\hat{\alpha}$-closed, since B^c is $^*g\hat{\alpha}$-closed, $f(C)$ is $^*g\hat{\alpha}$-closed. □

Proposition 3.2. *Let $f : (X,\tau) \to (Y,\sigma)$ be $^*g\alpha$-irresolute and $^*g\hat{\alpha}$-closed map and $A \in {}^*g\hat{\alpha}C(X,\tau)$. Then $f(A) \in {}^*g\hat{\alpha}C(Y,\sigma)$.*

Proof. Consider, $U \in {}^*g\hat{\alpha}O(Y,\sigma)$ such that $f(A) \subseteq U$. Now, $f^{-1}(U) \in {}^*g\alpha O(X,\tau)$ containing A, Since f is $^*g\alpha$- irresolute map. Since $A \in {}^*g\alpha C(X,\tau)$, $cl(A) \subseteq f^{-1}(U)$. From the statement of the theorem, f is a $^*g\hat{\alpha}$-closed map, $f(cl(A)) \in$

${}^*g\hat{\alpha}C(Y,\sigma)$, $cl(f(cl(A))) \subseteq U$ which implies that $cl(f(A)) \subseteq U$. Therefore $f(A) \in {}^*g\hat{\alpha}C(Y,\sigma)$. □

Remark 3.1. *Consider the maps $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$. Then $f \circ g$ need not be a ${}^*g\hat{\alpha}$-closed map where the maps f and g are ${}^*g\hat{\alpha}$-closed.*

Corollary 3.1. *Consider the maps $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$. Then $g \circ f$ is ${}^*g\hat{\alpha}$-closed where f is ${}^*g\hat{\alpha}$-closed, g is ${}^*g\hat{\alpha}$-closed and ${}^*g\alpha$-irresolute.*

Proof. Let $V \in \tau^c$. From the statement of the theorem, $f(V) \in {}^*g\hat{\alpha}C(Y,\sigma)$. Now we have $g \circ f$ is ${}^*g\hat{\alpha}$-closed map, because the mapping g is both ${}^*g\hat{\alpha}$-closed and ${}^*g\alpha$-irresolute and by the previous Proposition. □

Proposition 3.3. *Consider the maps $f:(X,\tau) \to (Y,\sigma)$, $g:(Y,\sigma) \to (Z,\eta)$. The composite map $g \circ f$ is a ${}^*g\hat{\alpha}$-closed map when f is closed, g is ${}^*g\hat{\alpha}$-closed.*

Proof. Consider $A \in \tau^c$. Then by hypothesis, $f(A) \in \sigma^c$. Now, we get $g(f(A)) = (g \circ f)(A) \in {}^*g\hat{\alpha}C(Z,\eta)$, since g is ${}^*g\hat{\alpha}$-closed. Therefore the composition $g \circ f$ is ${}^*g\hat{\alpha}$-closed. □

Remark 3.2. *Consider a ${}^*g\hat{\alpha}$-closed map $f:(X,\tau) \to (Y,\sigma)$, closed map $g:(Y,\sigma) \to (Z,\eta)$. Then $g \circ f$ is not ${}^*g\hat{\alpha}$-closed in general.*

Theorem 3.2. *Consider the maps $f:(X,\tau) \to (Y,\sigma)$, $g:(Y,\sigma) \to (Z,\eta)$ such that $g \circ f$ is ${}^*g\hat{\alpha}$-closed. Then we get the bellow statements.*

(1) *The map g is ${}^*g\hat{\alpha}$-closed, if the surjective map f is continuous.*
(2) *The map f is ${}^*g\hat{\alpha}$-closed, if the injective map g is ${}^*g\hat{\alpha}$-irresolute, then*
(3) *g is ${}^*g\hat{\alpha}$-closed, if (X,τ) is $T_{1/2}$ and the surjective map f is generalized continuous.*
(4) *The map f is closed, if the injective map g is strongly ${}^*g\hat{\alpha}$-continuous.*

Proof. (1) consider the set $V \in \sigma^c$. Now, $f^{-1}(V) \in \tau^c$, since f is continuous. Then we have $(g \circ f)(f^{-1}(V)) \in {}^*g\hat{\alpha}C(Z,\eta)$, because $g \circ f$ is ${}^*g\hat{\alpha}$-closed. Since f is surjective, $g(V) \in {}^*g\hat{\alpha}C(Z,\eta)$. Therefore g is ${}^*g\hat{\alpha}$-closed.
(2) consider the set $A \in \tau^c$. Now, $(g \circ f)(A) \in {}^*g\hat{\alpha}C(Z,\eta)$, because $g \circ f$ is ${}^*g\hat{\alpha}$-closed. Also we have $g^{-1}((g \circ f)(A) \in {}^*g\hat{\alpha}C(Y,\sigma)$, because g is ${}^*g\hat{\alpha}$-irresolute. Since g is injective, $f(A) \in {}^*g\hat{\alpha}C(Y,\sigma)$. Therefore f is a ${}^*g\hat{\alpha}$-closed map.

(3) Consider the set $B \in \sigma^c$. Now, $f^{-1}(B) \in {}^*g\hat{\alpha}C(X,\tau)$, Since f is a generalized-continuous map. Also we get $f^{-1}(B) \in {}^*g\hat{\alpha}C(X,\tau)$ and by hypothesis (i), g is a $^*g\hat{\alpha}$-closed map, because X is a $T_{1/2}$ space.
(4) Consider the set $C \in \tau^c$. Noe, $(g \circ f)(C)\ {}^*g\hat{\alpha}C(Z,\eta)$, because $g \circ f$ is a $^*g\hat{\alpha}$-closed map. Also we get $g^{-1}((g \circ f)(C) \in {}^*g\hat{\alpha}C(Y,\sigma)$, since g is a strongly $^*g\hat{\alpha}$-continuous map. Since g is injective, $f(C) \in \sigma^c$. Therefore the map f is closed.

□

Definition 3.2. *Consider, a map $f : (X,\tau) \to (Y,\sigma)$. A $^*g\hat{\alpha}$-open map f is defined by $f(U) \in {}^*g\hat{\alpha}O(Y,\sigma)$ for every $U \in \tau$.*

Theorem 3.3. *Consider a map $f : (X,\tau) \to (Y,\sigma)$. Then f is $^*g\hat{\alpha}$-open iff for any set A of (Y,σ), $K \in \tau^c$ and $f^{-1}(A) \subset K$, there $B \in {}^*g\hat{\alpha}C(Y,\sigma)$ containing the set A such that $K \supseteq f^{-1}(B)$.*

Proof. The proof is same as proof of Theorem 3.1. □

Corollary 3.2. *Let a map $f : (X,\tau) \to (Y,\sigma)$. Then f is $^*g\hat{\alpha}$-open iff $f^{-1}({}^*g\hat{\alpha} - cl(A)) \subseteq cl(f^{-1}(A))$ for each $A \in Y$.*

Proof. Assume that f is $^*g\hat{\alpha}$-open. Then, we have for a subset A of Y, $f^{-1}(A) \subseteq cl(f^{-1}(A))$. By the above theorem, there exists $C \in {}^*g\hat{\alpha}C(Y,\sigma)$ such that $A \subseteq C$, $f^{-1}(C) \subseteq cl(f^{-1}(A))$. Now, $f^{-1}({}^*g\hat{\alpha} - cl(A)) \subseteq f^{-1}(C) \subseteq cl(f^{-1}(A))$, because $C \in {}^*g\hat{\alpha}C(y,\sigma)$.
Conversely, Consider B be a subset of (Y,σ) and $D \in \tau^c$ and $f^{-1}(B) \subseteq D$. Which implies $C =^* g\hat{\alpha} - cl(B)$. Then, we have $C \in {}^*g\hat{\alpha}C(X,\tau)$, $B \subseteq C$. From the statement of the theorem, $f^{-1}(C) = f^{-1}({}^*g\hat{\alpha} - cl(B)) \subseteq cl(f^{-1}(B)) = D$. Hence by the previous theorem, f is $^*g\hat{\alpha}$-open. □

Definition 3.3. *Consider a mapping $f : (X,\tau) \to (Y,\sigma)$. A strongly $^*g\hat{\alpha}$-closed map is defined by for each $A \in {}^*g\hat{\alpha}C(X,\tau)$, $f(A) \in {}^*g\hat{\alpha}C(Y,\sigma)$.*

4. $^*g\hat{\alpha}$-HOMEOMORPHISM

Definition 4.1. *Consider a bijection $f : (X,\tau) \to (Y,\sigma)$. A $^*g\hat{\alpha}$-homeomorphism f is defined by f is both $^*g\hat{\alpha}$-open map, $^*g\hat{\alpha}$-continuous map.*

Example 1. *Consider the Topological spaces* (X,τ) *and* (Y,σ)*, where* $X=\{a,b,c\}=Y$*,* $\tau=\{\emptyset,\{a\},\{b,c\},X\}$ *and* $\sigma=\{\emptyset,\{a\},\{b\},\{a,b\},\{b,c\},Y\}$*. Let the bijection* $f:(X,\tau)\to(Y,\sigma)$ *be identity map. Now,* f *is both* $^*g\hat{\alpha}$*-open,* $^*g\hat{\alpha}$*-continuous. Therefore, the bijective mapping* f *is* $^*g\hat{\alpha}$*-homeomorphism.*

Proposition 4.1. *Homeomorphism implies* $^*g\hat{\alpha}$*-homeomorphism .*

The proof is obvious from the consequences of the notions of homeomorphism and $^*g\hat{\alpha}$*-homeomorphism.*

Converse of the above proposition is not true in general. Consider the mapping f defined in the Example 1. Because f is not continuous, we have f is a $^*g\hat{\alpha}$-homeomorphism but not a homeomorphism.

Proposition 4.2. $^*g\hat{\alpha}$*-homeomorphism implies* g*-homeomorphism.*

Proof. The proposition follows by the following results: both $^*g\hat{\alpha}$-implies g-continuous, $^*g\hat{\alpha}$-open map implies generalized open map. □

Converse of the above proposition is not true in general.

Example 2. *Consider the Topological spaces* (X,τ) *and* (Y,σ)*, where* $X=\{p,q,r\}=Y$*,* $\tau=\{\emptyset,\{p\},X\}$ *and* $\sigma=\{\emptyset,\{q\},Y\}$*. Define* $f:(X,\tau)\to(Y,\sigma)$ *by* $f(p)=r$*,* $f(q)=p$ *and* $f(r)=q$*. Then, we have* f *is not a* $^*g\hat{\alpha}$*-homeomorphism but it is a* g*-homeomorphism.*

Proposition 4.3. *Consider a bijection* $f:(X,\tau)\to(Y,\sigma)$ *such that* f *is* $^*g\hat{\alpha}$*-continuous. Then the below results are equal:*

(1) *The bijection* f *is* $^*g\hat{\alpha}$*-open.*
(2) *The bijection* f *is* $^*g\hat{\alpha}$*-homeomorphism.*
(3) *The bijection* f *is* $^*g\hat{\alpha}$*-closed.*

Proof. From the Proposition 4.2, the proof follows. □

Remark 4.1. *Consider two* $^*g\hat{\alpha}$*-homeomorphic maps* f *and* g*. Then,* $f\circ g$ *need not be a* $^*g\hat{\alpha}$*-homeomorphism.*

From the below example, the above remark follows.

Example 3. *Let the topological spaces,* (X,τ)*,* (Y,σ) *and* (Z,η)*, where* $X=Y=Z=\{a,b,c\}$*,* $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},X\}$*,* $\sigma=\{\emptyset,\{a,b\},Y\}$ *and* $\eta=$

$\{\emptyset, \{a\}, \{a,b\}, Z\}$. *Now, define mappings* $f : (X,\tau) \to (Y,\sigma)$, $g : (Y,\sigma) \to (Z,\eta)$ *be identity. Then we have* $g \circ f$ *is not a* $^*g\hat{\alpha}$*-homeomorphism and* f *and* g *are* $^*g\hat{\alpha}$*-homeomorphism, since the set* $\{b\} \in \tau$, $(g \circ f)(\{b\}) = \{b\} \notin {}^*g\hat{\alpha}O(Z,\eta)$.

Definition 4.2. *Consider a bijective mapping* $f : (X,\tau) \to (Y,\sigma)$. *A strongly* $^*g\hat{\alpha}$*-homeomorphism* f *is defined by both* f *and its inverse map* f^{-1} *are* $^*g\hat{\alpha}$*-irresolute.*

For our convenience, we give the following notions

(1) the collection of all $^*g\hat{\alpha}$-homeomorphism of (X,τ) onto (X,τ) by $^*g\hat{\alpha}$-$h(X,\tau)$,
(2) the collection of all strongly $^*g\hat{\alpha}$-homeomorphism of (X,τ) onto (X,τ) by s-$^*g\hat{\alpha}$-$h(X,\tau)$.

Proposition 4.4. *Strongly* $^*g\hat{\alpha}$*-homeomorphism implies* $^*g\hat{\alpha}$*-homeomorphism.*

Proof. From the Proposition 2.1 the proof follows. □

The converse part of the above proposition is not true. we can justify from the following example. Let the map g which is defined in Example 3 is not strongly $^*g\hat{\alpha}$-homeomorphism and it is $^*g\hat{\alpha}$-homeomorphism, because $\{a,c\} \in {}^*g\hat{\alpha}C(Y,\sigma)$, $(g^{-1})^{-1}(\{a,c\}) = g(\{a,c\}) = \{a,c\} \notin {}^*g\hat{\alpha}C(Z,\eta)$. Hence g is not a strongly $^*g\hat{\alpha}$-homeomorphism.

Proposition 4.5. *Strongly* $^*g\hat{\alpha}$*-homeomorphism implies generalized-homeomorphism.*

Proof. From Propositions 4.2 and 4.4, the proof follows. □

The converse part of the above proposition is not true. we can justify from the following example. Let the map f defined in Example 2. Then f is not strongly $^*g\hat{\alpha}$-homeomorphism but it is generalized-homeomorphism.

Proposition 4.6. *Consider two are strongly* $^*g\hat{\alpha}$*-homeomorphisms:* $f : (X,\tau) \to (Y,\sigma)$, $g : (Y,\sigma) \to (Z,\eta)$ *, then* $g \circ f$ *is a strongly* $^*g\hat{\alpha}$*-homeomorphism.*

Proof. Consider $V \in {}^*g\hat{\alpha}O(Z,\eta)$. Then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(U)$, $U = g^{-1}(V)$. From the assumption, $U \in {}^*g\hat{\alpha}O(Y,\sigma)$ also we have $f^{-1}(U) \in {}^*g\hat{\alpha}O(X,\tau)$. Hence $g \circ f$ is a $^*g\hat{\alpha}$-irresolute mapping. Consider $K \in {}^*g\hat{\alpha}O(X,\tau)$. Then we get $(g \circ f)(K) = g(f(K)) = g(F)$, $F = g(K)$. From the assumption $f(K) \in {}^*g\hat{\alpha}O(Y,\sigma)$ and by hypothesis, $g(f(K)) \in {}^*g\hat{\alpha}O(Z,\eta)$. i.e., $(g \circ f)(K) \in {}^*g\hat{\alpha}O(Z,\eta)$. Thus $(g \circ f)^{-1}$ is a $^*g\hat{\alpha}$-irresolute mapping. Hence composition of g and f is strongly $^*g\hat{\alpha}$-homeomorphism. □

Theorem 4.1. *Under composition of maps, the collection s-$^*g\hat{\alpha}$-$h(X,\tau)$ form a group.*

Proof. A binary operation $*$ defined by $g * h = h \circ g$ for each $g, h \in s$-$^*g\hat{\alpha}$-$h(X,\tau)$, where $*$:s-$^*g\hat{\alpha}$-$h(X,\tau) \times s$-$^*g\hat{\alpha}$-$h(X,\tau) \to s$-$^*g\hat{\alpha}$-$h(X,\tau)$, and $\circ$ is composition of mappings. Now, $h \circ g \in s$-$^*g\hat{\alpha}$-$h(X,\tau)$, since by Proposition 4.6. Consider the identity map $I : (X,\tau) \to (X,\tau)$. Then $I \in s$-$^*g\hat{\alpha}$-$h(X,\tau)$ is the identity element, since the operation $\circ$ is associative. If $g \in s$-$^*g\hat{\alpha}$-$h(X,\tau)$, then we have $g^{-1} \in s$-$^*g\hat{\alpha}$-$h(X,\tau)$ where $g \circ g^{-1} = g^{-1} \circ g = I$. Therefore, under the operation $\circ$ the collection $(s$-$^*g\hat{\alpha}$-$h(X,\tau), \circ)$ form a group. ☐

Theorem 4.2. *Consider set of all topological space. Then the collection s-$^*g\hat{\alpha}$-homeomorphism satisfies the three conditions of equivalence relation.*

Proof. From Proposition 4.6, the proof follows. ☐

Theorem 4.3. *Consider a map $f : (X,\tau) \to (Y,\sigma) \in s$-$^*g\hat{\alpha}$-homeomorphism, then $^*g\hat{\alpha}$-$cl(f^1(A)) = f^{-1}(^*g\hat{\alpha}$-$cl(A)$ for every $A \subseteq Y$.*

Proof. The map f is $^*g\hat{\alpha}$-irresolute, since f is a s-$^*g\hat{\alpha}$-homeomorphism. Also we have $f^{-1}(^*g\hat{\alpha}) \in {}^*g\hat{\alpha}C(X,\tau)$, because $^*g\hat{\alpha}$-$cl(f(A)) \in {}^*g\hat{\alpha}C(Y,\sigma)$. Also, $f^{-1}(A) \subseteq f^{-1}(^*g\hat{\alpha})$-$cl(A)$ and $^*g\hat{\alpha}$-$cl(f^{-1}(A)) \subseteq f^{-1}(^*g\hat{\alpha}$-$cl(A))$. Now, f^{-1} is $^*g\hat{\alpha}$-irresolute, because $f \in s$-$^*g\hat{\alpha}$-$h(X,\tau)$. Again we have $(f^{-1})^{-1}(^*g\hat{\alpha}$-$cl(f^1(A)) = f(^*g\hat{\alpha}$-$cl(f^{-1}(A)) \in {}^*g\hat{\alpha}C(Y,\sigma)$, because $^*g\hat{\alpha}$-$cl(f^{-1}(A)) \in {}^*g\hat{\alpha}C(X,\tau)$. Now, $A \subseteq (f^{-1})^{-1}(f^{-1}(A)) \subseteq (f^{-1})^{-1}(^*g\hat{\alpha}$-$cl(f^{-1}(A)) = f(^*g\hat{\alpha}$-$cl(f^{-1}(A))$ and so $^*g\hat{\alpha}$-$cl(A) \subseteq f(^*g\hat{\alpha}$-$cl(f^{-1}(A))$. Thus $f^{-1}(^*g\hat{\alpha}$-$cl(A)) \subseteq f^{-1}(f(^*g\hat{\alpha}$-$cl(f^{-1}(A)))) \subseteq^* g\hat{\alpha}$-$cl(f^{-1}(A))$. Therefore, the equality holds. ☐

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