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FIXED POINT THEOREM FOR SOME GENERALIZED CONTRACTION IN b-METRIC SPACE

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ABSTRACT. The aim of this paper is to obtain uniqueness and completeness of fixed point theorem on b-metric space. In this paper, we show that different rational contractive type maps exist in b-metric space and prove some fixed point theorem for these types of maps in b-metric space.

1. INTRODUCTION

In 1989, Bakhtin [3] introduced the notion of b-metric space and Czerwik [9] introduced the notion of b metric space with coefficient 2. This notion was generalized later with coefficient $s \geq 1$ in [10]. Since then many authors obtained fixed point theorem in b-metric space [1] [2] [4] [5] [6] [7] [8]. Kir [11] in 2013 extended Kannan, Chatterjea's contraction mapping on b-metric space, After that in 2014 Mishra [12] obtained Reich and Hardy Rogers contraction mapping in b-metric space. In this paper, we generalized some well-known fixed point theorems in b metric space.

2. PRELIMINARIES

We recall some definitions and properties for b-metric spaces.

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Definition 2.1. *[10] Let X be a non-empty set and $s \in [1, \infty)$,then a mapping $D : X \times X \to R^+$ is called b-metric space if the following conditions are satisfied:*

(i) $D(x, y) = 0$ *if and only if* $x = y$*;*
(ii) $D(x, y) = D(y, x)$*;*
(iii) $D(x, z) \leq s[D(x, y) + D(y, z)]$ *for all* $x, y, z \in X$*.*

The pair (X, D, s) is called a b-metric space.

We note that if $s = 1$, then every b-metric space is reduced into usual metric space.

Example 1. *[4] Let $X = 0, 1, 2$ and $D(0, 2) = D(2, 0) = m \geq 2, D(1, 0) = D(0, 1) = D(1, 2) = D(2, 1) = 1$ and$D(0, 0) = D(1, 1) = D(2, 2) = 0$, then $D(x, y) \leq \frac{m}{2}[D(x, z) + D(z, y)]$ for all $x, y, z \in X$. If $m > 2$ then the triangle inequality does not hold.*

Example 2. *Let $X = R$ and $D(x, y) = |x - y|^2$ for all $x, y \in X$, then (X, D, s) is a b-metric space with coefficient $s = 2$.*

Example 3. *[6] Let $0 < p \leq 1$, $l_p = \big\{\{x_n\} : x_n \in R, n \in N \sum_{n=1}^{\infty} |x_n|^p < \infty\big\}$ and $D(x, y) = \big(\sum_{n=1}^{\infty} |x_n - y_n|^p\big)^{\frac{1}{p}}$for all $x = \{x_n\}, y = \{y_n\} \in l_p$. Then D is a b-metric with coefficient $s = 2^{1/p}$.*

Definition 2.2. *[4] Let (X,D,s) be a b-metric space. Then a sequence $\{x_n\} \subset X$ is called a Cauchy sequence if and only if for all $\epsilon > 0, \exists n(\epsilon) \in N$ such that for each $n, m \geq n(\epsilon)$, we have $D(x_n, x_m) < \epsilon$.*

Definition 2.3. *[4] Let (X, D, s) be a b-metric space. Then a sequence $\{x_n\} \subset X$ is called convergent sequence if for all $\epsilon > 0$ and $n \geq N$ we have $D(x_n, x) < \epsilon$. In this case, we write $\lim_{n\to\infty} x_n = x$.*

Definition 2.4. *[4] A b-metric space (X, D, s) is said to be complete if every Cauchy sequence in X converges to a point of X.*

3. Main Result

Now we present our main theorem.

Theorem 3.1. *Let (X, D, s) be a complete b-metric space with coefficient $s \geq 1$ and T be a self mapping $T : X \to X$ such that*

$$\begin{aligned} D(Tx,Ty) &\leq a_1 D(x,Tx) + a_2 D(y,Ty) + a_3 D(x,Ty) + a_4 D(y,Tx) \\ &\quad + a_5 \frac{D(x,Tx)D(x,Ty) + D(y,Ty)D(y,Tx)}{D(x,Ty) + D(y,Tx)} \end{aligned}$$

for all $x, y \in X$, a_1, a_2, a_3, a_4, a_5 are non-negative numbers and $D(x,Ty)+D(y,Tx) \neq 0$ with condition $(a_1 + a_3 + a_5)s + a_2 + a_3 s^2 < 1$. Then T has a unique fixed point.

Proof. Let $x_0 \in X$ be arbitrary and $\{x_n\}_{n=1}^{\infty}$ be a sequence in X, defined by the recursion $x_0, x_1 = Tx_o, x_2 = Tx_1, ..., x_{n+1} = Tx_n \; \forall n \in N$

$$\begin{aligned} D(x_n, x_{n+1}) &= D(Tx_{n-1}, Tx_n) \\ &\leq a_1 D(x_{n-1}, Tx_{n-1}) + a_2 D(x_n, Tx_n) + a_3 D(x_{n-1}, Tx_n) + a_4 D(x_n, Tx_{n-1}) \\ &\quad + a_5 \frac{D(x_{n-1}, Tx_{n-1}) D(x_{n-1}, Tx_n) + D(x_n, Tx_n) D(x_n, Tx_{n-1})}{D(x_{n-1}, Tx_n) + D(x_n, Tx_{n-1})} \\ &\leq a_1 D(x_{n-1}, x_n) + a_2 D(x_n, x_{n+1}) + a_3 D(x_{n-1}, x_{n+1}) + a_4 D(x_n, x_n) \\ &\quad + a_5 \frac{D(x_{n-1}, x_n) D(x_{n-1}, x_{n+1}) + D(x_n, x_{n+1}) D(x_n, x_n)}{D(x_{n-1}, x_{n+1}) + D(x_n, x_n)} \\ &\leq a_1 D(x_{n-1}, x_n) + a_2 D(x_n, x_{n+1}) + a_3 s D(x_{n-1}, x_n) + a_3 s D(x_n, x_{n+1}) \\ &\quad + a_5 D(x_{n-1}, x_n) \\ D(x_n, x_{n+1}) &\leq \left(\frac{a_1 + a_3 s + a_5}{1 - a_2 - a_3 s} \right) D(x_{n-1}, x_n) \\ D(x_n, x_{n+1}) &\leq \lambda D(x_{n-1}, x_n)\,, \end{aligned}$$

where

$$\lambda = \left(\frac{a_1 + a_3 s + a_5}{1 - a_2 - a_3 s} \right).$$

Similarly

$$\begin{aligned}
D(x_{n-1},x_n) &= D(Tx_{n-2},Tx_{n-1})\\
&\leq a_1D(x_{n-2},Tx_{n-2})+a_2D(x_{n-1},Tx_{n-1})+a_3D(x_{n-2},Tx_{n-1})+a_4D(x_{n-1},Tx_{n-2})\\
&\quad +a_5\frac{D(x_{n-2},Tx_{n-2})D(x_{n-2},Tx_{n-1})+D(x_{n-1},Tx_{n-1})D(x_{n-1},Tx_{n-2})}{D(x_{n-2},Tx_{n-1})+D(x_{n-1},Tx_{n-2})}\\
&\leq a_1D(x_{n-2},x_{n-1})+a_2D(x_{n-1},x_n)+a_3D(x_{n-2},x_n)+a_4D(x_{n-1},x_{n-1})\\
&\quad +a_5\frac{D(x_{n-2},x_{n-1})D(x_{n-2},x_n)+D(x_{n-1},x_n)D(x_{n-1},x_{n-1})}{D(x_{n-2},x_n)+D(x_{n-1},x_{n-1})}\\
&\leq a_1D(x_{n-2},x_{n-1})+a_2D(x_{n-1},x_n)+a_3sD(x_{n-2},x_{n-1})+a_3sD(x_{n-1},x_n)\\
&\quad +a_5D(x_{n-2},x_{n-1})\\
D(x_{n-1},x_n) &\leq \left(\frac{a_1+a_3s+a_5}{1-a_2-a_3s}\right)D(x_{n-2},x_{n-1})\\
D(x_{n-1},x_n) &\leq \lambda D(x_{n-2},x_{n-1})\\
D(x_n,x_{n+1}) &\leq \lambda^2 D(x_{n-2},x_{n-1})\,.
\end{aligned}$$

Continuing this process, we get $D(x_n,x_{n+1}) \leq \lambda^n D(x_0,x_1)$, since $\lambda=\left(\frac{a_1+a_3s+a_5}{1-a_2-a_3s}\right)<1$. Hence T is contraction mapping.

Now we show that $\{x_n\}$ is a Cauchy sequence in X.

Let $m,n\in N$ and $m>n$

$$\begin{aligned}
D(x_n,x_m) &\leq s[D(x_n,x_{n+1})+D(x_{n+1},x_m)]\\
&\leq sD(x_n,x_{n+1})+s^2D(x_{n+1},x_{n+2})+s^3D(x_{n+2},x_{n+3})+...\\
&\leq s\lambda^nD(x_0,x_1)+s^2\lambda^{n+1}D(x_0,x_1)+s^3\lambda^3D(x_0,x_1)+...\\
&\leq s\lambda^n[1+s\lambda+s^2\lambda^2+...]D(x_0,x_1)\\
&\leq \left(\frac{s\lambda^n}{1-s\lambda}\right)D(x_0,x_1)\,.
\end{aligned}$$

Since $s\lambda<1$, taking $m,n\to\infty$ We have $\lim_{n\to\infty}D(x_n,x_m)=0$. Hence $\{x_n\}$ is a Cauchy sequence in complete b-metric space X. Consider $\{x_n\}$ converges to x^*.

Now we have to show that x^* is a fixed point of T.

$$\begin{aligned} D(Tx^*, Tx_n) &\leq a_1 D(x^*, Tx^*) + a_2 D(x_n, Tx_n) + a_3 D(x^*, Tx_n) + a_4 D(x_n, Tx^*) \\ &\quad + a_5 \frac{D(x^*, Tx^*) D(x^*, Tx_n) + D(x_n, Tx_n) D(x_n, Tx^*)}{D(x^*, Tx_n) + D(x_n, Tx^*)} \\ &\leq a_1 D(x^*, Tx^*) + a_2 D(x_n, x_{n+1}) + a_3 D(x^*, x_{n+1}) + a_4 D(x_n, Tx^*) \\ &\quad + a_5 \frac{D(x^*, Tx^*) D(x^*, x_{n+1}) + D(x_n, x_{n+1}) D(x_n, Tx^*)}{D(x^*, x_{n+1}) + D(x_n, Tx^*)}, \end{aligned}$$

taking $n \to \infty$

$$D(Tx^*, x^*) \leq (a_1 + a_4) D(x^*, Tx^*).$$

The above inequality hold if $D(Tx^*, x^*) = 0$. Hence x^* is fixed point of T. Uniqueness of fixed point: Let $x^* \neq x'$ be two fixed point of T, then:

$$\begin{aligned} D(x^*, x') &\leq a_1 D(x^*, Tx^*) + a_2 D(x', Tx') + a_3 D(x^*, Tx') + a_4 D(x', Tx^*) \\ &\quad + a_5 \frac{D(x^*, Tx^*) D(x^*, Tx') + D(x', Tx') D(x', Tx^*)}{D(x^*, Tx') + D(x', Tx^*)} \\ &\leq a_1 D(x^*, x^*) + a_2 D(x', x') + a_3 D(x^*, x') + a_4 D(x', x^*) \\ &\quad + a_5 \frac{D(x^*, x^*) D(x^*, x') + D(x', x') D(x', x^*)}{D(x^*, x') + D(x', x^*)} \\ &\leq (a_3 + a_4) D(x^*, x'). \end{aligned}$$

The above inequality hold if $D(x^*, x') = 0$.Hence fixed point of T is unique in X. □

Corollary 3.1. *Let (X, D, s) be a complete b-metric space with $s \geq 1$ and T be a self mapping $T : X \to X$ such that*

$$D(Tx, Ty) \leq \mu[D(x, Tx) + D(y, Ty)], \forall x, y \in X, \text{ and } \mu \in [0, 1/2),$$

then T has a unique fixed point.

Proof. By putting $a_1 = a_2 = \mu$ and $a_3 = a_4 = a_5 = 0$ in the above theorem, we get the required result. □

Corollary 3.2. *Let (X, D, s) be a complete b-metric space with $s \geq 1$ and T be a self mapping $T : X \to X$ such that*

$$D(Tx, Ty) \leq \lambda[D(x, Ty) + D(y, Tx)], \forall x, y \in X \text{ and } s\lambda \in [0, 1/2),$$

then T has a unique fixed point.

Proof. By putting $a_3 = a_4 = \lambda$ and $a_1 = a_2 = a_5 = 0$ in the above theorem, we get the required result. □

Theorem 3.2. *Let (X, D, s) be a complete b-metric space with $s \in [1, \infty)$ and T be a mapping $T : X \to X$ satisfying the condition*

$$D(Tx, Ty) \leq a_1 D(x, Tx) + a_2 D(y, Ty) + a_3 D(x, Ty) + a_4 D(y, Tx) + a_5 D(x, y) + a_6 \frac{D(y, Ty)[1 + D(x, Tx)]}{1 + D(x, y)} + a_7 \frac{D(y, Ty) + D(y, Tx)}{1 + D(y, Ty) D(y, Tx)}$$

for all $x, y \in X$ and $a_1, a_2, ..., a_7$ are non-negative real number such that $a_1 s + a_2 + (s + s^2) a_3 + a_5 s + a_6 + a_7 < 1$, Then T has a unique fixed point.

Proof. Let $x_0 \in X$ be arbitrary and $\{x_n\}_{n=1}^{\infty}$ be a sequence in X, defined by the recursion $x_0, x_1 = Tx_o, x_2 = Tx_1, ..., x_{n+1} = Tx_n \ \forall n \in N$.

$$\begin{aligned}
&D(x_n, x_{n+1}) = D(Tx_{n-1}, Tx_n) \\
&\leq a_1 D(x_{n-1}, Tx_{n-1}) + a_2 D(x_n, Tx_n) + a_3 D(x_{n-1}, Tx_n) + a_4 D(x_n, Tx_{n-1}) + a_5 D(x_{n-1}, x_n) \\
&+ a_6 \frac{D(x_n, Tx_n)[1 + D(x_{n-1}, Tx_{n-1})]}{1 + D(x_{n-1}, x_n)} + a_7 \frac{D(x_n, Tx_n) + D(x_n, Tx_{n-1})}{1 + D(x_n, Tx_n) D(x_n, Tx_{n-1})} \\
&\leq a_1 D(x_{n-1}, x_n) + a_2 D(x_n, x_{n+1}) + a_3 D(x_{n-1}, x_{n+1}) + a_4 D(x_n, x_n) + a_5 D(x_{n-1}, x_n) \\
&\quad + a_6 \frac{D(x_n, x_{n+1})[1 + D(x_{n-1}, x_n)]}{1 + D(x_{n-1}, x_n)} + a_7 \frac{D(x_n, x_{n+1}) + D(x_n, x_n)}{1 + D(x_n, x_{n+1}) D(x_n, x_n)} \\
&\leq a_1 D(x_{n-1}, x_n) + a_2 D(x_n, x_{n+1}) + a_3 s D(x_{n-1}, x_n) + a_3 s D(x_n, x_{n+1}) + a_5 D(x_{n-1}, x_n) \\
&\quad + a_6 D(x_n, x_{n+1}) + a_7 D(x_n, x_{n+1})
\end{aligned}$$

$$\begin{aligned}
D(x_n, x_{n+1}) &\leq \left(\frac{a_1 + a_3 s + a_5}{1 - a_2 - a_3 s - a_6 - a_7} \right) D(x_{n-1}, x_n) \\
D(x_n, x_{n+1}) &\leq \lambda D(x_{n-1}, x_n),
\end{aligned}$$

where

$$\lambda = \left(\frac{a_1 + a_3 s + a_5}{1 - a_2 - a_3 s - a_6 - a_7} \right).$$

Similarly

$$D(x_{n-1}, x_n) = D(Tx_{n-2}, Tx_{n-1})$$

$$\leq a_1D(x_{n-2},Tx_{n-2})+a_2D(x_{n-1},Tx_{n-1})+a_3D(x_{n-2},Tx_{n-1})+a_4D(x_{n-1},Tx_{n-2})$$
$$+a_5D(x_{n-2},x_{n-1})+a_6\frac{D(x_{n-1},Tx_{n-1})[1+D(x_{n-2},Tx_{n-2})]}{1+D(x_{n-2},x_{n-1})}$$
$$+a_7\frac{D(x_{n-1},Tx_{n-1})+D(x_{n-1},Tx_{n-2})}{1+D(x_{n-1},Tx_{n-1})D(x_{n-1},Tx_{n-2})}$$
$$\leq a_1D(x_{n-2},x_{n-1})+a_2D(x_{n-1},x_n)+a_3D(x_{n-2},x_n)+a_4D(x_{n-1},x_{n-1})+a_5D(x_{n-2},x_{n-1})$$
$$+a_6\frac{D(x_{n-1},x_n)[1+D(x_{n-2},x_{n-1})]}{1+D(x_{n-2},x_{n-1})}+a_7\frac{D(x_{n-1},x_n)+D(x_{n-1},x_{n-1})}{1+D(x_{n-1},x_n)D(x_{n-1},x_{n-1})}$$
$$\leq a_1D(x_{n-2},x_{n-1})+a_2D(x_{n-1},x_n)+a_3sD(x_{n-2},x_{n-1})+a_3sD(x_{n-1},x_n)$$
$$+a_5D(x_{n-2},x_{n-1})+a_6D(x_{n-1},x_n)+a_7D(x_{n-1},x_n)$$
$$D(x_{n-1},x_n)\leq\left(\frac{a_1+a_3s+a_5)}{1-a_2-a_3s-a_6-a_7}\right)D(x_{n-2},x_{n-1})$$
$$D(x_{n-1},x_n)\leq\lambda D(x_{n-2},x_{n-1})$$
$$D(x_n,x_{n+1})\leq\lambda^2D(x_{n-2},x_{n-1})\,.$$

Continuing this process, we get $D(x_n,x_{n+1}) \leq \lambda^nD(x_0,x_1)\,,$ since $\lambda=\left(\frac{a_1+a_3s+a_5)}{1-a_2-a_3s-a_6-a_7}\right)<1\,.$ Hence T is contraction mapping.

Now we show that $\{x_n\}$ is a Cauchy sequence in X.

Let $m,n\in N$ and $m>n$

$$\begin{aligned}
D(x_n,x_m) &\leq s[D(x_n,x_{n+1})+D(x_{n+1},x_m)]\\
&\leq sD(x_n,x_{n+1})+s^2D(x_{n+1},x_{n+2})+s^3D(x_{n+2},x_{n+3})+\ldots\\
&\leq s\lambda^nD(x_0,x_1)+s^2\lambda^{n+1}D(x_0,x_1)+s^3\lambda^3D(x_0,x_1)+\ldots\\
&\leq s\lambda^n[1+s\lambda+s^2\lambda^2+\ldots]D(x_0,x_1)\\
&\leq \left(\frac{s\lambda^n}{1-s\lambda}\right)D(x_0,x_1)\,.
\end{aligned}$$

Since $s\lambda<1$, taking $m,n\to\infty$, we have $\lim_{n\to\infty}D(x_n,x_m)=0$. Hence $\{x_n\}$ is a Cauchy sequence in complete b-metric space X. Consider $\{x_n\}$ converges to x^*. Now we have to show that x^* is a fixed point of T.

$$\begin{aligned}
D(Tx^*,Tx_n) \quad &\leq \quad a_1D(x^*,Tx^*)+a_2D(x_n,Tx_n)+a_3D(x^*,Tx_n)+a_4D(x_n,Tx^*) \\
+a_5D(x^*,x_n)+a_6\frac{D(x_n,Tx_n)[1+D(x^*,Tx^*)]}{1+D(x^*,x_n)} &+a_7\frac{D(x_n,Tx_n)+D(x_n,Tx^*)}{1+D(x_n,Tx_n)D(x_n,Tx^*)} \\
&\leq \quad a_1D(x^*,Tx^*)+a_2D(x_n,x_{n+1})+a_3D(x^*,x_{n+1})+a_4D(x_n,Tx^*) \\
+a_5D(x^*,x_n)+a_6\frac{D(x_n,x_{n+1})[1+D(x^*,Tx^*)]}{1+D(x^*,x_n)} &+a_7\frac{D(x_n,x_{n+1})+D(x_n,Tx^*)}{1+D(x_n,x_{n+1})D(x_n,Tx^*)},
\end{aligned}$$

taking $n\to\infty$

$D(Tx^*,x^*)\leq(a_1+a_4+a_7)D(x^*,Tx^*)$.

The above inequality hold if $D(Tx^*,x^*)=0$, Hence x^* is fixed point of T.

Uniqueness of fixed point: Let $x^*\neq x'$ be two fixed point of T, then

$$\begin{aligned}
D(x^*,x')\leq a_1D(x^*,Tx^*)&+a_2D(x',Tx')+a_3D(x^*,Tx')+a_4D(x',Tx^*)+a_5D(x^*,x') \\
&+a_6\frac{D(x',Tx')[1+D(x^*,Tx^*)]}{1+D(x^*,x')}+a_7\frac{D(x',Tx')+D(x',Tx^*)}{1+D(x',Tx')D(x',Tx^*)} \\
&\leq a_1D(x^*,x^*)+a_2D(x',x')+a_3D(x^*,x')+a_4D(x',x^*)+a_5D(x^*,x') \\
&+a_6\frac{D(x',x')[1+D(x^*,x^*)]}{1+D(x^*,x')}+a_7\frac{D(x',x')+D(x',x^*)}{1+D(x',x')D(x',x^*)} \\
&\leq(a_3+a_4+a_5+a_7)D(x^*,x').
\end{aligned}$$

The above ineq. hold if $D(x^*,x')=0$. Hence fixed point of T is unique in X. □

Corollary 3.3. *Let (X,D,s) be a complete b-metric space with $s\geq 1$ and T be a self mapping $T:X\to X$ satisfying the condition*

$$D(Tx,Ty)\leq aD(x,Tx)+bD(y,Ty)+cD(x,y)$$

$\forall x,y\in X$, where a,b,c are non-negative numbers such that $a+s(b+c)<1$, then T has a unique fixed point.

Proof. By putting $a_1=a,a_2=b,a_5=c$ and $a_3=a_4=a_6=a_7=0$ in the above theorem, we get the required result. □

Corollary 3.4. *Let (X,D,s) be a complete b-metric space with $s\geq 1$ and T be a self mapping $T:X\to X$ satisfying the condition*

$$D(Tx,Ty)\leq a_1D(x,Tx)+a_2D(y,Ty)+a_3D(x,Ty)+a_4D(y,Tx)+a_5D(x,y)$$

such that $a_1s+a_2+(s+s^2)a_3+a_5s<1$ for all $x,y\in X$ and a_1,a_2,a_3,a_4,a_5 are non-negative numbers. Then T has a unique fixed point.

Proof. By putting $a_6 = a_7 = 0$ in the above theorem, we get the required result. □

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