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THE NUMBER OF LOCATING INDEPENDENT DOMINATING SET ON GENERALIZED CORONA PRODUCT GRAPHS

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ABSTRACT. A set of vertices in a graph in which no two of vertices are adjacent is called independent set. A set of vertices D of graph $G = (V, E)$ is called dominating set if every vertices from $u \in V(G) - D$ is adjacent to a vertex $v \in D$. An independent dominating set of graph G is a set that is both dominating and independent. A set of vertices D of graph G is called locating dominating set with the additional characteristics that for $u, v \in (V(G) - D)$ satisfies $N(u) \cap D \neq N(v) \cap D$. Locating independent dominating set of graph G is a set of vertices D of graph G with satisfies characteristics of independent dominating set and locating dominating set. Locating independent domination number is the minimum cardinality of locating independent dominating set and denoted $\gamma_{Li}(G)$. In this paper, we have analyzed the locating independent domination number of corona product on path, cycle, helm, wheel, sun flower, ladder graph, and its results attain the lower bound. Furthermore we determine the generalized corona product of graphs.

1. INTRODUCTION

Along with this paper, we assume that all graphs are simple, nontrivial, and undirected. A graph G is pairs of $(V(G), E(G))$ with $V(G) = \{v_1, v_2, \ldots, v_n\}$ are finite and not empty set of vertex and $E(G) = \{e_1, e_2, \ldots, e_n\}$ is a set of irregular

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pairs $\{v_1, v_2\}$ from $v_1, v_2 \in V(G)$ called an edge. For more detail definition of graph, we refer to [3,4].

Graph theory develops very rapidly, thus it gives a lot of new study in graph theory. Graph study provide useful mathematical models for a wide range of applications. Especially in the early of this 21st century, where the first years of it was marked by the rise of disruptive technology. One of the disruptive technology is designing a massive parallel processing systems. The complexity in building massive scale parallel processing systems has resulted in a growing interest in the study of interconnection networks design. Network design affects the performance, cost, scalability, and availability of parallel computers. The dominating set (DS) study is the one of activities to discover a good structure of the network is one of the basic issues.

A set of vertices D of graph $G = (V, E)$ is called a dominating set if every vertex from $u \in V(G) - D$ is adjacent to a vertex $v \in D$. The minimum cardinality of dominating set is called domination number, denoted by $\gamma(G)$. Some natural extensions derived from dominating set study have been found, it implies this topic splits into several type of dominating set such as independent dominating set (IDS), locating dominating set (LDS), locating independent dominating set (LIDS). The dominating set is called LDS if for any two vertices $v, w \epsilon V - D$, $N(v) \cap D \neq N(w) \cap D$. The value of $\gamma_L(G)$ is the minimum cardinality of LDS, called a locating domination number. Slater [1,2,5,6] firstly studied the concept of LDS. The local dominating set is a dominating set with an additional characteristics that D is an independent set and every vertex not in D is adjacent to a vertex in D. The $i(G)$ is the minimum number of vertices in an independent set $D \subseteq V(G)$ such that D also dominates $V(G)$. The LIDS of graph G is a set that both locating dominating and independent set. The locating independent domination number is the minimum cardinality of LIDS and denoted by $\gamma_{Li}(G)$. The lower bound of locating independent domination number of graph Amal(G, v, m) has determined by Wardani et. al. [7] .

In this research we will study the locating independent dominating set and we will determine the locating independent domination number of corona product of graphs. Wardani et. al. [8] have determined the lower bound of LIDS of corona product of graph and obtained two Lemmas in the following.

Lemma 1.1. *[8] Let H_1, H_2 be a simple, connected, and undirected graphs. If $H_1 \odot H_2$, then locating independent dominating set of $H_1 \odot H_2$ is located in H_2.*

Lemma 1.2. *[8] For any graph H_1 of order n and H_2 of order m, so*

$$\gamma_{Li} \geq \begin{cases} p(H_1)(\gamma_{L_i}(H_2)) & ;\text{for } diam(H_2) > 2 \\ \sim & ;\text{for } diam(H_2) \leq 2 \\ & ;\gamma_{Li}(H_2) = \sim \end{cases}$$

Furthermore, we will extend our study for a generalized corona product of graphs, namely $H_1 \odot^l H_2$. Lai [9] determine the $corona$ of two graphs H_1 and H_2, denoted by $H_1 \odot H_2$. This graph is developed from one copy of H_1 and $\mid V(H_1) \mid$ copies of H_2 where the ith vertex of H_1 is adjacent to every vertex in the ith copy of H_2. For any integer $l \geq 2$, we establish the graph $H_1 \odot^l H_2$ recursively from $H_1 \odot H_2$ as $H_1 \odot^l H_2 = (H_1 \odot^{l-1} H_2) \odot H_2$. The graph $H_1 \odot^l H_2$ is also called as $l - corona\ \ product$ of H_1 and H_2.

2. Results

We are now ready to present our results on the locating independent dominating set of generalized corona product of graphs. First, we will show the lower bound of locating independent domination number of corona product of graph $G \odot H$ in the following theorem.

Theorem 2.1. *Let G be a corona product of ladder graph L_u and helm graph H_k. For $u \geq 2$ and $k \geq 3$, the locating independent domination number of G is $\gamma_{Li(L_u \odot H_k)} = 2u(k+1)$.*

Proof. The corona graph $L_u \odot H_k$ is a connected graph with vertex set $V(L_u \odot H_k) = \{x_r, y_r; r = 1..u\} \cup \{a_r^t; r = 1..u, t = 1 \text{ and } 2\} \cup \{x_{r,s}^t, y_{r,s}^t; r = 1..u, s = 1..t, t = 1 \text{ and } 2\}$ and edge set $E(W_u \odot P_k) = \{x_r x_{r+1}, y_r y_{r+1}; r = 1..u-1\} \cup \{x_r y_r; r = 1..u\} \cup \{x_r x_{r,s}^t, y_r y_{r,s}^t; r = 1..u, s = 1..t, t = 1 \text{ and } 2\} \cup \{x_r a_r^1, y_r a_r^2; r = 1..u\} \cup \{x_{r,s}^1 x_{r,s+1}^1, y_{r,s}^1 y_{r,s+1}^1; r = 1..u, s = 1..t-1\} \cup \{a_r^1 x_{r,s}^1, a_r^2 y_{r,s}^1; r = 1..u; s = 1..t\} \cup \{x_{r,s}^1 x_{r,s}^2, y_{r,s}^1 y_{r,s}^2, ; r = 1..u, s = 1..t\} \cup \{x_{r,1}^1 x_{r,u}^1, y_{r,1}^1 y_{r,u}^1; r = 1..u\}$. Thus $|V(L_u \odot H_k)| = 4u + 4uk$, $|E(L_u \odot H_k)| = 5u + 10uk - 2$, and $diam(H_k) = k$. $\gamma_{Li}(H_k) = k + 1$. Figure 1 shows the examples of corona graph of wheel and path graphs. Based on Lemma 1.2, we have $\gamma_{Li}(L_u \odot H_k) \geq 2u(k+1)$.

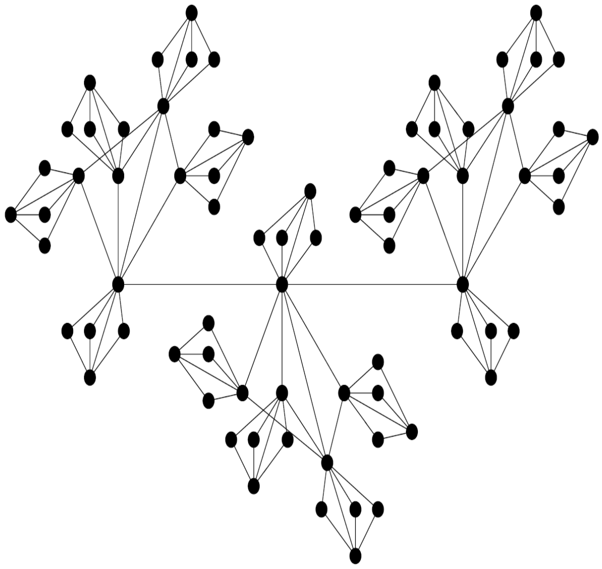

FIGURE 1. Corona of L_2 and H_3.

Now, We will prove that $\gamma_{Li}(L_u \odot H_k) \leq 2u(k+1)$ with choose $D = \{x^2_{r,s}; r = 1..u, s = 1..k\} \cup \{y^2_{r,s}; r = 1..u, s = 1..k\} \cup \{a^t_r; r = 1..u, t = 1 \text{ and } 2\}$ as the dominator set of $L_u \odot H_k$, for $u \geq 2$ and $k \geq 3$, thus $|D| = 2u(k+1)$. Choose $V - D = \{x^1_{r,s}; r = 1..u, s = 1..k\} \cup \{y^1_{r,s}; r = 1..u, s = 1..k\} \cup \{x_r, y_r; r = 1..u\}$ as the non-dominator set of $L_u \odot H_k)$ for $u \geq 2$ and $k \geq 3$. Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

$$\begin{aligned} N(x^1_{r,s}) \cap D &= \{x^2_{r,s}, a^1_r\}, r = 1..u, s = 1..k \\ N(y^1_{r,s}) \cap D &= \{y^2_{r,s}, a^2_r\}, r = 1..u, s = 1..k \\ N(x_r) \cap D &= \{x^2_{r,s}; s = 1..k\} \cup \{a^1_r\}, \\ & \quad r = 1..u \\ N(y_r) \cap D &= \{y^2_{r,s}; s = 1..k\} \cup \{a^2_r\}, \\ & \quad r = 1..u \end{aligned}$$

It can be shown that the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and the element of D are all different, and it is not empty set. The set D does dominate all vertices in $V(L_u \odot H_k)$. So, we can conclude that for $\gamma_{Li}(L_u \odot H_k) \leq 2u(k+1)$ satisfies the upper bound of LIDS. Thus, $\gamma_{Li}(L_u \odot H_k) \leq 2u(k+1)$. Therefore, $\gamma_{Li}(L_u \odot H_k) = 2u(k+1)$.

Figure 2 describes the LIDS of corona of wheel graph and path graph. □

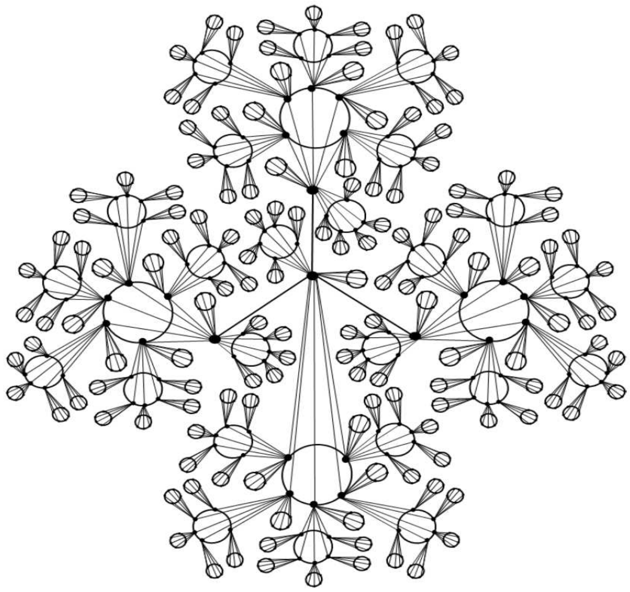

FIGURE 2. LIDS of $L_3 \odot H_3$.

Theorem 2.2. *Let G be a corona product of graph of ladder L_u and sunflower SF_k. For $u \geq 2$ and $k \geq 4$, the locating independent domination number of G is $\gamma_{Li(L_u \odot SF_k)} = 2uk$.*

Proof. The corona graph $L_u \odot F_k$ is a connected graph with vertex and edge set are $V(L_u \odot SF_k) = \{x_r, y_r; r = 1..u\} \cup \{x^t_{r,s}, y^t_{r,s}; r = 1..u, s = 1..k, t = 1$ and $2\}$ and $E(L_u \odot SF_k) = \{x_r x_{r+1}, y_r y_{r+1}; r = 1..u-1\} \cup \{x_r y_r; r = 1..u\} \cup \{x_r x^t_{r,s}, y_i y^t_{r,s}; i = 1..u, s = 1..k, t = 1$ and $2\} \cup \{x^1_{r,s} x^1_{r,s+1}, y^1_{r,s} y^1_{r,s+1}; r = 1..u, s = 1..k-1\} \cup \{x^1_{r,s} x^2_{r,s}, y^1_{r,s} y^2_{r,s}; r = 1..u; s = 1..k-1\} \cup \{x^1_{r,s} x^2_{r,s+1}, y^1_{r,s} y^2_{r,s+1}; r = 1..u, s = 1..k-1\} \cup \{x^1_{r,u} x^1_{r,1}, y^1_{r,u} y^1_{r,1}; r = 1..u\} \cup \{x^1_{r,u} x^2_{r,1}, y^1_{r,u} y^2_{r,1}; r = 1..u\}$. Thus $|V(L_u \odot SF_k)| = 2u + 4uk$, $|E(L_u \odot SF_k)| = 10uk + 3u - 2$, and $diam(SF_k) = \lfloor \frac{u}{2} + 1 \rfloor$. The locating independent domination number of sunflower graph is $\gamma_{Li}(SF_k) = k$. Based on Lemma 1.2, we have $\gamma_{Li}(L_u \odot SF_k) \geq 2uk$.

Now, we will prove that $\gamma_{Li}(L_u \odot SF_k) \leq 2uk$ with choose $D = \{x^2_{r,s}, y^2_{r,s}; r = 1..u, s = 1..k\}$ as the dominator set of $L_u \odot SF_k$, for $u \geq 2$ and $k \geq 4$, thus $|D| = 2uk$. Thus, the non-dominator set of $L_u \odot SF_k$ for $u \geq 2$ and $k \geq 4$ is $V - D = \{x^1_{r,s}, y^1_{r,s}; r = 1..u, s = 1..k\} \cup \{x_r, y_r; r = 1..u\}$. Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

$$\begin{array}{rcl} N(x^1_{r,s}) \cap D &=& \{x^2_{r,s}, x^2_{r,s+1}\}, r = 1..u, \\ && s = 1..k-1 \\ N(x^1_{r,k}) \cap D &=& \{x^2_{r,1}, x^2_{r,k}\}, r = 1..u \\ N(y^1_{r,s}) \cap D &=& \{y^2_{r,s}, y^2_{r,s+1}\}, r = 1..u, \\ && s = 1..k-1 \\ N(y^1_{r,k}) \cap D &=& \{y^2_{r,1}, y^2_{r,k}\}, r = 1..u \\ N(x_r) \cap D &=& \{x^2_{r,s}; s = 1..k\}, r = 1..u \\ N(y_r) \cap D &=& \{y^2_{r,s}; s = 1..k\}, r = 1..u \end{array}$$

It can be seen that the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and D are all different, and it is not empty set. The set D does dominate all vertices in $V(L_u \odot SF_k)$. It concludes that $\gamma_{Li}(L_u \odot SF_k) \leq 2uk$ satisfies the upper bound of LIDS. Thus $\gamma_{Li}(L_u \odot SF_k) \leq 2uk$. Therefore, $\gamma_{Li}(L_u \odot SF_k) = 2uk$. □

Theorem 2.3. *Let G be a corona product of path graphs P_u and P_k. For $u \geq 2$ and $k \geq 6$, the locating independent domination number of G is $\gamma_{Li(P_u \odot P_k)} = u\lceil \frac{2k}{5} \rceil$.*

Proof. The corona graph $P_u \odot P_k$ is a connected graph with vertex set $V(P_u \odot P_k) = \{x_r; r = 1..u\} \cup \{x_{r,s}; r = 1..u, s = 1..k\}$ and edge set $E(P_u \odot P_k) = \{x_r x_{r+1}; r = 1..u-1\} \cup \{x_r x_{r,s}; r = 1..u, s = 1..k\} \cup \{x_{r,s} x_{r,s+1}; r = 1..u; s = 1..k-1, \}$. Thus $|V(P_u \odot P_k)| = u + uk$, $|E(P_u \odot P_k)| = 2uk - 1$, and $diam(P_k) = k - 1$. The locating independent domination number of $\gamma_{Li}(P_k) = \lceil \frac{2k}{5} \rceil$. Based on Lemma 1.2 $\gamma_{Li}(P_u \odot P_k) \geq u\lceil \frac{2k}{5} \rceil$.

Now, we will prove that $\gamma_{Li}(P_u \odot P_k) \leq u\lceil \frac{2k}{5} \rceil$ as the upper bound of LIDS. Choose the dominator set as follows.

1) $k \equiv 0$ mod 10
   $\{x_{r,s}; r = 1..u, s \equiv 2 \text{ mod } 5 \cup s \equiv 4 \text{ mod } 5\}$
2) another m
   $\{x_{r,s}; r = 1..u, s \equiv 0 \text{ mod } 2\}$

thus, we obtain the non-dominator set in the following.

1) for $k \equiv 0$ mod 10
   $V - D = \{x_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 5 \cup s \equiv 3 \text{ mod } 5 \cup s \equiv 0 \text{ mod } 5\} \cup \{x_r; r = 1..u\}$

2) otherwise k

$V - D = \{x_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 2\} \cup \{x_r; r = 1..u\}$

Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

1) for $k \equiv 0$ mod 10

$$\begin{aligned} N(x_{r,s\equiv 1mod5}) \cap D &= \{x_{r,s+1}\}, r = 1..u \\ N(x_{r,s\equiv 3mod5}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u \\ N(x_{r,s\equiv 0mod5}) \cap D &= \{x_{r,s-1}\}, r = 1..u \\ N(x_r) \cap D &= \{x_{r,s}; s \equiv 2 \text{ mod } 5 \cup \\ & \quad s \equiv 4 \text{ mod } 5\}, r = 1..u \end{aligned}$$

2) for k even

$$\begin{aligned} N(x_{r,1}) \cap D &= \{x_{r,2}\}, r = 1..u \\ N(x_{r,s\equiv 1mod2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u, \\ & \quad s = 2..k \\ N(x_r) \cap D &= \{x_{r,s}; s \equiv 0 \text{ mod } 2\}, \\ & \quad r = 1..u \end{aligned}$$

3) for k odd

$$\begin{aligned} N(x_{r,1}) \cap D &= \{x_{r,2}\}, r = 1..u \\ N(x_{r,s\equiv 1mod2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u, \\ & \quad s = 2..k-1 \\ Since N(x_{r,k}) \cap D &= \{x_{r,k-1}\}, r = 1..u \\ N(x_r) \cap D &= \{x_{r,s}; s \equiv 0 \text{ mod } 2\}, \\ & \quad r = 1..u \end{aligned}$$

Based on the result of intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, It can be shown that the intersection are all different, and it is not empty set. The set D dominate all vertices in $V(P_u \odot P_k)$. Thus, we can conclude that for $\gamma_{Li}(P_u \odot P_k) \leq 2u(k+1)$ satisfies the properties of LIDS. Since $\gamma_{Li}(P_u \odot P_k) \geq u\lceil\frac{2k}{5}\rceil$ and $\gamma_{Li}(P_u \odot P_k) \leq u\lceil\frac{2k}{5}\rceil$, thus $\gamma_{Li}(P_u \odot P_k) = 2u(k+1)$. □

Theorem 2.4. *Let G be a corona product of path graph P_u and cycle graph C_k. For $u \geq 2$ and $k \geq 6$, the locating independent domination number of G is* $\gamma_{Li(P_u \odot C_k)} = u\lceil\frac{2k}{5}\rceil$.

Proof. The corona graph $P_u \odot C_k$ is a connected graph with vertex set $V(P_u \odot C_k) = \{x_r; r = 1..u\} \cup \{x_{r,s}; r = 1..u, s = 1..k\}$ and edge set $E(P_u \odot C_k) =$

$\{x_rx_{r+1}; r = 1..u-1\} \cup \{x_rx_{r,s}; r = 1..u, s = 1..k\} \cup \{x_{r,s}x_{r,s+1}; r = 1..u, s = 1..k-1\} \cup \{x_{r,1}, x_{r,k}; r = 1..u\}$. Thus $|V(P_u \odot C_k)| = u+uk$, $|E(P_u \odot C_k)| = 2uk-1$, and $diam(C_k) = \lfloor \frac{u}{2} \rfloor$. $\gamma_{Li}(C_k) = \lceil \frac{2k}{5} \rceil$. Based on Lemma 1.2 $\gamma_{Li}(P_u \odot C_k) \geq u\lceil \frac{2k}{5} \rceil$.

Now, we will prove that $\gamma_{Li}(P_u \odot C_k) \leq u\lceil \frac{2k}{5} \rceil$ with choose the dominator set in the following.

1) $k \equiv 0 \text{ mod } 10$
   $\{x_{r,s}; r = 1..u, s \equiv 2 \text{ mod } 5 \cup s \equiv 4 \text{ mod } 5\}$
2) another k
   $\{x_{r,s}; r = 1..u, s \equiv 0 \text{ mod } 2\}$.

Thus, we obtain the non-dominator set in the following.

1) $k \equiv 0 \text{ mod } 10$
   $\{x_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 5 \cup s \equiv 3 \text{ mod } 5 \cup s \equiv 0 \text{ mod } 5\} \cup \{x_r; r = 1..u\}$
2) another k
   $\{x_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 2\} \cup \{x_r; 1 \leq r \leq u\}$.

Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

1) for $k \equiv 0 \text{ mod } 10$
$$\begin{aligned} N(x_{r,s\equiv 1 mod 5}) \cap D &= \{x_{r,s+1}\}, r = 1..u \\ N(x_{r,s\equiv 3 mod 5}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u \\ N(x_{r,s\equiv 0 mod 5}) \cap D &= \{x_{r,s-1}\}, r = 1..u \\ N(x_r) \cap D &= \{x_{r,s}; s \equiv 2 \text{ mod } 5 \cup \\ & \quad s \equiv 4 \text{ mod } 5\}, r = 1..u \end{aligned}$$
2) for k even neq $k \equiv 0 \text{ mod } 10$
$$\begin{aligned} N(x_{r,1}) \cap D &= \{x_{r,2}, x_{r,k}\}, r = 1..u \\ N(x_{r,s\equiv 1 mod 2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u, \\ & \quad s = 2..k \end{aligned}$$
$$N(x_r) \cap D = \{x_{r,s}; s \equiv 0 \text{ mod } 2\}, r = 1..u$$
3) for k odd
$$\begin{aligned} N(x_{r,1}) \cap D &= \{x_{r,2}\}, r = 1..u \\ N(x_{r,s\equiv 1 mod 2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u, \\ & \quad s = 2..k-1 \\ N(x_{r,s}) \cap D &= \{x_{r,k-1}\}, r = 1..u \\ N(x_r) \cap D &= \{x_{r,s}; s \equiv 0 \text{ mod } 2\}, \\ & \quad r = 1..u \end{aligned}$$

Based on the result of intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, It can be shown that the intersection are all different, and it is not empty set. The set D dominate all vertices in $V(P_u \odot C_k)$. Thus, we can conclude that for $\gamma_{Li}(P_u \odot C_k) \leq u\lceil\frac{2k}{5}\rceil$ satisfies the properties of LIDS. Since $\gamma_{Li}(P_u \odot C_k) \geq u\lceil\frac{2k}{5}\rceil$ and $\gamma_{Li}(P_u \odot C_k) \leq u\lceil\frac{2k}{5}\rceil$, Thus $\gamma_{Li}(P_u \odot P_k) = u\lceil\frac{2k}{5}\rceil$. □

Theorem 2.5. *Let G be a corona product of cycle graph C_u and cycle graph C_k. for $u \geq 3$ and $k \geq 6$, the locating independent domination number of G is $\gamma_{Li(C_u \odot C_k)} = u\lceil\frac{2k}{5}\rceil$.*

Proof. The corona graph $C_u \odot C_k$ is a connected graph with vertex set $V(C_u \odot C_k) = \{x_r; r = 1..u\} \cup \{x_{r,s}; r = 1..u, s = 1..k\}$ and edge set $E(C_u \odot C_k) = \{x_r x_{r+1}; r = 1..u-1\} \cup \{x_u x_1\} \cup \{x_r x_{r,s}; r = 1..u, s = 1..k\} \cup \{x_{r,s} x_{r,s+1}; r = 1..u, s = 1..k-1\} \cup \{x_{r,k}, x_{r,1}; r = 1..u\}$. Thus $|V(C_u \odot C_k)| = u+uk$, $|E(C_u \odot C_k)| = 2uk+u$, and $diam(C_k) = \lfloor\frac{u}{2}\rfloor$. Locating independent domination number of cycle graph is $\gamma_{Li}(C_k) = \lceil\frac{2k}{5}\rceil$. Based on Lemma 1.2 $\gamma_{Li}(C_u \odot C_k) \geq u\lceil\frac{2k}{5}\rceil$.

Now, we will prove that $\gamma_{Li}(C_u \odot C_k) \leq u\lceil\frac{2k}{5}\rceil$ with choose the dominator set in the following.

1) $k \equiv 0 \bmod 10$
   $\{x_{r,s}; r = 1..u, s \equiv 2 \bmod 5 \cup s \equiv 4 \bmod 5\}$
2) another k
   $\{x_{r,s}; r = 1..u, s \equiv 0 \bmod 2\}$.

Thus, we obtain the non-dominator set in the following.

1) $k \equiv 0 \bmod 10$
   $\{x_{r,s}; r = 1..u, s \equiv 1 \bmod 5 \cup s \equiv 3 \bmod 5 \cup s \equiv 0 \bmod 5\} \cup \{x_r; r = 1..u\}$
2) another k
   $\{x_{r,s}; r = 1..u, s \equiv 1 \bmod 2\} \cup \{x_r; r = 1..n\}$.

Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

1) for $k \equiv 0$ mod 10

$$\begin{aligned} N(x_{r,s\equiv 1 mod 5}) \cap D &= \{x_{r,s+1}\}, r = 1..u \\ N(x_{r,s\equiv 3 mod 5}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u \\ N(x_{r,s\equiv 0 mod 5}) \cap D &= \{x_{r,s-1}\}, r = 1..u \\ N(x_r) \cap D &= \{x_{r,s}; s \equiv 2 \text{ mod } 5 \cup \\ & \quad s \equiv 4 \text{ mod } 5\}, r = 1..u \end{aligned}$$

2) for k even $\neq k \equiv 0$ mod 10

$$\begin{aligned} N(x_{r,1}) \cap D &= \{x_{r,2}, x_{r,k}\}, r = 1..u \\ N(x_{r,s\equiv 1 mod 2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u, \\ & \quad s = 2..k \end{aligned}$$

$$N(x_r) \cap D = \{x_{r,s}; s \equiv 0 \text{ mod } 2\}, r = 1..u$$

3) for k odd

$$\begin{aligned} N(x_{r,1}) \cap D &= \{x_{r,2}\}, r = 1..u \\ N(x_{r,s\equiv 1 mod 2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}\}, r = 1..u, \\ & \quad s = 2..k-1 \\ N(x_{r,k}) \cap D &= \{x_{r,k-1}\}, r = 1..u \\ N(x_r) \cap D &= \{x_{r,s}; s \equiv 0 \text{ mod } 2\}, \\ & \quad r = 1..u \end{aligned}$$

Based on the result of intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, It can be shown that the intersection are all different, and it is not empty set. The set D dominate all vertices in $V(C_u \odot C_k)$. Thus, we can conclude that for $\gamma_{Li}(C_u \odot C_k) \leq u\lceil \frac{2k}{5} \rceil$ satisfies the properties of LIDS. Since $\gamma_{Li}(C_u \odot C_k) \geq u\lceil \frac{2k}{5} \rceil$ and $\gamma_{Li}(C_u \odot C_k) \leq u\lceil \frac{2k}{5} \rceil$, thus $\gamma_{Li}(C_u \odot C_k) = u\lceil \frac{2k}{5} \rceil$. □

Theorem 2.6. *Let G be a corona product of path graph P_u and ladder graph L_k. For $u \geq 2$ and $k \geq 4$, the locating independent domination number of G is $\gamma_{Li(P_u \odot L_k)} = uk$.*

Proof. The corona graph $P_u \odot L_k$ is a connected graph with vertex set $V(P_u \odot L_k) = \{x_r; r = 1..u\} \cup \{x_{r,s}, y_{r,s}y; r = 1..u, s = 1..k\}$ and edge set $E(P_u \odot L_k) = \{x_r x_{r+1}; r = 1..u-1\} \cup \{x_r x_{r,s}, x_r y_{r,s}; r = 1..u, s = 1..k\} \cup \{x_{r,s}x_{r,s+1}, y_{r,s}y_{r,s+1}; r = 1..u; s = 1..k-1\} \cup \{x_{r,s}y_{r,s}; r = 1..u; s = 1..k\}$. thus $|V(P_u \odot L_k)| = u + 2uk$, $|E(P_u \odot L_k)| = 5uk - u - 1$, and $diam(L_k) = k$. $\gamma_{Li}(L_k) = k$. Based on Lemma 1.2 we have $\gamma_{Li(P_u \odot L_k)} \geq uk$.

Now, we will prove that $\gamma_{Li(P_u \odot L_k)} \leq uk$ with choose the dominator set $D = \{x_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 2\} \cup \{y_{r,s}; r = 1..u, s \equiv 0 \text{ mod } 2\}$ and the non-dominator set of corona product $P_u \odot L_k$ is $V - D = \{x_{r,s}; r = 1..u, s \equiv 0 \text{ mod } 2\} \cup \{y_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 2\} \cup \{x_r; r = 1..u\}$. Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

1) for k even

$$\begin{aligned}
N(x_{r,s\equiv 0\text{mod}2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}, y_{r,s}\},\\
&\quad r = 1..u, s = 1..k-1\\
N(x_{r,k}) \cap D &= \{x_{r,k-1}, y_{r,k}\}, r = 1..u\\
N(y_{r,s\equiv 1\text{mod}2}) \cap D &= \{y_{r,s-1}, y_{r,s+1}, x_{r,s}\},\\
&\quad r = 1..u, s = 2..k\\
N(y_{r,1}) \cap D &= \{y_{r,2}, x_{r,1}\}, r = 1..u\\
N(x_r) \cap D &= \{x_{r,s}; s \equiv 1 \text{ mod } 2\} \cup\\
&\quad \{y_{r,s}; s \equiv 0 \text{ mod } 2\}, r = 1..u
\end{aligned}$$

2) for k odd

$$\begin{aligned}
N(x_{r,s\equiv 0 mod 2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}, y_{r,s}\},\\
&\quad r = 1..u\\
N(y_{r,s\equiv 1 mod 2}) \cap D &= \{y_{r,s-1}, y_{r,s+1}, x_{r,s}\},\\
&\quad r = 1..u, s = 2..k-1\\
N(y_{r,1}) \cap D &= \{y_{r,2}, x_{r,1}\}, r = 1..u\\
N(y_{r,k}) \cap D &= \{y_{r,k-1}, x_{r,k}\}, r = 1..u\\
N(x_r) \cap D &= \{x_{r,s}; s \equiv 1 \text{ mod } 2\} \cup\\
&\quad \{y_{r,s}; s \equiv 0 \text{ mod } 2\}, r = 1..u
\end{aligned}$$

Based on the result of intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, It can be shown that the intersection are all different, and it is not empty set. The set D dominate all vertices in $V(P_u \odot L_k)$. Thus, we can conclude that $\gamma_{Li(P_u \odot L_k)} \leq uk$ satisfies the properties of LIDS. Since $\gamma_{Li(P_u \odot L_k)} \geq uk$ and $\gamma_{Li(P_u \odot L_k)} \leq uk$, thus $\gamma_{Li(P_u \odot L_k)} = uk$. $\square$

Theorem 2.7. *Let G be a corona product of wheel graph W_u and ladder graph L_k for $u \geq 2$ and $k \geq 4$, the locating independent domination number of G is $\gamma_{Li(W_u \odot L_k)} = (u+1)k$.*

Proof. The corona graph $W_u \odot L_k$ is a connected graph with vertex set $V(W_u \odot L_k) = \{x_r; 1 \leq r \leq u\} \cup \{a\} \cup \{x_{r,s}, y_{r,s}; r = 1..u+1, s = 1..k\}$ and edge set

$E(W_u \odot L_k) = \{ax_r; r = 1..u\} \cup \{x_r x_{r+1}; r = 1..u-1\} \cup \{x_u x_1\} \cup \{x_r x_{r,s}, x_r y_{r,s}; r = 1..u, s = 1..k\} \cup \{ax_{u+1,s}, ay_{u+1,s}; s = 1..k\} \cup \{x_{r,s}x_{r,s+1}, y_{r,s}y_{r,s+1}; r = 1..u+1; s = 1..k-1\} \cup \{x_{r,s}y_{r,s}; r = 1..u+1; s = 1..k\}$. Thus $|V(W_u \odot L_k)| = u + 2uk + 2k + 1$, $|E(W_u \odot L_k)| = 5uk + 5k$, and $diam(L_k) = k$. $\gamma_{Li}(L_k) = k$. Based on Lemma 1.2 $\gamma_{Li}(W_u \odot L_k) \geq (u+1)k$.

Now, we will prove that $\gamma_{Li}(W_u \odot L_k) \leq (u+1)k$ with choose the dominator set $D = \{x_{r,s}; r = 1..u+1, s \equiv 1 \text{ mod } 2\} \cup \{y_{r,s}; r = 1..u+1, s \equiv 0 \text{ mod } 2\}$ and the non-dominator set of corona product $W_u \odot L_k$ is $V - D = \{x_{r,s}; r = 1..u+1, s \equiv 0 \text{ mod } 2\} \cup \{y_{r,s}; r = 1..u+1, s \equiv 1 \text{ mod } 2\}$. Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

1) for k even

$$\begin{aligned}
N(x_{r,s\equiv 0 \text{mod} 2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}, y_{r,s}\}, \\
&\quad r = 1..u+1, s = 1..k-1 \\
N(y_{r,s\equiv 1 \text{mod} 2}) \cap D &= \{y_{r,s-1}, y_{r,s+1}, x_{r,s}\}, \\
&\quad r = 1..u+1, s = 2..k \\
N(x_{r,k}) \cap D &= \{x_{r,k-1}, y_{r,k}\}, r = 1..u+1 \\
N(y_{r,1}) \cap D &= \{y_{r,2}, x_{r,1}\}, r = 1..u+1 \\
N(x_r) \cap D &= \{x_{r,s}; s \equiv 1 \text{ mod } 2\} \cup \\
&\quad \{y_{r,s}; s \equiv 0 \text{ mod } 2\}, \\
&\quad r = 1..u \\
N(a) \cap D &= \{x_{u+1,s}; s \equiv 1 \text{ mod } 2\} \cup \\
&\quad \{y_{u+1,s}; s \equiv 0 \text{ mod } 2\}
\end{aligned}$$

2) for k odd

$$\begin{aligned}
N(x_{r,s\equiv 0 \text{mod} 2}) \cap D &= \{x_{r,s-1}, x_{r,s+1}, y_{r,s}\}, \\
&\quad r = 1..u+1 \\
N(y_{r,s\equiv 1 \text{mod} 2}) \cap D &= \{y_{r,s-1}, y_{r,s+1}, x_{r,s}\}, \\
&\quad r = 1..u+1, s = 2..k-1 \\
N(y_{r,k}) \cap D &= \{y_{r,k-1}, x_{r,k}\}, r = 1..u+1 \\
N(y_{r,1}) \cap D &= \{y_{r,2}, x_{r,1}\}, r = 1..u+1 \\
N(x_r) \cap D &= \{x_{r,s}; s \equiv 1 \text{ mod } 2\} \cup \\
&\quad \{y_{r,s}; s \equiv 0 \text{ mod } 2\}, \\
&\quad r = 1..u \\
N(a) \cap D &= \{x_{u,s}; s \equiv 1 \text{ mod } 2\} \cup \\
&\quad \{y_{u+1,s}; j \equiv 0 \text{ mod } 2\}
\end{aligned}$$

Based on the result of intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, It can be shown that the intersection are all different, and it is not empty set. The set D dominate all vertices in $V(W_u \odot L_k)$. Thus, we can conclude that $\gamma_{Li}(W_u \odot L_k) \leq (u+1)k$ indicates that it satisfies the properties of LIDS. Since $\gamma_{Li}(W_u \odot L_k) \geq (u+1)k$ and $\gamma_{Li}(W_u \odot L_k) \leq (u+1)k$, thus $\gamma_{Li}(W_u \odot L_k) = (u+1)k$. □

Theorem 2.8. *Let G be a corona product of sun flower graph SF_u and ladder graph L_k for $u \geq 3$ and $k \geq 4$, the locating independent domination number of G is $\gamma_{Li(SF_u \odot L_k)} = 2uk$.*

Proof. The corona graph $SF_u \odot L_k$ is a connected graph with vertex set $V(SF_u \odot L_k) = \{x_r, y_r; r = 1..u\} \cup \{x^t_{r,s}, y^t_{r,s}; r = 1..u, s = 1..k, t = 1$ and $2\}$ and edge set$E(SF_u \odot L_k) = \{x_r x_{r+1}; r = 1..u-1\} \cup \{x_u x_1\} \cup \{x_r y_r; r = 1..u\} \cup \{x_r y_{r+1}; r = 1..u-1\} \cup \{x_u y_1\} \cup \{x_r x^t_{r,s}, x_r y^t_{r,s}; r = 1..n, s = 1..k, t = 1$ and $2\} \cup \{x^t_{r,s} x^t_{r,s+1}, y^t_{r,s} y^t_{r,s+1}; r = 1..u; s = 1..k-1, t = 1$ and $2\} \cup \{x^1_{r,s} x^2_{r,s}, y^1_{r,s} y^2_{r,s}; r = 1..u; s = 1..k\}$. Thus $|V(SF_u \odot L_k)| = 2u + 4uk$, $|E(SF_u \odot L_k)| = 10uk$, and $diam(L_k) = k$. $\gamma_{Li}(L_k) = k$. Based on Lemma 1.2 $\gamma_{Li}(SF_u \odot L_k) \geq 2uk$.

Now, we will prove that $\gamma_{Li}(SF_u \odot L_k) \leq 2uk$ with choose the dominator set $D = \{x^1_{r,s}, y^1_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 2\} \cup \{x^2_{r,s}, y^2_{r,s}; r = 1..u, s \equiv 0 \text{ mod } 2\}$ and the non-dominator set of corona product $SF_u \odot L_k$ is$V - D = \{x^1_{r,s}, y^1_{r,s}; r = 1..u, s \equiv 0 \text{ mod } 2\} \cup \{x^2_{r,s}, y^2_{r,s}; r = 1..u, s \equiv 1 \text{ mod } 2\} \cup \{x_r, y_r; r = 1.u\}$. Furthermore, we will obtain the intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D in the following.

1) for k even

$$\begin{aligned}
N(x^1_{r,s\equiv 0 \text{mod} 2}) \cap D &= \{x^1_{r,s-1}, x^1_{r,s+1}, x^2_{r,s}\}, \\
&\quad r = 1..u, s = 1..k-1 \\
N(x^2_{r,s\equiv 1 \text{mod} 2}) \cap D &= \{x^2_{r,s-1}, x^2_{r,s+1}, x^1_{r,s}\}, \\
&\quad r = 1..u, s = 2..k \\
N(x^1_{r,k}) \cap D &= \{x^1_{r,k-1}, x^2_{r,m}\}, r = 1..u \\
N(x^2_{r,1}) \cap D &= \{x^2_{r,2}, x^1_{r,1}\}, r = 1..u \\
N(x_r) \cap D &= \{x^1_{r,s}; s \equiv 1 mod 2\} \cup \\
&\quad \{x^2_{r,s}; s \equiv 0 mod 2\},
\end{aligned}$$

$$
\begin{aligned}
& & & r = 1..u \\
N(y^1_{r,s\equiv 0\mathrm{mod}2}) \cap D &=& & \{y^1_{r,s-1}, y^1_{r,s+1}, y^2_{r,s}\} \\
& & & r = 1..u, s = 1..k-1 \\
N(y^2_{r,s\equiv 1\mathrm{mod}2}) \cap D &=& & \{y^2_{r,s-1}, y^2_{r,s+1}, y^1_{r,s}\}, \\
& & & r = 1..u, s = 2..k \\
N(y^1_{r,k}) \cap D &=& & \{y^1_{r,k-1}, y^2_{r,k}\}, r = 1..u \\
N(y^2_{r,1}) \cap D &=& & \{y^2_{r,2}, y^1_{r,1}\}, r = 1..u \\
N(y_r) \cap D &=& & \{y^1_{r,s}; s \equiv 1 mod 2\} \cup \\
& & & \{y^2_{r,s}; s \equiv 0 mod 2\}, \\
& & & r = 1..u,
\end{aligned}
$$

2) for k odd

$$
\begin{aligned}
N(x^1_{r,s\equiv 0\mathrm{mod}2}) \cap D &=& & \{x^1_{r,s-1}, x^1_{r,s+1}, x^2_{r,s}\}, \\
& & & r = 1..u \\
N(x^2_{r,s\equiv 1\mathrm{mod}2}) \cap D &=& & \{x^2_{r,s-1}, x^2_{r,s+1}, x^1_{r,s}\}, \\
& & & r = 1..u, s = 2..k-1 \\
N(x^2_{r,k}) \cap D &=& & \{x^2_{r,k-1}, x^1_{r,k}\}, r = 1..u \\
N(x^2_{r,1}) \cap D &=& & \{x^2_{r,2}, x^1_{r,1}\}, r = 1..u \\
N(x_r) \cap D &=& & \{x^1_{r,s}; s \equiv 1 \text{ mod } 2\} \cup \\
& & & \{x^2_{r,s}; s \equiv 0 \text{ mod } 2\}, \\
& & & r = 1..u \\
N(y^1_{r,s\equiv 0\mathrm{mod}2}) \cap D &=& & \{y^1_{r,s-1}, y^1_{r,s+1}, y^2_{r,s}\}, \\
& & & r = 1..u\} \\
N(y^2_{r,s\equiv 1\mathrm{mod}2}) \cap D &=& & \{y^2_{r,s-1}, y^2_{r,s+1}, y^1_{r,s}\}, \\
& & & r = 1..u, s = 2..k-1\} \\
N(y^2_{r,k}) \cap D &=& & \{y^2_{r,k-1}, y^1_{r,k}\}, r = 1..u \\
N(y^2_{r,1}) \cap D &=& & \{y^2_{r,2}, y^1_{r,1}\}, r = 1..u \\
N(y_r) \cap D &=& & \{y^1_{r,s}; s = 1 \text{ mod } 2\} \cup \\
& & & \{y^2_{r,s}; s \equiv 0 \text{ mod } 2\}, \\
& & & r = 1..u
\end{aligned}
$$

Based on the result of intersection among the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, It can be shown that the intersection are all different, and it is not empty set. The set D dominate all vertices in $V(SF_u \odot L_k)$. Thus, we can conclude that, for $\gamma_{Li}(SF_u \odot L_k) \leq 2uk$, it satisfies the properties of LIDS. Since $\gamma_{Li}(SF_u \odot L_k) \geq 2uk$ and $\gamma_{Li}(SF_u \odot L_k) \leq 2uk$, thus $\gamma_{Li}(SF_u \odot L_k) = 2uk$. $\square$

In the following results, we study the locating independent domination number of any graph. Along with this result we show three corollaries.

Theorem 2.9. *For $l \geq 2$, H_1 order n and H_2 order m, the locating independent domination number of $H_1 \odot^l H_2$ is:*

$$\gamma_{Li} = \begin{cases} (\Sigma_{i=1}^{l-1}\lceil \frac{i}{2} \rceil m^{l-i} + \lceil \frac{l-1}{2} \rceil + 1)(\gamma_{L_i}(H_2))|V(H_1)|; \\ \text{for } l \,\text{odd};\ diam(H_2) > 2 \\ (\Sigma_{i=1}^{l-1}\lceil \frac{i}{2} \rceil m^{l-i} + \lceil \frac{l-2}{2} \rceil + 1)(\gamma_{L_i}(H_2))|V(H_1)|; \\ \text{for } l \,\text{even};\ diam(H_2) > 2 \\ \sim; \text{for } diam(H_2) \leq 2; \gamma_{Li}(H_2) = \sim \end{cases}$$

Proof. Based on the definition, it can be seen that the corona $H_1 \odot^l H_2$ is developed from multiple copies of H_2 along with number of vertices of path P_l attached to the vertices of graph H_1 till the last vertex of the coronation of H_2 of degree l. Thus, to determine the the locating independent domination number of $H_1 \odot^l H_2$ is by considering the LIDS of path. According to [7], the LIDS of path lays at x_i with i is an odd number, thus the dominator of the path as corona graph builder is at l odd. Each l has a graph H_2, for corona of degree 1 has one graph H_2, corona of degree 2 has two graphs H_2 and soo on, where $2 \leq l \leq l-1$. Thus, for the graph coronation consisting one graph H_2 excludes the coronation of degree 1. It implies the addition 1 at the end of the process is done. Since the H_2 graphs of degree 1 are also as dominators, based on Lemma 1.1, the dominator is located in LIDS on graphs H_2. Based on Lemma 1.2, $\gamma_{Li}(H_2) > 2$. Finally, if $\gamma_{Li} \leq 2$ and $\gamma_{Li}(H_2) = \sim$ then $\gamma_{Li}(H_1 \odot^l H_2) = \sim$. It conclude the proof. $\square$

Corollary 2.1. *Let G be a corona of order two of path graphs P_n and P_m. For $n \geq 3$ and $m \geq 3$, the locating independent domination number of is $\gamma_{Li(P_n \odot^2 P_m)} = (m+1)(\lceil \frac{2m}{5} \rceil)(n)$.*

Proof. The graph $P_n \odot^2 P_m$ has vertex set $V(P_n \odot^2 P_m) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{x_{i,j,k}; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq m\} \cup \{y_{i,j}; 1 \leq i \leq$

$n, 1 \leq j \leq m\}$ and edge set $E(P_n \odot^2 P_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_{i,j} y_{i,j+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_i x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{x_i x_{i,j+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{i,j} x_{i,j,k}; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq m\} \cup \{x_{i,j} x_{i,j,k+1}; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq m-1\}$. The vertex and edge cardinality of graph are $\mid V(P_n \odot^2 P_m) \mid = n(m^2+2m+1)$, $\mid E(P_n \odot^2 P_m) \mid = 2nm^2 + 3nm - n - 1$ respectively. Thus, $\Delta(P_n \odot^2 P_m) = 2p(H_2) + 2 = 2m + 2$, $\delta(P_n \odot^2 P_m) = 2$, $diam(P_m) = n - 1$, and $\gamma_{Li}(P_m) = \lceil \frac{2m}{5} \rceil$.

Based on Theorem 2.9 the locating independent domination number of $P_n \odot^2 P_m = (m+1)(\lceil \frac{2m}{5} \rceil)(n)$. Figure 3 shows LIDS of corona order two of path graph and path graph.

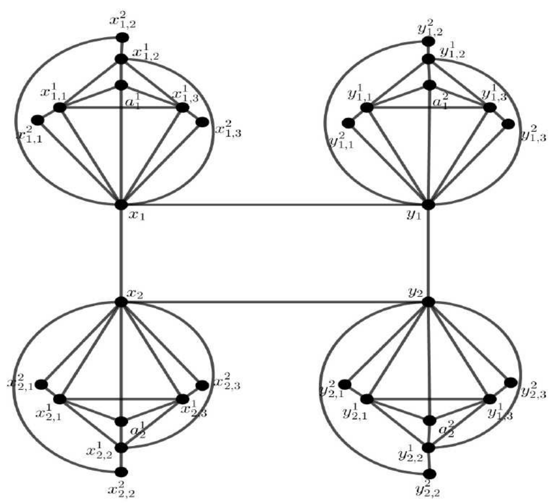

FIGURE 3. Example of LIDS $P_3 \odot^2 P_3$.

$\square$

Corollary 2.2. *If G is graph of corona order two of path graph P_n and Star graph S_m for $n \geq 3$ and $m \geq 3$, thus $\gamma_{Li(P_n \odot^2 S_m)} = \sim$.*

Proof. The Graph $P_n \odot^2 S_m$ have vertex set $V(P_n \odot^2 S_m) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{x^k_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq m\} \cup \{y_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(P_n \odot^2 S_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_{i,j} y_{i,m}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_i x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{x_{i,j} x_{i,m}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{i,j} x_{i,j,k}; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq$

$k \leq m\} \cup \{x_{i,j,k}x_{i,j,k}; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq m-1\}$. The vertex and edge cardinality of graph are $| V(P_n \odot^2 S_m) |= nm^2 + 2nm + n$, $| E(P_n \odot^2 S_m) |= 2nm^2 + 3nm - n - 1$ respectively. Thus, $\Delta(P_n \odot^2 S_m) = 2p(H_2) + 2 = 2m + 2$, $\delta(P_n \odot^2 S_m) = 2$, $diam(S_m) = 2$, and $\gamma_{Li}(S_m) = m$. Based on Lemma 1.2 the locating independent domination number of $\gamma_{Li(P_n \odot^2 S_m)} = \sim$. Figure 4 shows the illustration of LIDS of corona of degree two of path graph and star graph.

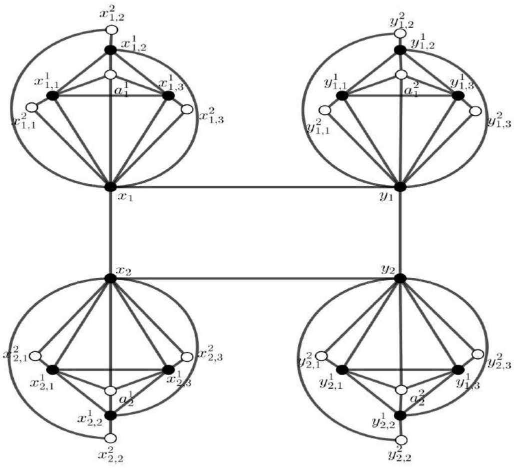

FIGURE 4. Example of LIDS $P_3 \odot^2 S_3$.

□

Corollary 2.3. *If G is graph of corona of degree two of star graph S_n and cycle graph C_m for $n \geq 3$ and $m \geq 5$, then* $\gamma_{Li(S_n \odot^3 C_m)} = (m^2 + m + 2)(\lceil \frac{2m}{5} \rceil)(n)$.

Proof. Based on Lemma 1.1, the dominator of graph $S_n \odot^3 C_m$ lays on the outer leaf of graph H_2. Based on Theorem 2.9, the graph $S_n \odot^3 C_m$ has a path as many as 3, thus $\gamma_{Li(S_n \odot^3 C_m)} = (m^2 + m + 2)(\lceil \frac{2m}{5} \rceil)(n)$. Figure 5 shows the illustration of LIDS of corona of degree tree of star graph and cycle graph.

□

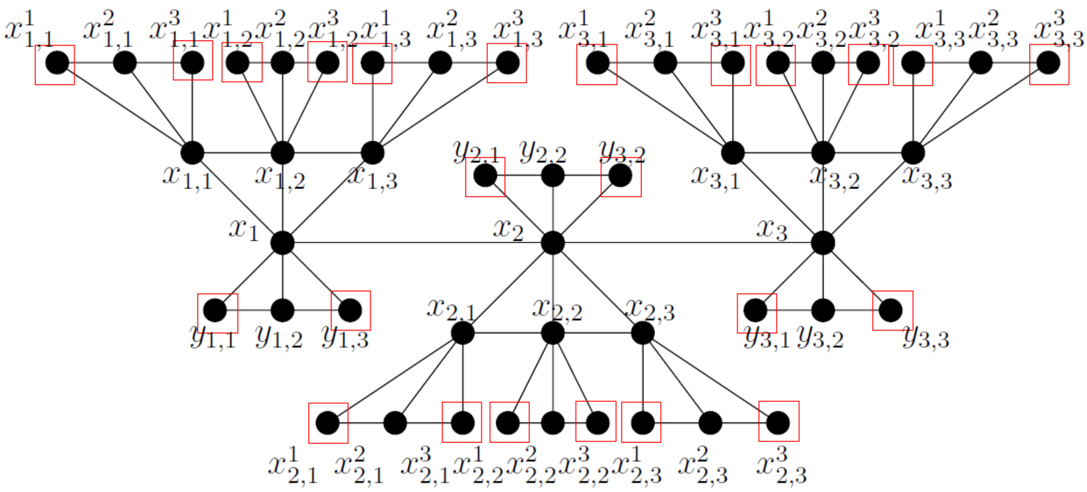

FIGURE 5. Example of LIDS $S_3 \odot^2 C_5$.

3. CONCLUDING REMARKS

In this paper, we have determined the locating independent dominating number of some coronation of graphs. However, to determine the locating independent dominating number is considered to be a NP-complete problem. Thus, finding the locating independent dominating number of other graph is still strongly recommended. Therefore, we propose the following open problem.

Open Problem 3.1. *Let G and H be a connected graph, determine the locating independent dominating number of other operation of the two graphs.*

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