ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal **9** (2020), no.7, 4907–4916
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.7.55

BULK VISCOUS FLUID BIANCHI TYPE-I STRING COSMOLOGICAL MODEL WITH NEGATIVE CONSTANT DECELERATION PARAMETER

KANGUJAM PRIYOKUMAR SINGH AND JITEN BARO[1]

ABSTRACT. Here we have studied a Bianchi Type-I string cosmological model with bulk viscous fluid and negative constant deceleration parameter in general relativity. To solve the survival field equations here we assumed that the shear scalar and scalar expansion are directly proportional to each other $\sigma\infty\theta$. The geometrical as well as physical features of the model are obtained and discussed. The model universe starts at initial epoch $t = 0$ with 0 volume and then expand with accelerated rate. The model universe obtained here is non shearing. The coefficient of bulk viscosity plays an important role in the cosmological consequences.The tension density diminishes with faster rate than particle density in the evolution of universe which shows that the present day universe is particle dominated.

1. INTRODUCTION

One of the tough problem for the researcher is to obtained the actual physical state of the universe at the very early days of its formation. Strings cosmological models are studied widely in present days due to their major contribution in the study of the evolution of the universe in early stages after the big bang. According to the grand unified theories (Everett [1], Vilenkin [2]), those strings was formed during the transition of phases when the temperature went down

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 85A40,58D30,83F05.

Key words and phrases. Cloud String, Bianchi Type-I, bulk viscosity, general relativity.

beneath some critical temperature soon after the explosion of big-bang . Letelier [3] and Stachel [4] are the two prominent authors who initiated to study about the strings. Letelier solved the Einstein's field equation and obtained the solutions for a cloud of strings with the plane, spherical, and cylindrical symmetry. He also solved the same equation for the cloud of massive strings in the year 1983, and constructed the cosmological models in the Bianchi Type-I Space-time.

In the evolution of the universe the bulk viscosity contribute a significance role. It can arise in different circumstances and can lead to constructive mechanism of the formation of galaxies. The amplitude of the bulk viscous stress with respect to the expansion can be determined by means of the coefficients of bulk viscosity. The homogeneous and anisotropic Bianchi type-I cosmological models are considered to understand the evolution in the early stages of the universe. Many authors have tried to find the precise solutions of field equations by the way of considering viscous consequences in general relativity in isotropic as well as anisotropic cosmological model. Misner [5, 6], studied about the consequences of bulk viscosity in the evolution of cosmological model. Nightingale [7] has obtained the importance of viscosity in cosmology within the evolution from the early epoch of the universe. Wang [8] constructed a Bianchi type-III cosmological models with string within the framework of general relativity considering the bulk viscous fluid. The behavior of the Bianchi type-III cosmological models with strings in general relativity with and without bulk viscosity are discussed by Bali and Pradhan [9]. Kandalkar et al. [10] constructed cosmic string in Bianchi type-I cosmological model with bulk viscosity. Also, Kandalkar et al. [11] investigated a Bianchi-V cosmological models in general relativity with constant deceleration parameter and viscous fluid. Humad et al [12] constructed a string cosmological model in Bianchi type-I space time with the help of Bulk viscosity in context of general relativity. Rao et al. [13], Singh [14], Tripathi et al. [15], Pradhan and Jaiswal [16], Dubey et al. [17], Singh and Daimary [18], are some of the prominent authors who have investigated various string cosmology in the Bianchi models with bulk viscosity in different space-time.

Here we have attempted to find a model in cosmology with string with Bianchi type-I space-time by considering the deceleration parameter(q) as a constant quantity in general relativity with bulk viscous fluid. In the first Section of this paper we discussed a brief introduction of Bianchi type string cosmological models, In second Section a Bianchi type-I metric is presented and the field equations in general relativity are derived. We determined the solutions of the survival field equations in the Section 3. In Section 4, physical and geometrical behavior of our model are discussed and then conclusions are given in last Section.

2. METRIC AND FIELD EQUATIONS

Here we take the Bianchi type-I metric as

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2) + b^2 dz^2 \, , \tag{2.1}$$

where a(t) and b(t) are the metric functions of 't'.

The Einstein's field equation $(8\pi G = 1, C = 1)$in general relativity is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \, . \tag{2.2}$$

For a cloud string with bulk viscous fluid, the energy-momentum tensor is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j) \, . \tag{2.3}$$

Here, $\lambda = \rho - \rho_p$ is tension density, ρ is energy density and ρ_p is particle density, u^i is four velocity vector of particles and x^i is unit space-like vector which gives the direction of string, given by

$$u^i = (0,0,0,1) \; and \; x^i = (a^{-1}0,0,0) \tag{2.4}$$

such that,

$$u_i u^j = -1 = -x_i x^j \; and \; u_i x^i = 0 \, . \tag{2.5}$$

The spatial volume, scalar expansion, Hubble parameter, shear scalar and mean anisotropy parameter are respectively given by

$$V = a^2 b = R^3 \tag{2.6}$$

$$\begin{aligned}
\theta &= u^i_{,i} = 2\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \\
H &= \frac{1}{3}(2\frac{\dot{a}}{a} + \frac{\dot{b}}{b}) \\
\sigma^2 &= \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}[2(\frac{\dot{a}}{a})^2 + (\frac{\dot{b}}{b})^2] - \frac{\theta^2}{6} \\
\Delta &= \frac{1}{3}\sum_{i=1}^{3}(\frac{H_i - H}{H})^2 \, ,
\end{aligned}$$

where, H_i (i=x,y,z)are defined as $H_x = H_y = \frac{\dot{a}}{a}$, and $H_z = \frac{\dot{b}}{b}$ for the metric (2.1).

Using (2.3)-(2.5) in (2.2) yields

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = \xi\theta \tag{2.7}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \lambda + \xi\theta \tag{2.8}$$

$$\frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}\dot{b}}{ab} = \rho \, , \tag{2.9}$$

where the overhead dots denotes the order of derivative w.r.t. time ‘t’.

3. Solution of the Field Equations

We have 3 highly nonlinear independent differential equations (2.7)-(2.9) with five unknowns variables a, b,ρ, λ and ξ. So to find exact solution we must used two extra plausible conditions. So here we used the following assumptions:

The shear scalar and scalar expansion are directly proportional to each other, $\sigma \propto \theta$ leading to the equation:

$$a = b^n \, , \tag{3.1}$$

where $n \neq 0$ is a constant.

The above assumption is based on observations of velocity and red-shift relation for an extragalactic source which predicted that the Hubble expansion is 30 percent isotropic, which is supported by the works of Thorne [19], Kantowski and Sachs [20], Kristian and Sachs [21]. In particular, it can be said that $\dfrac{\sigma}{H} \geq 0.30$, where σ and H are respectively shear scalar and Hubble constant. Also, Collins et al. [22] has shown that if the normal to the spatially homogeneous line element is congruent to the homogeneous hyper-surface then $\dfrac{\sigma}{\theta}$ =constant, θ being the expansion factor.

Also, Berman's [23] suggestion regarding variation of Hubble's parameter H provides us a model universe that expands with constant deceleration parameter. So for the determinate solution, let us take deceleration parameter to be a negative constant-

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = h \; (constant) \,. \tag{3.2}$$

It is well known that when q is negative then the model universe expand with acceleration, and when q is positive then it explains a decelerating(contracting) universe. Although the present observations like CMBR and SNe Ia suggested the negative value of q but it can be remarkably state that they are not able to deny about the decelerating expansion(positive q) of universe.

Solving (3.2), we get

$$R(t) = (lt + m)^{\frac{1}{1+h}} \; h \neq -1 \,, \tag{3.3}$$

where l, m, are constants of integration.

Using (2.6), (3.1) and (3.3) we get,

$$a = (lt + m)^{\frac{3n}{(1+h)(2n+1)}}, \; b = (lt + m)^{\frac{3}{(1+h)(2n+1)}} \,.$$

With the suitable choice of coordinates and constant we can take (l=1 and m=0) and then

$$a = t^{\frac{3n}{(1+h)(2n+1)}}, \; b = t^{\frac{3}{(1+h)(2n+1)}} \,.$$

The metric (2.1) reduces to

$$ds^2 = -dt^2 + t^{\frac{6n}{(1+h)(2n+1)}}(dx^2 + dy^2) + t^{\frac{6}{(1+h)(2n+1)}} dz^2 \,, \tag{3.4}$$

which gives the geometry of the metric (2.1).

4. Physical and Geometric Parameters

The tension density, energy density and particle density of the model (3.4) are obtained as-

$$\begin{aligned}
\lambda &= \frac{3(2-h)(n-1)}{(1+h)^2(2n+1)t^2}\,, \\
\rho &= \frac{9n(n+2)}{(1+h)^2(2n+1)^2t^2}\,, \\
\rho_p &= \frac{3[h(2n^2-n-1)-(n^2-8n-2)]}{(1+h)^2(2n+1)^2t^2}\,.
\end{aligned}$$

The spatial volume is

$$V = t^{\frac{3}{(1+h)}}\,.$$

The expansion scalar is

$$\theta = \frac{3}{(1+h)t}\,.$$

Hubble parameter is

$$H = \frac{1}{(1+h)t}\,.$$

The bulk viscosity of the model is

$$\xi = \frac{-h(2n^2+3n+1)+2}{(1+h)(2n+1)^2t}\,.$$

The shear scalar of the model is

$$\sigma = \frac{\sqrt{3}(n-1)}{(1+h)(2n+1)t}\,. \tag{4.1}$$

The mean anisotropy parameter is

$$\Delta = \frac{2(n-1)^2}{(2n+1)^2} = Constant. \tag{4.2}$$

5. Physical Interpretations

The model given by the equation (3.4) is a Bianchi type-I cosmological model with string in general relativity with constant deceleration parameter(q=constant) and bulk viscosity. The variation of some of the features with time for the model are shown below by taking $n=2, h=-0.5$.

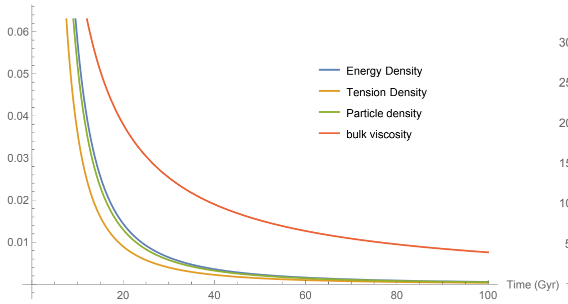

FIGURE 1. Variation of ρ, λ, ρ_p, ξ vs. t.

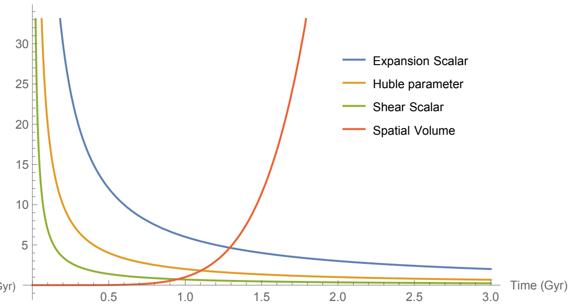

FIGURE 2. Variation of θ, H, σ, V vs. t.

The geometrical and physical behaviors of the model universe for $(1 + h > 0)$ are discussed as

(i) The tension density (λ), energy density (ρ) and particle density (ρ_p) all are infinite as $t = 0$, and are decreasing functions of time t and they all become 0 as $t \to \infty$, Figure 1 , which indicates that the universe starts at $t = 0$ and expand with time. Hence the model admits initial singularity at $t = 0$. This model satisfies the energy density conditions $\rho \geq 0$ and $\rho_p \geq 0$. It is also observed that $\dfrac{\rho_p}{|\lambda|} > 1$ which shows that tension density of string diminishes more quickly than particle density, so the late universe is particle dominated.

(ii) The bulk viscosity $\xi \to \infty$ when t=0 and it decreases with the increasers of time and finally when $t \to \infty$ bulk viscosity ξ vanishes, Figure 1.

(iii) Initially at $t = 0$ the spatial volume is 0 for this model and as time increases the volume also increases, Figure 2. It reaches to infinite value at $t \to \infty$ and so the model represents an expanding universe with respect to time.

(iv) At the initial epoch $t = 0$, the scalar expansion θ as well as Hubble parameter H both are infinite and as the time progresses gradually they decreases and finally they become 0 when $t \to \infty$, Figure 2. Hence the model shows that the universe is expanding with time but the rate of expansion become slow with the increases of time and the expansion end at $t \to \infty$. Since $\dfrac{dH}{dt}$ is negative quantity which also explained that our model universe is expanding with acceleration.

(v) From equation (4.1) and Figure 2 it seen that the value of the shear scalar σ is infinite at initial epoch and decreases with time and become zero at late universe showing that the universe obtained here is shear free in the late time.

(vi) From (4.2) the mean anisotropy parameter $\Delta = constant \neq 0$ for $n \neq 1$ and $\Delta = 0$ for $n = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = constant \neq 0$ for $n \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $n = 1$. From both statements we can conclude at late time the universe is anisotropic, when $n \neq 1$ but it is isotropic for $n = 1$ throughout evolution.

6. Conclusion

Here, we have constructed a Bianchi type-I string cosmological models with the help of bulk viscosity and constant deceleration parameter in general relativity. The parameters which are very important in the study of cosmological models are obtained and discussed. The model is expanding,non shearing, anisotropic for $n \neq 1$ and isotropic for $n = 1$. The present universe starts at initial epoch at $t = 0$ with 0 volume and then expand with accelerated rate and the rate of expansion becomes slow with increase of time. The bulk viscosity coefficient plays a significance role in the cosmological consequences. The tension density diminishes with the faster rate than particle density which shows that present day universe is dominated by particles.

References

[1] A. E .Everett: *Cosmic Strings in Unified Gauge Theories*, Phys. Rev. D, **24** (1981), 858.

[2] A. Vilenkin: *Cosmic Strings*, Phys. Rev. D, **24** (1981), 2082-2089.

[3] P. S. Letelier: *Clouds of Strings in General Relativity*, Phys.Rev.D, **20** (1979), 1249.

[4] J .Stachel: *Thickening the String. I. The String Perfect Dust*, Phys Rev D, **21** (1980), 2171-2181.

[5] C. W. Misner: *Transport processes in the primordial fireball*, Nature, **214** (1967), 40.

[6] C. W. Misner: *The isotropy of the universe*, Astrophys.J, **151** (1968), 431.

[7] J. D. Nightingale: *Independent investigations concerning bulk viscosity in relativistic homogeneous isotropic cosmologies*, Astrophys.J., **185** (1973), 105-114.

[8] X. X. WANG: *Bianchi type-III string cosmological model with bulk viscosity in general relativity*, Chin.phys.lett, **22** (2005), 29-32.

[9] R. BALI, A. PRADHAN: *Bianchi type-III string cosmological models with time dependent bulk viscosity*, Chin.Phys.Lett., **24** (2007), 585-588.

[10] S. P. KANDALKAR, S. P. WASNIK, S. P. GAWANDE: *Some Bianchi type-I string Cosmological models with Bulk Viscosity*, Rom.Jour.Phys, **56** (2011), 289-296.

[11] S. KANDALKAR, P. KHADE, M. GAIKWAD: *Bianchi-V cosmological models with viscous fluid and constant deceleration parameter in general relativity*, Turk J Phys. **36** (2012), 141-154.

[12] V. HUMAD, V. NAGAR, S. SHRIMALI: *Bulk viscous fluid Bianchi type-I string cosmological model in general relativity*, IOSR-JM, **12** (2016), 11-15.

[13] V. U. M. RAO, T. VINUTHA, M. V. SANTHI: *Bianchi Type-V Cosmological Model with perfect fluid using Negative Constant Deceleration Parameter in a Scalar Tensor Theory based on Lyra Manifold*, Astrophys. Space Sci, **314** (2008), 213-216.

[14] C. P. SINGH: *String Cosmology with Magnetized Bulk Viscous Fluid in Bianchi I Universe*, Astrophys. Space Sci., **343** (2013), 773-781.

[15] B. R. TRIPATHI, A. TYAGI, S. PARIKH: *Bianchi Type-I Inhomogeneous String Cosmological Model with Electromagnetic Field in General relativity*, Prespacetime J, **8** (2017), 474-483.

[16] A. PRADHAN, R. JAISWAL: *Magnetized String Cosmological Models of Acceleration*, Int. J. Geom. Meth. Mod. Phys., **15**(5) (2018), ID1850076.

[17] R. K. DUBEY, B. V. SHUKLA, N. YADAV: *On mathematical analysis for bianchi typeâĂŞI string cosmological model in modified theory of relativity*, Phys Astron Int J, **2** (2018), 143-146.

[18] K. P. SINGH, M.DAIMARY: *Anisotropic Cloud String Cosmological Model with Bianchi Type-I Space-Time in General Relativity*, The African Rev. of Phys., **14** (2019), 94-99.

[19] K. S. THORNE: *Primordial Element Formation, Primrdial Magnectic Fields and Isotropy of the Universe,* Astrophys., J., **148** (1967), 51.

[20] R. KANTOWSKI, R. K. SACHS: *Some Spatially homogeneous anisotropic relativistic cosmological models*. J.Math. Phys., **7** (1966), 443-446.

[21] J. KRISTIAN, R. K. SACHS: *Observations in Cosmology*, Astrophys.J., **143** (1966), 379-399 .

[22] C. B. COLLINS, E. N. GLASS, D. A. WILKINSON: *Exact spatially homogeneous cosmologies* Gen. Relative.Gravit., **12** (1980), 805-823.

[23] M. S. BERMAN: *A special law of variation for Hubble's parameter*, Nuov Cim B, **74** (1983), 182-186.

Department of Mathematical Sciences
Bodoland University
Kokrajhar-783370, BTC, Assam, India
Department of Mathematics
Manipur University
Imphal-795003 Manipur, India
E-mail address: *pk_mathematics@yahoo.co.in*

Department of Mathematical Sciences
Bodoland University
Kokrajhar-783370, BTC, Assam, India
E-mail address: barojiten5@gmail.com