Advances in Mathematics: Scientific Journal **9** (2020), no.7, 4917–4922
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.7.56

SOME PROPERTIES OF G-SPACES ACTED WITH TOPOLOGICAL GROUPS

S. SIVAKUMAR AND D. LOHANAYAKI[1]

ABSTRACT. In this paper, We mainly discuss some properties of O-soft and the character of limit G-space and extend several results of subspectrum. We also introduce some basic concepts and elementary results on actions of topological groups on G-Spaces.

1. INTRODUCTION

On G-Spaces of topological groups on topological spaces provide a natural general background dynamics. At the heart of it is the fundamental idea of a topological group of transformations and the roots of the notion of O-Soft and we also introduce the fundamental concept of a topological group on G-space. In this paper we define the basic definition of G-space and more details on the topology of the limit G-Space of inverse spectrum. Since 2008, Arhangel'skill's has done a series of sinificant work on Actions of topological groups with topological spaces.

For further reference see [1–6].

Definition 1.1. *Let G be a topological group and X be a topological space. An action θ of G on X is called continuous if θ is continuous as a mapping of $G \times X$ to X. The space X is called a G-space.*

Definition 1.2. *A continuous mapping $p : A \to B$ is called O-soft if for every zero-dimensional compact G-space C, every continuous mapping $q : C \to B$ and a*

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 08A99,20J99.

Key words and phrases. G-space, O-soft, limit G-Space, Subspectrum.

$$\begin{tikzcd} X \arrow[r, hook] \arrow[d] & C \arrow[d, "q"] \arrow[dl, "\psi"'] \\ r \arrow[r, "p"'] & B \end{tikzcd}$$

FIGURE 1

continuous mapping $r : X \to A$ of a closed subset X of C satisfying q $\lceil X = p \circ r$, there exists a continuous mapping $\psi : C \to A$ extending r which makes the above diagram commutative.

Definition 1.3. *Consider the product G-Space $\prod = \prod_{\beta<\gamma} A_\beta$ and denote by $\prod_\beta$ the projection of $\prod$ to the factor A_β where $\beta < \gamma$. Let A be the G-Subspace of $\prod$ defined by $A = \{(\alpha_\beta)_{\beta<\gamma} \in_{\prod}: S^\alpha_\beta(\alpha_\alpha) = a_\beta$ whenever $\beta < \alpha < \gamma\}$ then A is the limit G-Space of the Spectrum P.*

Definition 1.4. *Let $P = A_\beta, S^\alpha_\beta : \beta < \alpha < \gamma$ be a well ordered inverse spectrum with continuous projections S^α_β, and X be the limit G-Space of P. For every $\alpha < K$, denote by S_β the limit projection of A to A'_β. If $B \subset A$ is a G-Space of A, we put $B_\beta = S_\beta(b)$ for each $\beta < \gamma$ and define W^α_β as the restriction of S^α_β to B_α, where $\beta < \alpha < \gamma$. Clearly, $P_B = \{b_\beta, W^\alpha_\beta : \beta < \alpha < \gamma\}$ is a well-ordered inverse spectrum; this spectrum is called the subspectrum of P generated by B.*

2. MAIN RESILTS

Theorem 2.1. *The composition of O-soft mapping of compact G-Spaces is O-soft.*

Proof. Let $P_1 : A_1 \to A_2$ and $P_2 : A_2 \to A_3$ be O-soft mapping of compact G-Spaces A_1, A_2, A_3, and $P = P_2 \circ P_1$. suppose that $q : C \to A_3$ and $r : X \to A_1$ are a continuous mapping such that $P \circ r = q \circ s$, where X is a closed subsetof a zero-dimensional compact G-Space C, and $j : X \to C$ is natural embedding.

Put $K_1 = P_1 \circ K$. Since P_2 is O-soft, K_1 can be extended to a continuous mapping $\psi_1 : C \to A_2$ such that $P_2 \circ \psi_2 = q$.

Clearly, the mapping ψ satisfies the equality $P_1 \circ \psi = P_2 \circ \psi_1$, So the mapping

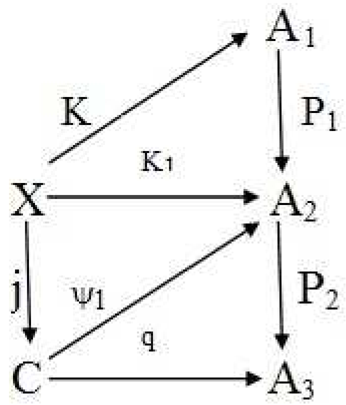

FIGURE 2

Again, since P_1 is O-soft, we can find a continuous mapping $\psi : C \to A$, extending K such that the diagram below commutes.

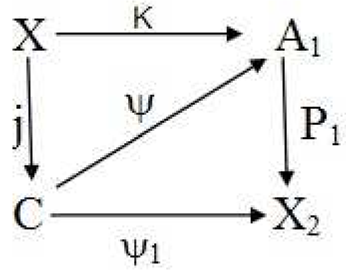

FIGURE 3

P is O-soft. □

Theorem 2.2. *Let A be the limit G-Space of a well-ordered inverse spectrum $P = \{A_\beta, S_\beta^\alpha : \beta < \alpha < \gamma\}$ with Hausdorff G-Space A_β. Then A is closed G-subSpace of the product G-Space $\prod = \prod_{\beta<\gamma} A_\beta$. In addition, the sets of the from $(S_\beta)^{-1}(O)$, where $\beta < \gamma$, $S_\beta : A \to A_\beta$ is the limit projection and O is open in A_β, constitute a base for the topology of A.*

Proof. Take an arbitrary $a = (a_\beta)_{\beta<\gamma}$ in $\prod \backslash A$
By the definition of A, there exist α, β with $\beta < \alpha < \gamma$ such that $S_\beta^\alpha(a_\alpha) \neq a_\beta$. Since A_β is a Hausdorff G-Space and the connecting mapping S_β^α is continuous, we can find open neighbourhoods X and Y of a_α and a_β in A_α and A_β respectively,such that $S_\beta^\alpha(X) \cap Y = \phi$. It is clear that the set $C = (\prod_\alpha)^{-1}(X) \cap$

$(\prod_\beta)^{-1}(Y)$ is open in $\prod$ and $C \cap A = \phi$ where $\prod_\beta$ and $\prod_\alpha$ are the projections of $\prod$ to the factors A_β and A_α respectively, therefore the complement $\prod \backslash A$ is open in $\prod$.

To prove the second part of the theorem, take any $a \in A$ and any neighbourhood X of a in A . Since A is a G-Subspace of $\prod$, there exists a canonical open set Y in $\prod$ such that $a \in Y \cap A \subset X$. Choose ordinals $\beta_1 < \cdots < \beta_n < K$ and open sets $Y_1, Y_2, \ldots, Y_n$ in $A_{\beta_1}, \ldots\ldots A_{\beta_n}$ respectively, such that $Y = (\prod_{\beta_1})^{-1}(Y_1) \cap \cdots \cap (\prod_{\beta_n})^{-1}(Y_n)$. Since each limit projection $S_\beta : A \to A_\beta$ is the restriction of $\prod_\beta$ to A, the open neighbourhood $O = (S_{\beta_1})^{-1}(Y_1) \cap \cdots \cap (\prod_{\beta_n})^{-1}(Y_n)$ of a in A is contained in X. Clearly the set $O = (S_{\beta_1}^{\beta_n})^{-1}(Y_1) \cap \cdots \cap (S_{\beta_n - 1}^{\beta_n})^{-1}(Y_{n-1}) \cap Y_n$ is open in Y_n is open in A_{β_n} and it follows that $a \in T = (S_{\beta_n})_{-1}(S) \subset X$. □

Theorem 2.3. *Let $P = \{A_\beta, S_\beta^\alpha : \beta < \alpha < \gamma\}$ be an inverse spectrum of G-Space A_β and continuous connecting mapping S_β^α then the limit G-Space A of P is compact.In addition, if each S_β^α is surjective, then so are the limit projections $S_\beta : A \to A_\beta$*

Proof. The product G-space $\prod = \prod_{\beta<\gamma} A_\beta$ is compact and X is a closed G-Subspace of $\prod$. Hence X is compact.

Suppose that each connecting mapping S_β^α is surjective. For every $\alpha < \gamma$, let $\prod_\alpha = \{(a \in \prod : S_\beta^\alpha(\prod_\alpha)(a)) = \prod_\beta(a)$ for each $\beta < \alpha\}$, where $\prod_\beta : \prod_\to A_\alpha$ is the natural projection of $\prod$ onto A_β. Clearly, $\prod_\alpha$ is compact, as a closed subset of $\prod$ and in addition,$\prod_\alpha(\prod_\alpha) = \prod_\alpha$, for each $\alpha < \gamma$. Indeed, let $b \in A_\alpha$, and take $a = (a_\beta)_{\beta<\gamma} \in \prod$ such that $a_\alpha = b, a_\beta = S_\beta^\alpha(b)$, for each $\beta < \alpha\}$, where the coordinates a_δ with $\alpha < \delta < K$ are chosen arbitrary. Then $a \in \prod_\alpha$ and $\prod_\alpha(a) = b$.

It follows from the definition of the the limit G-Space A of the spectrum P that $A = \cap_{\alpha<\gamma} \prod_\alpha$. Let $\beta < \gamma$ and $b \in A_\beta$ be arbitrary. Then $b \in \prod_\beta(\prod_\alpha)$ or, equivalantly, $(\prod_\beta)^{-1}(b) \cap \prod_\alpha \neq \phi$ for each α , where $\beta \leq \alpha < \gamma$. Since $\{\prod_\alpha : \beta \leq \alpha < \gamma\}$ is a decreasing sequence of compact subsets of $\prod$, and the set $(\prod_\beta)^{-1}(b)$ is closed in $\prod$. We conclude that the intersection $(\prod_\beta)^{-1}(b) \cap \cap_{\beta\leq\alpha<\gamma} \prod_\alpha = (\prod_\beta)^{-1}(b) \cap A$ is not empty.

Hence $b \in \prod_\beta(A) = S^\beta(A)$ and therefore $S_\beta(A) = A_\beta$. □

Theorem 2.4. *Suppose that A is a limit G-Space of a well-ordered inverse spectrum $P = \{A_\beta, S_\beta^\alpha : \beta < \alpha < \gamma\}$, and that B is a closed G-Subspce of A. Then the limit G-Space of the inverse spectrum P_β is naturally homeomorphic to B.*

Proof. By the definition of the subspectrum $P_B = \{b_\beta, W_\beta^\alpha : \beta < \alpha < \gamma\}$ of P that B can be identified with a G-Subspace of the limit G-Space $B^\star$ of P_B which is, in its turn, a G-Subspace of A. Therefore, it sufficies to show that $B^\star \subset B$. Suppose to the contrary that there exists a point $a \in B^\star \setminus B$. B is closed in A, So we can find an ordinal $\beta < \alpha$ and an open set $X \subset A_\beta$ such that $a \in (S_\beta)^{-1}(X) \subset A \setminus B$. Hence $(S_\beta)^{-1}(X) \cap B = \phi$ and $X \cap S_\beta(B) = \phi$, that is, does not intesect the set B_α , a contrdiction with $S_\beta(a) \in X \cap B_\beta$. □

3. CONCLUSION

In this paper, we have solved some properties of G-Spaces and related theorems.

ACKNOWLEDGMENT

The authors would like to thank the referee for the valuable comments, that helped us improve this article.

REFERENCES

[1] S. A. ANTONYAN, M. SANCHIS: *Extension of locally group actions*, Annali di Matematica Pura ed Applicata, **181** (2002), 239–246.

[2] A. V. ARHANGELSKII, M. HUSEK: *Extensions of topological and semitopological groups and the product operation,* Comment. Math. Univ. Carolin., **42**(1) (2001), 173âĂŞ186.

[3] A. V. ARHANGELSKII, M. TKACHENKO: *Atlantis Studies in Mathematics Series*, Editor: J. van Mill,VU University Amsterdam, the Netherlands, (ISSN: 1875-7634), 2008.

[4] S. SIVAKUMAR, P. PALANICHAMY: *On orbit space of B-compact group action*, Acta ciencia indica, **XXXVIIM**(4) (2011), 813–817.

[5] S. SIVAKUMAR, D. LOHANAYAKI: *Homotopy groups acted on fibration in algebraric topology*, Advances and applications in mathematical sciences, (in press).

[6] S. SIVAKUMAR, D. LOHANAYAKI: *Free group actions on finite metrizable spaces*, Journal of Xian university of architecture and technology, **12**(5) (2020), 1327-1335.

DEPARTMENT OF MATHEMATICS
GOVRNMENT ARTS COLLEGE
C. MUTLUR,CHIDAMBARAM TK 608102
E-mail address: ssk.2012@yahoo.in

DEPARTMENT OF MATHEMATICS
GOVRNMENT ARTS COLLEGE
C. MUTLUR,CHIDAMBARAM TK 608102
E-mail address: lohanayakiappar@gmail.com