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GEOMETRIC DECOMPOSITION OF GRAPH WITH VARIOUS COMMON RATIO

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ABSTRACT. Let $G = (V, E)$ be a simple connected graph with p vertices and q edges. A decomposition of G is a collection of edge disjoint subgraphs $G_1, G_2, \ldots, G_n$ of G such that every edge of G belongs to exactly one G_i. A decomposition $G_a, G_{ar}, G_{ar^2}, G_{ar^3}, ..., G_{ar^{n-1}}$ of G is said to be a Geometric Decomposition(GD) if each $G_{ar^{i-1}}$ is connected and $|E\left(G_{ar^{i-1}}\right)| = ar^{i-1}$, for every $i = 1, 2, 3, ...n$ and $a, r \in N$.Clearly $q = \frac{a(r^n-1)}{r-1}$. In this paper we give Geometric Decomposition of graphs with first term as 1 and 3 ie., $a = 1, 3$ along with various common ratio.The Geometric Decomposition of graphs such as Fan graph, $K_{1,m} + K_1$, Ladder graph, Book graph, Bistar graph, Friendship graph are found.

1. INTRODUCTION

All graphs in this paper are finite and simple.Graph decomposition problems rank among the most noticeable area of research in graph theory and combinatorics and further it has abundant applications in various fields such as bio informatics, block diagram and networking.A decomposition of G is a collection of edge disjoint subgraphs $G_1, G_2, ..., G_n$ of G such that every edge of G belongs to exactly one G_i.A decomposition $G_1, G_2, ..., G_n$ of G is said to be a Continuous

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Monotonic Decomposition if each G_i is connected and $|E(G_i)| = i$, for every $i = 1, 2, 3, ...n$. Various types of decomposition of graphs have been studied in literature by imposing suitabe conditions on the subgraphs G_i .The concept of Continuous Monotonic Decomposition of graphs was introduced by N. Gnana Dhas , J. Paulraj Joseph, [4]. We use terminology of [5]. Ebin Raja Merly and D.Subitha introduced the concept of Geometric Decomposition [2] and studied about the Geometric Decomposition of complete tripartite graph with $a = 1$ and $r = 2$, see [3]. If the subgraphs of the Geometric Decomposition is star (path) then the decomposition is said to be Geometric star (path) decomposition.Also the definition of Fan graph can be studied in [1,6]. In this paper we have studied that the Geometric Decomposition of graphs with first term as 1 and common ratio as 3, similarly $a = 1, r = 4$, and $a = 3, r = 2$.

Definition 1.1. *[2] A decomposition $G_a, G_{ar}, G_{ar^2}, G_{ar^3}, ..., G_{ar^{n-1}}$ of G is said to be a Geometric Decomposition if each $G_{ar^{i-1}}$ is connected and $|E(G_{ar^{i-1}})| = ar^{i-1}$, for every $i = 1, 2, 3, ...n$ and $a, r \in N$.Clearly $q = \frac{a(r^n-1)}{r-1}$.*

2. Main Results

Theorem 2.1. *The graph G is Geometric Decomposition$(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$ iff $q = \frac{4^n-1}{3}$ for each $n \in N$.*

Proof. Let G be a connected graph with $q = \frac{4^n-1}{3}$ .Let v, w be two vertices of G such that $d(v, w)$ is maximum.Let $N_r(v) = \{w \in V/d(v, w) = r\}$. If $d(v) = 4^{n-1}$, choose 4^{n-1} edges incident with v.Let $G_{4^{n-1}}$be a subgraph induced by these 4^{n-1} edges . If $d(v) < 4^{n-1}$, then choose 4^{n-1} edges incident with vertices of $N_1(v), N_2(v), ...$ successively such that the subgraph $G_{4^{n-1}}$ induced by these edges is connected . In both cases $G - G_{4^{n-1}}$ has a connected component C_1 with $\frac{4^n-1}{3} - 4^{n-1}$ edges.Now consider C_1 and proceed as above to get $G_{4^{n-2}}$ such that $C_1 - G_{4^{n-2}}$ has a connected component C_2 of size $\frac{4^n-1}{3} - 4^{n-1} - 4^{n-2}$ edges.Proceeding like this we get a connected subgraph G_4 such that $C_{4^{n-2}}$ is a graph with one edge taken as G_1.Thus $G_1, G_4, G_{4^2}, ..., G_{4^{n-1}}$is a GD of G.
Conversely suppose G admits Geometric Decomposition$(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$.
Then $q(G) = 1 + 4 + 4^2 + ... + 4^{n-1}$.
$q(G) = \frac{4^n-1}{3}$ for each $n \in N$. □

Theorem 2.2. *The fan graph* $P_m + K_1$ *accepts Geometric Decomposition*$(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$ *iff* $m = \frac{4^n+2}{6}$ *for each* $n \in N$.

Proof. Let $V = \{u, v_1, v_2, ...v_m\}$be the vertex set and $E = \{v_i, v_{i+1}/1 \le i \le m-1\} \cup \{uv_i/1 \le i \le m\}$ be the edge set.Assume that a fan graph $P_m + K_1$ accepts Geometric Decomposition$(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$. We have $q(P_m + K_1) = 2m - 1$. By above theorem 2.1 the graph P_m+K_1 accepts Geometric Decomposition$(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$ iff $q(P_m + K_1) = \frac{4^n-1}{3}$. $\Rightarrow 2m - 1 = \frac{4^n-1}{3} \Rightarrow m = \frac{4^n+2}{6}$.
Conversely assume $(P_m + K_1)$ with $m = \frac{4^n+2}{6}$.
Here $q(P_m + K_1) = 2\left(\frac{4^n+2}{6}\right) - 1 = \frac{4^n-1}{3}$.
This implies that $P_m + K_1$ can be decomposed into $(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$. □

Corollary 2.1. *The graph* $G \cong K_{1,m}+K_1$ *accepts Geometric Decomposition*$(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$ *iff* $m = \frac{4^n-4}{6}$ *for each* $n \in N$.

Theorem 2.3. *The graph* G *is Geometric Decomposition*$(G_1, G_3, G_{3^2}, ..., G_{3^{n-1}})$ *iff* $q = \frac{3^n-1}{2}$ *for each* $n \in N$.

Proof. Let G be a connected graph with $q = \frac{3^n-1}{2}$ .Let v, w be two vertices of G such that $d(v, w)$ is maximum.Let $N_r(v) = \{w \in V/d(v, w) = r\}$. If $d(v) = 3^{n-1}$, choose 3^{n-1} edges incident with v.Let $G_{3^{n-1}}$be a subgraph induced by these 3^{n-1} edges . If $d(v) < 3^{n-1}$, then choose 3^{n-1} edges incident with vertices of $N_1(v), N_2(v), ...$ successively such that the subgraph $G_{3^{n-1}}$ induced by these edges is connected . In both cases $G - G_{3^{n-1}}$ has a connected component C_1 with $\frac{3^n-1}{2} - 3^{n-1}$ edges.Now consider C_1 and proceed as above to get $G_{3^{n-2}}$ such that $C_1 - G_{3^{n-2}}$ has a connected component C_2 of size $\frac{3^n-1}{2} - 3^{n-1} - 3^{n-2}$ edges.Proceeding like this we get a connected subgraph G_3 such that $C_{3^{n-2}}$ is a graph with one edge taken as G_1.Thus $G_1, G_4, G_{4^2}, ..., G_{4^{n-1}}$is a GD of G.
Conversely suppose G admits Geometric Decomposition$(G_1, G_3, G_{3^2}, ..., G_{3^{n-1}})$.
Then $q(G) = 1 + 3 + 3^2 + ... + 3^{n-1}$.
$q(G) = \frac{3^n-1}{2}$ for each $n \in N$. □

Theorem 2.4. *The Ladder graph* $P_m \times P_2$ *accepts Geometric Decomposition* $(G_1, G_3, G_{3^2}, ..., G_{3^{n-1}})$ *iff* $m = \frac{3^n+3}{6}$ *for each* $n \in N$.

Proof. Let $V = \{v_i/1 \le i \le m\} \cup \{u_i/1 \le i \le m\}$be the vertex set and $E = \{u_i, u_{i+1}/1 \le i \le m-1\} \cup \{v_iv_{i+1}/1 \le i \le m-1\} \cup \{v_iu_i/1 \le i \le m\}$ be the edge set.nAssume that a ladder graph $P_m \times P_2$ accepts Geometric Decomposition $(G_1, G_3, G_{3^2}, ..., G_{3^{n-1}})$. We have $q(P_m \times P_2) = 3m - 2$.

By above theorem the graph $P_m \times P_2$ accepts Geometric Decomposition $(G_1, G_3, G_{3^2}, ..., G_{3^{n-1}})$ iff $q(P_m \times P_2) = \frac{3^n-1}{2}$. $\Rightarrow 3m-2 = \frac{3^n-1}{2} \Rightarrow m = \frac{3^n+3}{6}$.
Conversely assume $(P_m \times P_2)$ with $m = \frac{3^n+3}{6}$.
Here $q(P_m \times P_2) = 3\left(\frac{3^n+3}{6}\right) - 2 = \frac{3^n-1}{2}$.
This implies that $P_m \times P_2$ can be decomposed into $(G_1, G_4, G_{4^2}, ..., G_{4^{n-1}})$. □

Corollary 2.2. *The Book graph B_m accepts Geometric Decomposition$(G_1, G_3, G_{3^2}, ..., G_{3^{n-1}})$ iff $m = \frac{3^n-3}{6}$ for each $n \in N$.*

Theorem 2.5. *The graph G is Geometric Decomposition$\Big(G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}\Big)$ iff $q = 3(2^n - 1)$ for each $n \in N$.*

Proof. Let G be a connected graph with $q = 3(2^n-1)$. Let v, w be two vertices of G such that $d(v,w)$ is maximum.Let $N_r(v) = \{w \in V/d(v,w) = r\}$. If $d(v) = 3(2)^{n-1}$, choose $3(2)^{n-1}$ edges incident with v.Let $G_{3(2)^{n-1}}$be a subgraph induced by these $3(2)^{n-1}$ edges . If $d(v) < 3(2)^{n-1}$, then choose $3(2)^{n-1}$ edges incident with vertices of $N_1(v), N_2(v), ...$ successively such that the subgraph $G_{3(2)^{n-1}}$ induced by these edges is connected . In both cases $G - G_{3(2)^{n-1}}$ has a connected component C_1 with $3(2^n-1) - 3(2)^{n-1}$ edges.Now consider C_1 and proceed as above to get $G_{3(2)^{n-2}}$ such that $C_1 - G_{3(2)^{n-2}}$ has a connected component C_2 of size $3(2^n) - 3(2)^{n-1} - 3(2)^{n-2} - 3$ edges. Proceeding like this we get a connected subgraph G_4 such that $C_{3(2)^{n-2}}$ is a graph with one edge taken as G_3. Thus $G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}$is a GD of G.
Conversely suppose G admits Geometric Decomposition$\Big(G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}\Big)$. Then $q(G) = 3 + 3(2) + 3(2)^2 + ... + 3(2^{n-1})$.
$q(G) = 3(2^n - 1)$ for each $n \in N$. □

Theorem 2.6. *The Bistar graph $B_{m,m}$ accepts Geometric Decomposition $\Big(G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}\Big)$ iff $m = 3(2^{n-1}) - 2$ for each $n \in N$.*

Proof. Let $V = \{v_i/0 \leq i \leq m\} \cup \{u_i/0 \leq i \leq m\}$ be the vertex set and $E = \{u_0u_i/1 \leq i \leq m\} \cup \{v_0v_i/1 \leq i \leq m\} \cup \{v_0u_0\}$ be the edge set.Assume that a Bistar graph $B_{m,m}$ accepts Geometric Decomposition $\Big(G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}\Big)$.
We have $q(B_{m,m}) = 2m+1$. By above theorem the graph $B_{m,m}$ accepts Geometric Decomposition$\Big(G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}\Big)$ iff $q(B_{m,m}) = 3(2^n-1)$.
$\Rightarrow 2m + 1 = 3(2^n - 1) \Rightarrow m = 3(2^{n-1}) - 2$.

Conversely assume $(B_{m,m})$ with $m = 3\,(2^{n-1}) - 2$.
Here $q(B_{m,m}) = 2\,(3\,(2^{n-1}) - 2) + 1 = 3\,(2^n - 1)$.
This implies that $B_{m,m}$ can be decomposed into $\left(G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}\right)$.
$\square$

Corollary 2.3. *The Friendship graph F_m accepts Geometric Decomposition $\left(G_3, G_{3(2)}, G_{3(2)^2}, ..., G_{3(2)^{n-1}}\right)$ iff $m = 2^n - 1$ for each $n \in N$.*

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