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$\pi G^*\beta$-CONTINUOUS FUNCTION IN TOPOLOGICAL SPACES

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ABSTRACT. The current study deals with the new class of functions in a topological space called π generalized g star β -continuous function (briefly $\pi g^*\beta$ -continuous function). Further we analysis the concepts of almost $\pi g^*\beta$ -continuous function and $\pi g^*\beta$ -irresolute function.

1. INTRODUCTION

The study of sets called the generalized closed set in topological spaces initiated by Levine [1] in 1970. The class of topological spacesand -closed sets are known as quasi normal spaces institute by Zaitsev [2]. M.E.Abd EI-Monsef [3] introduced Open sets and continuous mapping in 1983. Recently Tahiliani [4] introduced the concept of $\pi g\beta$ -closed sets in topological spaces $\pi g^*\beta$ -obtain a characterizations.

In this proposed system, we institute the concepts of $\pi g^*\beta$ -continuous function, $\pi g^*\beta$ -irresolute function, almost $\pi g^*\beta$ -continuous function and some of its characteristics are studied.

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2. Main Result

The notion(X_1, τ) (Y_1, σ) (Z_1, η) represents topological spaces (if necessary separation axioms are assumed).The closure and interior of K is denoted by cls(K) and inte(K) inany subset K of a space (X_1, τ). Let us recollect the successive definitions which we shall require in sequel (X_1, τ) for each regular closed set $V_1$of (Y_1, σ).

Definition 2.1. *A subset K of a space (X_1, τ) is said to becls$(K) \subseteq K$*

(1) *A regular closed set [4] if K=cls(inte(K)).*
(2) *A g-closed set [4] if $cls(K) \subseteq V$ whenever $K \subseteq V$ and V is open in (X_1, τ).*
(3) *Aβ^*-closed set [3] if cls(inte(K)) $\subseteq$V whenever K $\subseteq$ V and V is open in (X_1, τ).*
(4) *A $\pi g^*\beta$ -closed set [5] if inte(β -cls(K)) $\subseteq$ V whenever K $\subseteq$ V and V is π -open in (X_1, τ).*
(5) *A space (X_1, τ) is called a $\pi g^*\beta$ -$T_{1/2}$ space [5] if each$\pi g^*\beta$ -closed set is β^*-closed.*

Definition 2.2. *A mapping $h : (X_1, \tau) \to (Y_1, \sigma)$ is said to be :*

(1) *continuous [4] if $h^{-1}(V_1)$ is open in (X_1, τ) for eachopen set V_1 of (Y_1, σ).*
(2) *irresolute [4] if $h^{-1}(V_1)$ is semi-open in (X_1, τ) whenever V_1 is semi-open in (Y_1, σ).*
(3) *$\pi g\beta$-continuous [4] if $h^{-1}(V_1)$ is $\pi g\beta$-open in (X_1, τ) for each open set V_1 of (Y_1, σ).*
(4) *$\pi g\beta$-irresolute [4] if $h^{-1}(V_1)$ is $\pi g\beta$-open in $(X_1, \tau)$for each -open set V_1 of (Y_1, σ).*
(5) *β^*-continuous if $h^{-1}(V_1)$ is β^*-open in (X_1, τ) for each open set V_1 of (Y_1, σ).*
(6) *β^*-irresolute if $h^{-1}(V_1)$ is β^*-open in (X_1, τ) for each β^*-open set V_1 of (Y_1, σ).*
(7) *almost continuous if $h^{-1}(V_1)$is open in (X_1, τ) for each regular open set $V_1$of (Y_1, σ).*
(8) *almostβ^*-continuous if $h^{-1}(V_1)$is β -open in (X_1, τ) for each regular open set V_1 of (Y_1, σ).*

Proposition 2.1. *Each$\pi g^*\beta$ -irresolute function is $\pi g^*\beta$ -continuous but not conversely.*

Proof. Consider:$(X_1,\tau) \to (Y_1,\sigma)$ be $\pi g^*\beta$ -irresolute function. Let V_1 be closed set in (Y_1,σ). Then V_1 is $\pi g^*\beta$ -closed in (Y_1,σ).We know that h is $\pi g^*\beta$ - irresolute function then $h^{-1}(V_1)$ is $\pi g^*\beta$ -closed in (X_1,τ). Hence h is $\pi g^*\beta$ -continuous. □

Example 1. *Consider $X_1 = Y_1 = \{p_1, q_1, r_1\}$, $\tau = \{\phi, X_1, \{p_1\}, \{q_1\}, \{p_1, q_1\}\}$, $\sigma = \{\phi, X_1, \{p_1\}\}$. The identity maph:$(X_1,\tau) \to (Y_1,\sigma)$ is $\pi g^*\beta$ -continuous but not $\pi g^*\beta$ -irresolute.*

Remark 2.1. *Composition of two $\pi g^*\beta$ -continuous functions need not be $\pi g^*\beta$ -continuous. It can be observable from the succeeding example.*

Example 2. *Let $X_1 = Y_1 = \{p_1, q_1, r_1\}$, $\tau = \{\phi, X_1, \{p_1\}, \{q_1\}, \{p_1, q_1\}\}$, $\sigma = \{\phi, X_1, \{q_1, r_1\}\}$, $\zeta = \{\phi, X_1, \{r_1\}\}$. Define h:$(X_1,\tau) \to (Y_1,\sigma)$ be the identity map and g:$(Y_1,\sigma) \to (Z_1,\zeta)$ as identity mapping.Both h and g are $\pi g^*\beta$ -continuous but $(gh)^{-1}\{p_1, q_1\} = h^{-1}(g^{-1}\{p_1, q_1\}) = \{p_1, q_1\}$ is not $\pi g^*\beta$ -closed in X_1.*

Theorem 2.1. *Let h:$(X_1,\tau) \to (Y_1,\sigma)$ be a function then,*

(1) *If h is $\pi g^*\beta$ -irresolute and $(X_1,\tau)$is $\pi g^*\beta$ -$T_{1/2}$ space, then h is β^* -irresolute.*
(2) *If h is $\pi g^*\beta$ -continuous and (X_1,τ) is $\pi g^*\beta$ -$T_{1/2}$ space, then h is β^*-continuous.*

Proof. (1) Let V_1 be β^*-closed in (Y_1,σ) then V_1 is $\pi g^*\beta$ -closed in (Y_1,σ). Since h is $\pi g^*\beta$ - irresolute,
(2) Let V_1 be a closed set in (Y_1,σ). We know that h is $\pi g^*\beta$ - continuous, $h^{-1}(V_1)$ is $\pi g^*\beta$ -closed in (X_1,τ). Since(X_1,τ) is $\pi g^*\beta$ -$T_{1/2}$ space, $h^{-1}(V_1)$ is β^*-closed in (X_1,τ).Hence h is β^*-continuous.

□

Theorem 2.2. *Let h:$(X_1,\tau) \to (Y_1,\sigma)$and g:$(Y_1,\sigma) \to (Z_1,\zeta)$. Then*

(1) *$g \circ h$ is $\pi g^*\beta$ - continuous, if g is continuous and h is $\pi g^*\beta$ - continuous.*
(2) *$g \circ h$ is $\pi g^*\beta$ - irresolute, if g is $\pi g^*\beta$ -irresolute and h is $\pi g^*\beta$ - irresolute.*
(3) *$g \circ h$ is $\pi g^*\beta$ - continuous, if g is $\pi g^*\beta$ - continuous and h is $\pi g^*\beta$ -irresolute.*

Theorem 2.3. *Each $\pi g^*\beta$ -continuous function is almost$\pi g^*\beta$ -continuous.*

Proof. Let h:$(X_1,\tau) \to (Y_1,\sigma)$ be $\pi g^*\beta$ -continuous function. Let V_1 be a regular closed set in(Y_1,σ). Then V_1 is closed in (Y_1,σ). We know that hs is

$\pi g^*\beta$ -continuous function and $h^{-1}(V_1)$ is $\pi g\beta^*$-closed in (X_1,τ). Hence h is almost$\pi g\beta^*$-continuous. □

Theorem 2.4. *Each almostβ^*-continuous function is almost$\pi g^*\beta$ -continuous.*

Proof. Let h:$(X_1,\tau) \to (Y_1,\sigma)$be almost$\beta^*$-continuous function and let V_1 be regular closed set in (Y_1,σ). Then $h^{-1}(V_1)$ is β^*-closed in (X_1,τ), we know that$h^{-1}(V_1)$ is $\pi g^*\beta$ -closed in (X_1,τ).Thereforeh is almost $\pi g^*\beta$ -continuous. □

Theorem 2.5. *Let (X_1,τ) be a $\pi g^*\beta$ -$T_{1/2}$ space. Then h:$(X_1,\tau) \to (Y_1,\sigma)$ is almost $\pi g^*\beta$ -continuous iffh is almost β^*-continuous.*

Proof. Suppose h:$(X_1,\tau) \to (Y_1,\sigma)$ is almost $\pi g^*\beta$ -continuous. Consider A be a regular closed subset of (Y_1,σ). Then $h^{-1}(A)$ is h:$(X_1,\tau) \to (Y_1,\sigma)$is $\pi g^*\beta$ -closed in(X_1,τ). We know that (X_1,τ) is $\pi g^*\beta$ -$T_{1/2}$ space, $h^{-1}(A)$ is β^*-closed in (X_1,τ). Therefore h is almostβ^*-continuous. Conversely,
Suppose $h:(X_1,\tau) \to (Y_1,\sigma)$ is almost $\pi g^*\beta$ -continuous and A be a regular closed subset of (Y_1,σ). Then $h^{-1}(A)$ is β -closed in (X_1,τ). We know that each β^*-closed is $\pi g^*\beta$ -closed, $h^{-1}(A)$ is $\pi g^*\beta$ -closed. Therefore h is almost $\pi g^*\beta$ -continuous. □

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