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BIFURCATION ANALYSIS FOR NEURAL NETWORKS IN STANDARD BOUNDED AND UNIFORM METRIC SPACES

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ABSTRACT. Theoretical study of neural dynamics is well exposed with the help of the powerful tool namely bifurcation and is well explained with evidently build heart function. A mathematical approach to bifurcation theory of Hopfield - type neural dynamics has been made by utilizing metric topological space. The theoretical measure is adopted to prove that the brain function is always greater than or equal to the heart function and the results are obtained by using basic underlying theorems of topology. In addition graphical results analyzing the two functions are explained.

1. INTRODUCTION

Hopf bifurcation type neural networks have been studied widely during the past decades. Anyone can see that the human brain is superior to a digital computer at many tasks. Electrical signals are utilized by neutrons and few cells for transmitting information in the nervous system such as brain and organs like heart are controlled.

Ramana Reddy, et al, see cite1, studied on time delay effects on coupled limit cycle oscillators at Hopf bifurcation. S. Mohamad and K. Gopalsamy, see [2],

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learnt on neuronal dynamics in time varying environments: Continuous and discrete time models. Complex nonlinear dynamics of Hodgkin - Huxley equations induced by time change scale was learnt by Shingi Doi,et al, see [3]. Impulsive functional-differential equations with nonlocal conditions was explained by H. Akca,et al, see [4]. Haydar Akca, et al. [5], explained on continuous-time additive Hopfield-type neural networks with impulses. Wen Wu Yu and Jinde Cao in [6] studied on Stability and Hopf bifurcation on a two-neuron system with time delay in the frequency domain. Junjie Wei and Chunrui Zhang in [7] explained bifurcation analysis of a class of neural networks with delays. Amirhossein Hajihosseini, et al. see [8], learnt on Hopf bifurcation analysis on a time-delay recurrent neural network in the frequency domain.

In the present work, the concepts of heart function and brain function in metric topology has been introduced using bifurcation theory in Hopfield - type neural dynamics.

2. Preliminaries

2.1. **Metric Space (Human Body).** The functional relationship between heart and brain function are defined by using metric space topology. The function satisfies the following properties:

(1) d(heart, brain) $\geq$ 0, if heart $\neq$ brain; d(heart, heart)= 0(or d(brain, brain)=0)
(2) d(heart, brain) = d(brain, heart)
(3) d(heart, brain) $\leq$ d(heart, brainstem) + d(brainstem, brain) .

2.2. **Complex Metric space (parts of the human body).** The definition states that every Cauchy's sequence is convergent. As far as the functions in all parts of the body (ie) blood circulation is concern, the function converges to heart function namely Cauchy's sequence.

2.3. **Standard bounded Metric (Heart function).** Let

$$\bar{d}(atrium, Ventricle) = min\{d(atrium, ventricle), 1\}$$

be the standard bounded metric corresponding to heart function (d).

2.4. **Uniform Metric (Brain function).** If $f : brain \rightarrow (heart)$ human body be a function then
$\bar{\rho}(f, g) = \{d(f(cerebrum), g(cerebellum)/cerebrum, cerebellumbelongstoHead\}$.

Theorem 2.1. *A heart (or brain) is working properly if every parts of the heart (or brain) (Cauchy sequence) has a proper working nerves system (subsequences).*

Proof. Let (x_n) be the sequence of parts of heart (or brain) in human body. We will show that if (x_n) has a proper working nerves (x_{ni}) that converges to a point x (function of heart or brain), then the sequence (x_n) itself converges to x. For a given $\epsilon > 0$ (blood circulation in any part of the body), choose N large enough that (heart function is maximum)

$$d(atrium, ventricle) < \epsilon/2\,, \tag{2.1}$$

for all atrium ,ventricle greater than or equal to heart function (or)

$$d(cerebrum, cerebellum) < \epsilon/2\,,$$

for all cerebrum , cerebellum greater than or equal to brain function. Using the fact that (x_n) is a Cauchy's sequence, we choose the number of nerves in the parts of the body (an integer i) such that

$$\begin{aligned} d(x_{ni}, x) &< \epsilon/2 \\ d(nervessystem, heart) &< \epsilon/2 \qquad (2.2) \end{aligned}$$

(or)

$$d(nervessystem, brain) < \epsilon/2\,.$$

From (2.1) and (2.2) we have:
d (atrium, heart) $\leq$ d(atrium,nerves system)+d(nerves system,heart)
(or)
d (cerebrum, brain) $\leq$ d(cerebrum, nerves system) + d(nerves system, brain) .

Hence function of heart (or brain) is good, if every parts of heart (or brain) in body has a good working nerves system. According to the theorem a metric space X is complete if every Cauchy sequence in X has a convergent subsequence. $\square$

3. Main Result

Theorem 3.1. *If the heart function is proper in the body then the brain function is proper in the head corresponding to heart function.*

Proof. If the heart function is proper in the body, then blood circulation in all parts is normal, where all the parts of the body depends upon the proper function of heart. Now suppose that pituitary (f_1), hypothalamus $(f_2)\ldots$ represents a sequence of brain parts, that is, a Cauchy's sequence relative to brain function. Let α be given- (blood circulation in nerves). We have that $\bar{d}(f_n(\alpha), f_m(\alpha)) \leq \bar{\rho}(f_n(\alpha), f_m(\alpha))$, for all m, n means that the sequence of parts. Then $f_1(\alpha), f_2(\alpha), \ldots$ is a Cauchy's sequence in brain. Hence this sequence converges, say to a point yα which literal means that heart purifies the blood and circulate to other parts of the body.

Let f:brain$\rightarrow$(heart)human body be the function from brain to heart defined by $f(\alpha) = y_\alpha$ representing function of sum of the different parts equal to the heart function. We will prove that the sequence f_n converges to f in the brain. For a given $\epsilon > 0$, any positive quantity, we choose:

$$\begin{aligned} \bar{\rho}(f_n, f_m) &< \frac{\epsilon}{2}, \forall m, n \text{ (sequence of different parts)} \\ \bar{d}(f_n(\alpha), f(\alpha)) &< \frac{\epsilon}{2}, \end{aligned}$$

since m is large. The inequality between two parts (Heart & Brain) for any α (blood circulation in brain), provided any part of the brain $\geq$ blood circulation in heart .

Therefore $\bar{\rho}(f_n, f) < \frac{\epsilon}{2} < \epsilon, \forall n \geq N$ as desired.

From the above inequality and by using bifurcation theory (qualitative changes in the behavior) and theorem 2.1, it is proved that the heart is not properly worked, so the brain is somewhat not worked.

According to the theorem 2.1. if Y is complete in the metric d, then the space Y^J is complete in the uniform metric $\bar{\rho}$ corresponding to d in metric topology. $\square$

TABLE 1. Different values of Heart Function and Brain Function

Heart Functions	Brain Functions
60	12000
65	13000
70	14000
75	15000
80	16000
85	17000
90	18000
95	19000

4. RESULT ANALYSIS

For better understanding of heart function and brain function, graph has been plotted with well known theoretical values using C programming. It is observed that for different values of brain function, the graph progresses sharply. Figure 1 depicts clearly that brain function is greater than equal to heart function.

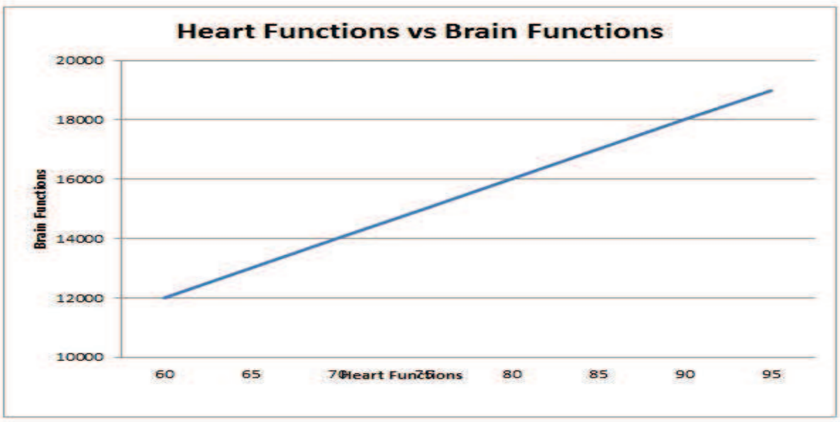

FIGURE 1. Graph of Heart Function against Brain Function

5. Conclusion

The present study elaborates about the behavior of heart and brain functions with help of the already defined theorems. In human body the increasing heart beat affects the brain function that is by reducing the memory power.

Using Hopfield bifurcation theory, It is proved that the brain function is always greater than heart function. Additionally graph has been given using C programming to calculate of heart function against brain function and its found that graphical results also predicts as desired.

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