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CARTESIAN PRODUCT OF MULTI $\mathcal{L}$-FUZZY IDEALS OF Γ-NEAR RING

K. HEMABALA [1] AND SRINIVASA KUMAR

ABSTRACT. In this paper, the perception of Cartesian Product of multi $\mathcal{L}$-fuzzy set is defined and some allied properties of Cartesian Product of multi $\mathcal{L}$-fuzzy ideals and multi level ideals of a Γ-near ring are defined. We establish a one association between multi $\mathcal{L}$-fuzzy left ideals of a Γ-near ring and crisp ideals of a Γ-near ring.

1. INTRODUCTION

Fuzzy set theory is introduced by L.A. Zadeh [4] In 1965. Later, A. Rosenfeld [1] developed the concepts of fuzzy groups. After that several authors developed the concepts of fuzzy sets. In that way Bh. Satyanarayana [2,3] introduced the concepts of Γ-near ring and authors like S. Ragamai, Y. Bhargavi ,T. Eswarlal [10] verified the properties of $\mathcal{L}$-fuuzy ideals of a Γ-near ring . After few years authors focused on multi fuzzy sets. In fuzzy set, membership function of an element is a single value between 0 and 1. However some vagueness is there how faraway the element is. Merely Fuzzy set is not enough to study some reality problems. Therefore multi fuzzy sets in terms of multi dimensional membership function was introduced by Sabu Sabastain [7,8,9]. Earlier, the concept of cartesian product of multi fuzzy set is defined by R.Muthuraj and C.Malarselvi [5]. Now, in this paper, I carry the results of R. Muthuraj [6] to multi $\mathcal{L}$-Fuzzy sets

[1] *corresponding author*

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of a Γ-near ring and also few properties of cartesian product of multi $\mathcal{L}$-fuzzy ideals and multi level ideals of a Γ-near ring are defined.

2. Basic Concepts

The basis of multi $\mathcal{L}$-fuzzy set is a branch of theories of $\mathcal{L}$-fuzzy sets. The membership function of a multi $\mathcal{L}$-fuzzy set is an ordered sequence of membership function of a fuzzy set. The concept of multi $\mathcal{L}$-fuzzy sets provides a innovative method to represent some problems which are difficult to explain in other branch of fuzzy set theory.

Definition 2.1. *A $\mathcal{L}$ fuzzy subset A_L of Γ is a mapping from Γ into $\mathcal{L}$ where $\mathcal{L}$ is a complete lattice fulfilling the infinite meet distribute law.*

Definition 2.2. *Let Γ be a non empty set. A multi $\mathcal{L}$-fuzzy set A_L in Γ is defined as a set of ordered sequence. $A_L = (\sqcap, A_{L1}(\sqcap), A_{L2}(\sqcap), \ldots A_{Li}(\sqcap)..) : \sqcap \in \mathcal{V}$ Where, $A_{Li} : \mathcal{V} \to \mathcal{L}$ for all $i = 1, 2, \ldots, m$.*

Remark 2.1.
1. *If the sequences of the membership functions have only m-terms (finite number of terms), m is called the dimension of A.*
2. *The set of all multi-fuzzy sets in Γ of dimension m is denoted by $M^mFS(\mathcal{V})$.*
3. *The multi fuzzy membership function A is a function from Γ to $[0,1]^m$ such that for all $\sqcap$ in Γ, $A(\sqcap) = (A_1(\sqcap), A_2(\sqcap) \ldots A_m(\sqcap))$.*
4. *For the sake of simplicity, we denote the multi fuzzy set $A = \{(\sqcap, A_1(\sqcap), A_2(\sqcap) \ldots A_m(\sqcap)) : \sqcap \in \mathcal{V}\}$ as $A = (A_1, A_2 \ldots A_m)$.*

Definition 2.3. *Let m be a positive integer and let A and B in $M^mFS(\mathcal{V})$, Where $A = (A_1, A_2, \ldots A_k)$ and $B = (B_1, B_2, \ldots B_k)$, then we have the following relations and operations*

1. $A \subseteq B$ *if and only if* $A_i \leq Bi$ *for all* $i = 1, 2, \ldots m$
2. $A = B$ *if and only if* $A_i = B_i$ *for all* $i = 1, 2, \ldots m$
3. $A \vee B = (A_1 \vee B_1, \ldots A_m \vee B_m) = \{(\sqcap, max(A_1(\sqcap), B_1(\sqcap)), \ldots max(A_m(\sqcap), B_m(\sqcap))) : \sqcap \in \mathcal{V}\}$
4. $A \wedge B = (A_1 \wedge B_1, \ldots, A_m \wedge B_m) = \{(\sqcap, min(A_1(\sqcap), B_1(\sqcap)), \ldots, \min(A_k(\sqcap), B_k(\sqcap))) : \sqcap \in \mathcal{V}\}$

(5) *The multi-fuzzy complement of multi fuzzy set A is* $C(A) = \{ \sqcap, C(A_1(\sqcap)), C(A_2(\sqcap)), \ldots C(A_m(\sqcap)) : \sqcap \in \mathcal{V}\}$ *where* $C(A_i(\sqcap))$ *is the complement of* $A_i(\sqcap)$ *for all* $i = 1, \ldots m$ $C(A_i(\sqcap)) = 1 - A_i(\sqcap)$ *for all* $i = 1, \ldots m$.

Definition 2.4. *Let m be a positive integer and A and B be two multi fuzzy sets of dimension m on Γ then cartesian product $A \times B$ of two multi fuzzy sets A and B is defined by*
$A \times B = \{< (\sqcap,\ \sqsubseteq), min(A_1(\sqcap), B_1(\sqsubseteq)), \ldots min(A_m(\sqcap), B_m(\sqsubseteq)) > /(\ \sqcap,\ \sqsubseteq) \in \sqcap \times \sqcap\}$

Definition 2.5. *Let* $A = \{< \sqcap, A_1(\sqcap), A_2(\sqcap), A_3(\sqcap), \ldots A_m(\sqcap) >: \sqcap \in \mathcal{V}\}$ *where* $A_j(\sqcap) \in [0,1]$ *for all* $j = 1, 2, \ldots m$ *be a multi fuzzy set on* Γ *and* $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_m), \alpha_i \in I$ *for* $i = 1, 2, \ldots m$. *Then the set* $A_\alpha = \{ \sqcap \in \mathcal{V}/A(\sqcap) \geq \alpha\}$ *is called level subset of a multi fuzzy subset A.*
That is $A_i(\sqcap) \geq \alpha_i$ *for each* $i = 1, 2, \ldots m$.

Definition 2.6. *A fuzzy subset A of a Γ near ring M is called a fuzzy left (resp. right) ideal of M if for all* $\sqcap, \sqsubseteq, m_1, m_2 \in M, \alpha \in \Gamma$

(1) $A(\sqcap - \sqsubseteq) \geq min\{A(\sqcap), A(\sqsubseteq)\}$
(2) $A(\sqsubseteq + \sqcap - \sqsubseteq) \geq A(\sqcap)$
(3) $A(m_1\alpha(\sqcap + m_2) - m_1\alpha m_2) \geq A(\sqcap)$ *resp.* $A(\sqcap\alpha m_1) \geq A(\sqcap)$.

3. Multi $\mathcal{L}$-fuzzy ideals of a Γ near ring

In this part, we introduced the definition of multi $\mathcal{L}$-fuzzy left (resp. right) ideals of a Γ near ring. also, we defined the cartesian product of multi $\mathcal{L}$-fuzzy set and we proved that the some properties of cartesian product of multi $\mathcal{L}$-fuzzy set of a Γ-near ring.

Definition 3.1. *A multi $\mathcal{L}$ fuzzy subset A_L of M is called multi $\mathcal{L}$-fuzzy left (resp. right) ideal of M if for all* $\sqcap, \sqsubseteq, m_1, m_2 \in M, \alpha \in \Gamma$

(1) $A_L(\sqcap - \sqsubseteq) \geq A_L(\sqcap)\Lambda A_L(\sqsubseteq)$.
(2) $A_L(\sqsubseteq + \sqcap - \sqsubseteq) \geq A_L(\sqcap)$.
(3) $A_L(m_1\alpha(\sqcap + b) - m_1\alpha m_2) \geq A_L(\sqcap)$ *resp.* $A_L(\sqcap\alpha m_1) \geq A_L(\sqcap)$.

If A_L is a both multi $\mathcal{L}$-fuzzy left and right ideal of M then A_L is called a multi $\mathcal{L}$-fuzzy ideal of M.

Definition 3.2. *Let m be a positive integer and A and B be two multi $\mathcal{L}$-fuzzy sets of dimension m on Γ that is*

$$\begin{aligned} A_L &= \{(\sqcap, A_{L1}(\sqcap), A_{L2}(\sqcap), \ldots A_{Lm}(\sqcap)) : \sqcap \in \mathcal{V}\} \\ B_L &= \{(\sqcap, B_{L1}(\sqcap), B_{L2}(\sqcap), \ldots B_{Lm}(\sqcap)) : \sqcap \in \mathcal{V}\}. \end{aligned}$$

Then the cartesian product of $A \times B$ of two multi $\mathcal{L}$-fuzzy sets A and B is defined by

$$\begin{aligned} A_L \times B_L = \{ &< (\sqcap, \sqsubseteq), (A_{L1}(\sqcap) \sqcap B_{L1}(\sqsubseteq)), (A_{L2}(\sqcap) \sqcap B_{L2}(\sqsubseteq)), \\ &\ldots (A_{Lm}(\sqcap) \sqcap B_{Lm}(\sqsubseteq)) > (\sqcap, \sqsubseteq) \in U \times V\} \end{aligned}$$

Theorem 3.1. *If A_L and B_L are two multi $\mathcal{L}$-fuzzy left ideals of a Γ-near ring M. Then $(A_L \times B_L)$ is multi $\mathcal{L}$-fuzzy left ideal of $M \times M$.*

Proof. Let A_L and B_L be two multi $\mathcal{L}$-fuzzy left ideals of M.
Let $(\sqcap_1, \sqcap_2)$ and $(\sqsubseteq_1, \sqsubseteq_2) \in M \times M$

(1)

$$\begin{aligned} &(A_L \times B_L)(\sqcap_1 - \sqsubseteq_1, \sqcap_2 - \sqsubseteq_2) = ((A_{L1}(\sqcap_1 - \sqsubseteq_1) \sqcap B_{L1}(\sqcap_2 - \sqsubseteq_2)), \\ &\ldots (A_{Lm}(\sqcap_1 - \sqsubseteq_1) \sqcap B_{Lm}(\sqcap_2 - \sqsubseteq_2))) \\ \geq\ & < \{(A_{L1}(\sqcap_1) A_{L1}(\sqsubseteq_1)) \sqcap (B_{L1}(\sqcap_2) \sqcap B_{L1}(\sqsubseteq_2))\} \ldots . \{(A_{Lm}(\sqcap_1) \sqcap A_{Lm}(\sqsubseteq_1)) \\ & \sqcap (B_{Lm}(\sqcap_2) \sqcap B_{Lm}(\sqsubseteq_2))\} > \\ \geq\ & < \{(A_{L1}(\sqcap_1) \sqcap B_{L1}(\sqcap_2)) \sqcap (A_{L1}(\sqsubseteq_1) \sqcap B_{L1}(\sqsubseteq 2))\} \ldots \{(A_{Lm}(\sqcap_1) \sqcap B_{Lm}(\sqcap_2)) \\ & \sqcap (A_{Lm}(\sqsubseteq_1) \sqcap B_{Lm}(\sqsubseteq_2))\} > \\ \geq\ & < \{(A_{L1}(\sqcap_1) \sqcap B_{L1}(\sqcap_2)) \ldots (A_{Lm}(\sqcap_1) \sqcap B_{Lm}(\sqcap_2))\} \sqcap (A_{L1}(\sqsubseteq_1) \sqcap B_{L1}(\sqsubseteq_2)) \ldots \\ & (A_{Lm}(\sqsubseteq_1) \sqcap B_{Lm}(\sqsubseteq_2))\} \\ \geq\ & < (A_L \times B_L)(\sqcap_1, \sqcap_2) \sqcap (A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2) > . \end{aligned}$$

(2)

$$\begin{aligned} &(A_L \times B_L)(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1, \sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2) = < (A_{L1}(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1) \sqcap B_{L1}(\sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2)), \\ &(A_{L2}(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1) \sqcap B_{L2}(\sqsubseteq_2 + \sqcap 2 - \sqsubseteq_2)), \\ &\ldots (A_{Lm}(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq 1) \sqcap B_{Lm}(\sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2)) > \\ \geq\ & < (A_{L1}(\sqcap_1) \sqcap B_{L1}(\sqcap_2)), (A_{L2}(\sqcap_1) \sqcap B_{L2}(\sqcap_2)), \ldots (A_{Lm}(\sqcap_1) \sqcap (B_{Lm}(\sqcap_2)) > \\ \geq\ & (A_L \times B_L)(\sqcap_1, \sqcap_2). \end{aligned}$$

(3)

$$\begin{aligned}
& (A_L \times B_L)(m_1\alpha(\sqcap_1 + m2) - m_1\alpha m_2, m_1\alpha(\sqcap_2 + m2) - m_1\alpha m_2) \\
= \; & < (A_{L1}(m_1\alpha(\sqcap_1 + m2) - m_1\alpha m_2)\sqcap B_{L1}(m_1\alpha(\sqcap_2 + m2) - m_1\alpha m_2)), \ldots \\
& (A_{Lm}(m_1\alpha(\sqcap_1 + m2) - m_1\alpha m_2)\sqcap B_{Lm}(m_1\alpha(\sqcap_2 + m_2) - m_1\alpha m_2)) > \\
\geq \; & < (A_{L1}(\sqcap_1)\sqcap B_{L1}(\sqcap_2)), \ldots (A_{Lm}(\sqcap_1)\sqcap B_{Lm}(\sqcap_2)) > \; .
\end{aligned}$$

Therefore $\geq (A_L \times B_L)(\sqcap_1, \sqcap_2)\sqcap(A_L X B_L)$ is a multi $\mathcal{L}$-fuzzy left ideal of M.

□

Theorem 3.2. *If A_L and B_L are two multi $\mathcal{L}$-fuzzy right ideals of a Γ-near ring M. Then $(A_L \times B_L)$ is a multi $\mathcal{L}$-fuzzy right ideal of $M \times M$.*

Proof. Let A_L and B_L be two multi $\mathcal{L}$-fuzzy right ideals of M.

Let $(\sqcap_1, \sqcap_2)$ and $(\sqsubseteq_1, \sqsubseteq_2) \in M \times M$.

(1)

$$\begin{aligned}
& (A_L \times B_L)(\sqcap_1 - \sqsubseteq_1, \sqcap_2 - \sqsubseteq_2) =< (A_{L1}(\sqcap_1 - \sqsubseteq_1)\sqcap B_{L1}(\sqcap_2 - \sqsubseteq_2)), \ldots \\
& (A_{Lm}(\sqcap_1 - \sqsubseteq_1)\sqcap B_{Lm}(\sqcap_2 - \sqsubseteq_2)) > \\
\geq \; & < (A_{L1}(\sqcap_1)\sqcap A_{L1}(\sqsubseteq_1))\sqcap(B_{L1}(\sqcap_2)\sqcap B_{L1}(\sqsubseteq_2)) \ldots (A_{Lm}(\sqcap_1)\sqcap A_{Lm}(\sqsubseteq_1)) \\
& \sqcap(B_{Lm}(\sqcap_2)\sqcap B_{Lm}(\sqsubseteq_2)) > \\
\geq \; & < (A_{L1}(\sqcap_1)\sqcap B_{L1}(\sqcap_2))\sqcap(A_{L1}(\sqsubseteq_1)\sqcap B_{L1}(\sqsubseteq_2)) \ldots \sqcap(A_{Lm}(\sqcap_1)\sqcap B_{Lm}(\sqcap_2)) \\
& \sqcap(A_{Lm}(\sqsubseteq_1)\sqcap B_{Lm}(\sqsubseteq_2)) > \\
\geq \; & < (A_{L1}(\sqcap_1)\sqcap B_{L1}(\sqcap_2)), \ldots (A_{Lm}(\sqcap_1)\mathcal{B}_{Lm}(\sqcap_2))\sqcap(A_{L1}(\sqsubseteq_1)\sqcap B_{L1}(\sqsubseteq_2)), \ldots \\
& (A_{Lm}(\sqsubseteq_1)\sqcap B_{Lm}(\sqsubseteq_2)) > \\
\geq \; & < (A_L \times B_L)(\sqcap_1, \sqcap_2)\sqcap(A_L x B_L)(\sqsubseteq_1, \sqsubseteq_2) > \; .
\end{aligned}$$

(2)

$$\begin{aligned}
& (A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2) > (A_L \times B_L)(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1, \sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2) \\
= \; & < (A_{L1}(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1)\sqcap B_{L1}(\sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2)), (A_{L2}(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1)\sqcap B_{L2}(\sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2)), \ldots \\
& (A_{Lm}(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1)\sqcap B_{Lm}(\sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2)) > \\
\geq \; & < (A_{L1}(\sqcap_1)\sqcap B_{L1}(\sqcap_2)), (A_{L2}(\sqcap_1)\sqcap B_{L2}(\sqcap_2)), \ldots (A_{Lm}(\sqcap_1)\sqcap(B_{Lm}(\sqcap_2)) > \\
\geq \; & < (A_L \times B_L)(\sqcap_1, \sqcap_2) \, .
\end{aligned}$$

(3)

$$
\begin{aligned}
& (A_L \times B_L)(\sqcap_1 \alpha m_1, \sqcap_2 \alpha m_1) \\
= \; & < (A_{L1}(\sqcap_1 \alpha m_1) \sqcap B_{L1}(\sqcap_2 \alpha m_1)), (A_{L2}(\sqcap_1 \alpha m_1) \\
& \sqcap B_{L2}(\sqcap_2 \alpha m_1)), \dots (A_{Lm}(\sqcap_1 \alpha m_1) \sqcap B_{Lm}(\sqcap_2 \alpha m_1)) > \\
\geq \; & < (A_{L1}(\sqcap_1) \sqcap B_{L1}(\sqcap_2)), (A_{L2}(\sqcap_1) \sqcap B_{L2}(\sqcap_2)), \dots (A_{Lm}(\sqcap_1) \sqcap B_{Lm}(\sqcap_2)) > \\
\geq \; & < (A_L \times B_L)(\sqcap_1, \sqcap_2) \sqcap (A_L \times B_L)
\end{aligned}
$$

is a multi $\mathcal{L}$-fuzzy right ideal of M.

□

Theorem 3.3. *If A_L and B_L are two multi $\mathcal{L}$-fuzzy ideals of a Γ-near ring M then $(A_L \times B_L)$ is A_L so multi $\mathcal{L}$-fuzzy ideal of $M \times M$.*

Proof. It is obvious. □

Definition 3.3. *If $A_L \times B_L$ is the Cartesian product of two multi $\mathcal{L}$-fuzzy sets A_L and B_L and $\alpha = (\alpha_1, \alpha_2, \dots \alpha_m), \alpha_i \in \mathcal{L}, i = 1, 2, \dots m$ then the set $(A_L \times B_L)\alpha = \{(\sqcap, \sqsubseteq) \in U \times V / (A_L \times B_L)(\sqcap, \sqsubseteq) \geq \alpha\}$ is called a level subset of $A_L \times B_L$. That is all $A_{L_i}(\sqcap) \sqcap B_{L_i}(\sqsubseteq) \geq \alpha_i$ for $i = 1, 2, \dots m$.*

Theorem 3.4. *Let A_L and B_L are two multi $\mathcal{L}$-fuzzy subsets of a set V and $\alpha = (\alpha_1, \alpha_2, \dots \alpha_m), \alpha_i \in L, i = 1, 2, \dots m$ then $(A_L \times B_L)_\alpha = A_{L\alpha} \times B_{L\alpha}$.*

Proof. Let $(\sqcap, \sqsubseteq) \in A_{L\alpha} \times B_{L\alpha}$

$\Longleftrightarrow \sqcap \in A_{L\alpha}$ and $\sqsubseteq \in B_{L\alpha}$

$\Longleftrightarrow A_{Li}(\sqcap) \geq \alpha_i$ and $B_{Li}(\sqsubseteq) \geq \alpha_i$, for $i = 1, 2, \dots m$

$\Longleftrightarrow A_{Li}(\sqcap) \sqcap B_{Li}(\sqsubseteq) \geq \alpha_i$, for $i = 1, 2, \dots m$

$\Longleftrightarrow (A_L \times B_L)(\sqcap, \sqsubseteq) \geq \alpha$

$\Longleftrightarrow (\sqcap, \sqsubseteq) \in (A_L \times B_L)\alpha$

Hence $(A_L \times B_L)_\alpha = A_{L\alpha} \times B_{L\alpha}$. □

Theorem 3.5. *Let A_L and B_L be a multi $\mathcal{L}$-fuzzy subsets of Γ-near ring $M \times M$. Then $A_L \times B_L$ is a multi $\mathcal{L}$-fuzzy left ideal of $M \times M$ if and only if each non empty multi level subset $(A_L \times B_L)\alpha$ of $A_L \times B_L$ is left ideal of $M \times M$.*

Proof. Let $(\sqcap_1, \sqcap_2), (\sqsubseteq_1, \sqsubseteq_2) \in (A_L \times B_L)_\alpha$

$$(A_L \times B_L)(\sqcap_1, \sqcap_2) \geq \alpha,$$
$$(A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2) \geq \alpha\,.$$

Since $(A_L \times B_L)$ is a multi $\mathcal{L}$-fuzzy left ideal of $M \times M$,

$$\begin{aligned} & (A_L \times B_L)(\sqcap_1 - \sqsubseteq_1, \sqcap_2 - \sqsubseteq_2) \\ \geq\ & (A_L \times B_L)(\sqcap 1, \sqcap_2) \sqcap (A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2) \\ \geq\ & \alpha \sqcap \alpha \\ \geq\ & \alpha \\ \Longrightarrow\ & (\sqcap_1 - \sqsubseteq_1, \sqcap_2 - \sqsubseteq_2) \in (A_L \times B_L)\alpha\,. \end{aligned}$$

Let $(\sqcap_1, \sqcap_2) \in (A_L \times B_L)\alpha$, $(A_L \times B_L)(\sqcap_1, \sqcap_2) \geq \alpha$. Let $(\sqsubseteq_1, \sqsubseteq_2) \in M \times M$. Moreover $(A_L \times B_L)(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1, \sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2) \geq (A_L x B_L)(\ \sqcap_1, \sqcap_2)$.

Since $(A_L \times B_L)(\sqcap_1, \sqcap_2) \geq \alpha \sqcap, (A_L \times B_L)(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1, \sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2) \geq \alpha$

$$\Longrightarrow (\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1, \sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2) \in (A_L \times B_L)\alpha\,.$$

Let $(\sqcap_1, \sqcap_2) \in (A_L \times B_L)\alpha\ m_1, m_2 \in M, \alpha \in \sqcap$.

Since

$$\begin{aligned} & (A_L \times B_L)(m_1\alpha(\sqcap_1 + m_2) - m_1\alpha m_2, a\alpha(\sqcap_2 + m_2) - m_1\alpha m_2) \\ \geq\ & (A_L \times B_L)(\sqcap_1, \sqcap_2) \\ \geq\ & \alpha \\ \Longrightarrow\ & (m_1\alpha(\sqcap_1 + m_2) - m_1\alpha m_2, m_1\alpha(\sqcap_2 + m_2) - m_1\alpha m_2) \in (A_L \times B_L)\alpha\,, \end{aligned}$$

$\Longrightarrow (A_L \times B_L)\alpha$ is a left ideal of $M \times M$.

Conversely, suppose that $(A_L \times B_L)\alpha$ is a left ideal of $M \times M$. Now, we have to prove that $A_L \times B_L$ is a $\mathcal{L}$-fuzzy left ideal of $M \times M$.

Let $(\sqcap_1, \sqcap_2), (\sqsubseteq_1, \sqsubseteq_2) \in M \times M$, and $(A_L \times B_L)(\sqcap_1, \sqcap_2) = \alpha_1$.

$$
\begin{aligned}
&(A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2) = \alpha_2 \\
&\alpha = \alpha_1 \sqcap \alpha_2 \\
&(A_L \times B_L)(\sqcap_1, \sqcap_2) = \alpha_1 \geq \alpha \\
&(A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2) = \alpha_2 \geq \alpha\,.
\end{aligned}
$$

Since $(\sqcap_1, \sqcap_2), (\sqsubseteq_1, \sqsubseteq_2) \in (A_L \times B_L)\alpha, ,$

$$
\begin{aligned}
&(\sqcap_1 - \sqsubseteq_1, \sqcap_2 - \sqsubseteq_2) \in (A_L \times B_L)\alpha \\
&(A_L \times B_L)(\sqcap_1 - \sqsubseteq_1, \sqcap_2 - \sqsubseteq_2) \geq \alpha = \alpha_1 \sqcap \alpha_2 \\
&(A_L \times B_L)(\sqcap_1 - \sqsubseteq_1, \sqcap_2 - \sqsubseteq_2) \geq (A_L \times B_L)(\sqcap_1, \sqcap_2) \sqcap (A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2)\,.
\end{aligned}
$$

Let $(\sqcap_1, \sqcap_2), (\sqsubseteq_1, \sqsubseteq_2) \in M \times M$, and let $(A_L \times B_L)(\sqcap_1, \sqcap_2) = \alpha, (A_L \times B_L)(\sqsubseteq_1, \sqsubseteq_2) = \alpha\,.$

$$
\begin{aligned}
&\Longrightarrow (\sqcap_1, \sqcap_2), (\sqsubseteq_1, \sqsubseteq_2) \in (A_L \times B_L)\alpha \\
&\Longrightarrow (\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1, \sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2) \in (A_L \times B_L)\alpha \\
&\Longrightarrow (A_L \times B_L)(\sqsubseteq_1 + \sqcap_1 - \sqsubseteq_1, \sqsubseteq_2 + \sqcap_2 - \sqsubseteq_2) \geq \alpha \geq (A_L \times B_L)(\sqcap_1, \sqcap_2)\,.
\end{aligned}
$$

Let $m_1, m_2 \in M, \alpha \in \sqcap, (x_1, x_2) \in (A_L \times B_L)\alpha\,, (A_L \times B_L)(\sqcap_1, \sqcap_2) = \alpha\,,$

$$
\begin{aligned}
&(m_1\alpha(\sqcap_1 + m2) - m_1\alpha m_2, m_1\alpha(\sqcap_2 + m_2) - m_1\alpha m_2) \in (A_L \times B_L)\alpha \\
&(A_L \times B_L)(m_1\alpha(\sqcap_1 + m2) - m_1\alpha m_2, m_1\alpha(\sqcap_2 + m_2) - m_1\alpha m_2) \\
\geq\ &\alpha \geq (A_L \times B_L)(\sqcap_1, \sqcap_2) \\
&\sqcap(A_L \times B_L)
\end{aligned}
$$

is a $\mathcal{L}$-fuzzy left ideal of $M \times M$. □

Note: In this approach we can verify for right ideal.

REFERENCES

[1] A. ROSENFELD: *Fuzzy groups*, J.Math.Anal.Appl., **35** (1971), 512–517.

[2] B. SATYANARAYANA: *A note on* Γ*-near rings*, Indian J.Math., **41**(3) (1999), 427–433.

[3] B. SATYANARAYANA: *Contribution to near ring theory*, Doctoral Thesis, Nagarjuna University , India 1984.

[4] L. A. ZADEH: *Fuzzy sets*, Information and control, **8** (1965), 338–353.

[5] R. MUTHURAJ, C. MALARSELVI: *Cartesian product of multi fuzzy ideal of a ring*, International Journal of Computational and Applied Mathematics, **12**(1) (2017), 64–71.

[6] R. MUTHURAJ, S. BALAMURUGAN: *Multi fuzzy group and its level sub groups*, Gen.Math. Notes, **17**(1) (2013), 74–81.

[7] S. SABU, T. V. RAMAKRISHNAN: *Multi fuzzy topology*, Internatiional journal of Applied Mathematics, **24**(1) (2011), 117–129.

[8] S. SABU, T. V. RAMAKRISHNAN: *Multi fuzzy subgroups*, Int.J.Contemp.Math.Sciences, **6**(8) (2011), 365–372.

[9] S. SABU, T. V. RAMAKRISHNAN: *Multi fuzzy sets*, International mathematical Forum, **50** (2010), 2471–2476.

[10] S. RAGAMAYI, Y. BHARGAVI, T. ESWARLAL: *L-fuzzy ideals of a Γ-near ring,* International Journal of Pure and Applied Mathematics, **118**(2) (2018), 291–307.

DEPARTMENT OF MATHEMATICS
KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM, GUNTUR-522502, AP, INDIA

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VADDESWARAM, GUNTUR-522502, AP, INDIA