ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.10, 7933–7946
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.10.27 Spec. Issue on ACMAMP-2020

COMPLEX CUBIC INTUITIONISTIC FUZZY SET AND ITS DECISION MAKING

V. CHINNADURAI[1], S. THAYALAN, AND A. BOBIN

ABSTRACT. In this manuscript, we present complex cubic intuitionistic fuzzy set (CCIFS) which is a combination of complex cubic membership values and complex cubic non-membership values and define some related operations on these sets. Also, an algorithm and a realistic example are given to show its usefulness.

1. INTRODUCTION

Zadeh [1] defined fuzzy set (FS) which is the most important events in Mathematics. The notion of intuitionistic FS (IFS) was introduced by Atanassov [2]with some theoretical properties of IFS. Later, Atanassov and Gargov [3] presented the concept of interval-valued IFS an extension of IFS. Chinnadurai and Barkavi [4] introduced the notion of cubic matrix and its properties. Kaur and Garg [5] described the notion of generalized cubic fuzzy with t-norm operators. Ramot et al. [6] presented the concept of complex fuzzy sets by analyzing its membership values using complex numbers. Chinnadurai et al. [7] introduced the notion of complex cubic set. In this paper, we define CCIFS, a combination of complex IFS (CIFS) and complex interval valued IFS(CIVIFS). We also present some basic algebraic properties of CCIFS and its application in real life.

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 47S40.

Key words and phrases. Complex intuitionistic fuzzy set, Complex interval valued intuitionistic fuzzy set, Complex cubic intuitionistic fuzzy set.

2. Preliminaries

Definition 2.1. *[8] A complex IFS (CIFS) $\mathcal{S}$ in $\vec{X}$ is a set given by $\mathcal{S} = \{(z, \mu_{\mathcal{S}}(z), \nu_{\mathcal{S}}(z)) : z \in \vec{X}\}$, where $\mu_{\mathcal{S}} : \vec{X} \to \{a : a \in C, |a| \leq 1\}$ and $\nu_{\mathcal{S}} : \vec{X} \to \{a : a \in C, |a| \leq 1\}$ are complex valued membership and non-membership function respectively given by $\mu_{\mathcal{S}}(z) = r_{\mathcal{S}}(z)e^{i\theta_{r_{\mathcal{S}}}(z)}$ and $\nu_{\mathcal{S}}(z) = q_{\mathcal{S}}(z)e^{i\theta_{q_{\mathcal{S}}}(z)}$. Here $r_{\mathcal{S}}(z), q_{\mathcal{S}}(z) \in [0,1]$ such that $0 \leq r_{\mathcal{S}}(z) + q_{\mathcal{S}}(z) \leq 1$. Also $\theta_{r_{\mathcal{S}}}(z)$ and $\theta_{q_{\mathcal{S}}}(z) \in [0, 2\pi]$, which satisfy the condition $0 \leq \theta_{r_{\mathcal{S}}}(z) + \theta_{q_{\mathcal{S}}}(z) \leq 2\pi$, for all $z \in \vec{X}$.*

Definition 2.2. *[9] Let $\vec{X}$ be the universe set. A complex interval-valued IFS (CIVIFS) defined on $\vec{X}$ is a set given by $\mathcal{S} = \{(z, [\mu_{\mathcal{S}}(z)^-, \mu_{\mathcal{S}}(z)^+], [\nu_{\mathcal{S}}(z)^-, \nu_{\mathcal{S}}(z)^+]) : z \in \vec{X}\}$, where $\mu_{\mathcal{S}}(z)^-, \mu_{\mathcal{S}}(z)^+$ and $\nu_{\mathcal{S}}(z)^-, \nu_{\mathcal{S}}(z)^+$ is the degrees of lower and upper bound of the membership and non-membership which are defined as $\mu_{\mathcal{S}}(z)^- = |\omega_1^-| = r_{\mathcal{S}}^-(z)e^{i\theta_{r_{\mathcal{S}}}^-(z)}, \mu_{\mathcal{S}}(z)^+ = |\omega_1^+| = r_{\mathcal{S}}^+(z)e^{i\theta_{r_{\mathcal{S}}}^+(z)}$ such that $|\omega_1^-| \leq |\omega_1^+|$, while $\nu_{\mathcal{S}}(z)^- = |\omega_2^-| = q_{\mathcal{S}}^-(z)e^{i\theta_{q_{\mathcal{S}}}^-(z)}, \nu_{\mathcal{S}}(z)^+ = |\omega_2^+| = q_{\mathcal{S}}^+(z)e^{i\theta_{q_{\mathcal{S}}}^+(z)}$ be such that $|\omega_2^-| \leq |\omega_2^+|$. The amplitude terms $r_{\mathcal{S}}^-, r_{\mathcal{S}}^+, q_{\mathcal{S}}^-, q_{\mathcal{S}}^+ \in [0,1]$ and satisfy the inequality $r_{\mathcal{S}}^- \leq r_{\mathcal{S}}^+, q_{\mathcal{S}}^- \leq q_{\mathcal{S}}^+$ and $r_{\mathcal{S}}^+ + q_{\mathcal{S}}^+ \leq 1$, for all $z \in \vec{X}$. On the other hand, the phase terms $\theta_{r_{\mathcal{S}}}^-, \theta_{r_{\mathcal{S}}}^+, \theta_{q_{\mathcal{S}}}^-, \theta_{q_{\mathcal{S}}}^+ \in [0, 2\pi]$ and satisfy the inequality $\theta_{r_{\mathcal{S}}}^- \leq \theta_{r_{\mathcal{S}}}^+, \theta_{q_{\mathcal{S}}}^- \leq \theta_{q_{\mathcal{S}}}^+$ and $\theta_{r_{\mathcal{S}}}^+ + \theta_{q_{\mathcal{S}}}^+ \leq 2\pi$, for all $z \in \vec{X}$. Therefore $\mathcal{S}$ can be represented as*

$$\mathcal{S} = \left\{ \left(z, \left[r_{\mathcal{S}}^-(z)e^{i\theta_{r_{\mathcal{S}}}^-(z)}, r_{\mathcal{S}}^+(z)e^{i\theta_{r_{\mathcal{S}}}^+(z)} \right], \left[q_{\mathcal{S}}^-(z)e^{i\theta_{q_{\mathcal{S}}}^-(z)}, q_{\mathcal{S}}^+(z)e^{i\theta_{q_{\mathcal{S}}}^+(z)} \right] \right) : z \in \vec{X} \right\}.$$

3. Complex cubic intuitionistic fuzzy set

In this segment, we present the idea of CCIFS and their analogous with desirable set-theoretic operations.

Definition 3.1. *A CCIFS $\mathbb{C}$ over $\mathcal{X}$ is defined on*

$$\mathbb{C} = \left\{ \breve{z}, \left(\left\langle \left[\xi_{\mathbb{C}}^-(\breve{z}), \xi_{\mathbb{C}}^+(\breve{z}) \right], \left[\varrho_{\mathbb{C}}^-(\breve{z}), \varrho_{\mathbb{C}}^+(\breve{z}) \right] \right\rangle, \left\langle \xi_{\mathbb{C}}(\breve{z}), \varrho_{\mathbb{C}}(\breve{z}) \right\rangle \right); \breve{z} \in \mathcal{X} \right\},$$

where $\xi_{\mathbb{C}}^-(\breve{z}), \xi_{\mathbb{C}}^+(\breve{z})$ and $\varrho_{\mathbb{C}}^-(\breve{z}), \varrho_{\mathbb{C}}^+(\breve{z})$ represent the lower and upper bounds of the membership and non-membership degrees which are given by $\xi_{\mathbb{C}}^-(\breve{z}) = \delta_1^- = g_{\mathbb{C}}^-(\breve{z})e^{i\theta_{g_{\mathbb{C}}}^-(\breve{z})}, \xi_{\mathbb{C}}^+(\breve{z}) = \delta_1^+ = g_{\mathbb{C}}^+(\breve{z})e^{i\theta_{g_{\mathbb{C}}}^+(\breve{z})}$ such that $|\delta_1^-| \leq |\delta_1^+|$, while $\varrho_{\mathbb{C}}^-(\breve{z}) = \delta_2^- = h_{\mathbb{C}}^-(\breve{z})e^{i\theta_{h_{\mathbb{C}}}^-(\breve{z})}, \varrho_{\mathbb{C}}^+(\breve{z}) = \delta_2^+ = h_{\mathbb{C}}^+(\breve{z})e^{i\theta_{h_{\mathbb{C}}}^+(\breve{z})}$ such that $|\delta_2^-| \leq |\delta_2^+|$. Then $\xi_{\mathbb{C}}(\breve{z}), \varrho_{\mathbb{C}}(\breve{z})$ are the degrees of fuzzy membership and non-membership which are defined as $\xi_{\mathbb{C}}(\breve{z}) = \delta_1 = g_{\mathbb{C}}(\breve{z})e^{i\theta_{g_{\mathbb{C}}}(\breve{z})}, \varrho_{\mathbb{C}}(\breve{z}) = \delta_2 = h_{\mathbb{C}}(\breve{z})e^{i\theta_{h_{\mathbb{C}}}(\breve{z})}$ such that $|\delta_1| +$

$|\delta_2| \leq 1$. *The amplitude terms* $g_{\mathbb{C}}^{-}, g_{\mathbb{C}}^{+}, h_{\mathbb{C}}^{-}, h_{\mathbb{C}}^{+}, g_{\mathbb{C}}, h_{\mathbb{C}} \in [0,1]$ *and satisfy the inequality* $g_{\mathbb{C}}^{-} \leq g_{\mathbb{C}}^{+}, h_{\mathbb{C}}^{-} \leq h_{\mathbb{C}}^{+}, g_{\mathbb{C}}^{+} + h_{\mathbb{C}}^{+} \leq 1$ *and* $g_{\mathbb{C}} + h_{\mathbb{C}} \leq 1$. *On the other hand, the phase terms* $\theta_{g_{\mathbb{C}}}^{-}, \theta_{g_{\mathbb{C}}}^{+}, \theta_{h_{\mathbb{C}}}^{-}, \theta_{h_{\mathbb{C}}}^{+}, \theta_{g_{\mathbb{C}}}, \theta_{h_{\mathbb{C}}} \in [0, 2\pi]$ *and satisfy the inequality* $\theta_{g_{\mathbb{C}}}^{-} \leq \theta_{g_{\mathbb{C}}}^{+}, \theta_{h_{\mathbb{C}}}^{-} \leq \theta_{h_{\mathbb{C}}}^{+}, \theta_{g_{\mathbb{C}}}^{+} + \theta_{h_{\mathbb{C}}}^{+} \leq 2\pi$ *and* $\theta_{g_{\mathbb{C}}} + \theta_{h_{\mathbb{C}}} \leq 2\pi$ *for all* $\breve{z} \in \mathcal{X}$, *we shall denote CCIFS is*

$$\begin{aligned}\mathbb{C} =& \Big(\Big\langle \left[g_{\mathbb{C}}^{-}(\breve{z}) e^{i\theta_{g_{\mathbb{C}}}^{-}(\breve{z})}, g_{\mathbb{C}}^{+}(\breve{z}) e^{i\theta_{g_{\mathbb{C}}}^{+}(\breve{z})} \right], \\ & \left[h_{\mathbb{C}}^{-}(\breve{z}) e^{i\theta_{h_{\mathbb{C}}}^{-}(\breve{z})}, h_{\mathbb{C}}^{+}(\breve{z}) e^{i\theta_{h_{\mathbb{C}}}^{+}(\breve{z})} \right] \Big\rangle, \\ & \Big\langle g_{\mathbb{C}}(\breve{z}) e^{i\theta_{g_{\mathbb{C}}}(\breve{z})}, h_{\mathbb{C}}(\breve{z}) e^{i\theta_{h_{\mathbb{C}}}(\breve{z})} \Big\rangle \Big)\end{aligned}$$

for all $\breve{z} \in \mathcal{X}$.

Definition 3.2. *Given a CCIFS*

$$\mathbb{C}_k = \left(\left\langle \left[g_k^{-} e^{i\theta_{g_k}^{-}}, g_k^{+} e^{i\theta_{g_k}^{+}} \right], \left[h_k^{-} e^{i\theta_{h_k}^{-}}, h_k^{+} e^{i\theta_{h_k}^{+}} \right] \right\rangle, \left\langle g_k e^{i\theta_{g_k}}, h_k e^{i\theta_{h_k}} \right\rangle \right)$$

in $\mathcal{X}$, *(*$k = 1, 2, \ldots, n$*) then the complement of* $\mathbb{C}_k$ *is denoted by* $(\mathbb{C}_k)^c$ *and it is defined as*

$$(\mathbb{C}_k)^c = \left(\left\langle \left[h_k^{-} e^{i\theta_{h_k}^{-}}, h_k^{+} e^{i\theta_{h_k}^{+}} \right], \left[g_k^{-} e^{i\theta_{g_k}^{-}}, g_k^{+} e^{i\theta_{g_k}^{+}} \right] \right\rangle, \left\langle h_k e^{i\theta_{h_k}}, g_k e^{i\theta_{g_k}} \right\rangle \right),$$

for all $\breve{z} \in \mathcal{X}$.

Definition 3.3. *For a CCIFSs*

$$\mathbb{C}_k = \left(\left\langle \left[g_k^{-} e^{i\theta_{g_k}^{-}}, g_k^{+} e^{i\theta_{g_k}^{+}} \right], \left[h_k^{-} e^{i\theta_{h_k}^{-}}, h_k^{+} e^{i\theta_{h_k}^{+}} \right] \right\rangle, \left\langle g_k e^{i\theta_{g_k}}, h_k e^{i\theta_{h_k}} \right\rangle \right)$$

in $\mathcal{X}$, *(k=1,2) we define:*

a) Equality: $\mathbb{C}_1 = \mathbb{C}_2 \Leftrightarrow \left[g_1^{-} e^{i\theta_{g_1}^{-}}, g_1^{+} e^{i\theta_{g_1}^{+}} \right] = \left[g_2^{-} e^{i\theta_{g_2}^{-}}, g_2^{+} e^{i\theta_{g_2}^{+}} \right]$,
$\left[h_1^{-} e^{i\theta_{h_1}^{-}}, h_1^{+} e^{i\theta_{h_1}^{+}} \right] = \left[h_2^{-} e^{i\theta_{h_2}^{-}}, h_2^{+} e^{i\theta_{h_2}^{+}} \right], g_1 e^{i\theta_{g_1}} = g_2 e^{i\theta_{g_2}}$ *and* $h_1 e^{i\theta_{h_1}} = h_2 e^{i\theta_{h_2}}$.

b) P-order: $\mathbb{C}_1 \subseteq_P \mathbb{C}_2 \Leftrightarrow \left[g_1^{-} e^{i\theta_{g_1}^{-}}, g_1^{+} e^{i\theta_{g_1}^{+}} \right] \subseteq \left[g_2^{-} e^{i\theta_{g_2}^{-}}, g_2^{+} e^{i\theta_{g_2}^{+}} \right]$,
$\left[h_1^{-} e^{i\theta_{h_1}^{-}}, h_1^{+} e^{i\theta_{h_1}^{+}} \right] \supseteq \left[h_2^{-} e^{i\theta_{h_2}^{-}}, h_2^{+} e^{i\theta_{h_2}^{+}} \right], g_1 e^{i\theta_{g_1}} \leq g_2 e^{i\theta_{g_2}}$ *and* $h_1 e^{i\theta_{h_1}} \geq h_2 e^{i\theta_{h_2}}$.

b) R-order: $\mathbb{C}_1 \subseteq_P \mathbb{C}_2 \Leftrightarrow \left[g_1^{-} e^{i\theta_{g_1}^{-}}, g_1^{+} e^{i\theta_{g_1}^{+}} \right] \subseteq \left[g_2^{-} e^{i\theta_{g_2}^{-}}, g_2^{+} e^{i\theta_{g_2}^{+}} \right]$,
$\left[h_1^{-} e^{i\theta_{h_1}^{-}}, h_1^{+} e^{i\theta_{h_1}^{+}} \right] \supseteq \left[h_2^{-} e^{i\theta_{h_2}^{-}}, h_2^{+} e^{i\theta_{h_2}^{+}} \right], g_1 e^{i\theta_{g_1}} \geq g_2 e^{i\theta_{g_2}}$ *and* $h_1 e^{i\theta_{h_1}} \leq h_2 e^{i\theta_{h_2}}$.

Remark 3.1. *For two CCIFSs*

$$\mathbb{C}_k = \left(\left\langle \left[g_k^{-} e^{i\theta_{g_k}^{-}}, g_k^{+} e^{i\theta_{g_k}^{+}} \right], \left[h_k^{-} e^{i\theta_{h_k}^{-}}, h_k^{+} e^{i\theta_{h_k}^{+}} \right] \right\rangle, \left\langle g_k e^{i\theta_{g_k}}, h_k e^{i\theta_{h_k}} \right\rangle \right)$$

in $\mathcal{X}$, $(k=1,2)$ we define that:
i) $\mathbb{C}_1 \subseteq \mathbb{C}_2$ *if* $g_1^- \leq g_2^-, g_1^+ \leq g_2^+, h_1^- \geq h_2^-, h_1^+ \geq h_2^+$
and $\theta_{g_1}^- \leq \theta_{g_2}^-, \theta_{g_1}^+ \leq \theta_{g_2}^+, \theta_{h_1}^- \geq \theta_{h_2}^-, \theta_{h_1}^+ \geq \theta_{h_2}^+$.
ii) $\mathbb{C}_1 = \mathbb{C}_2$ *if and only if* $\mathbb{C}_1 \subseteq \mathbb{C}_2$ *and* $\mathbb{C}_1 \supseteq \mathbb{C}_2$.

Definition 3.4. *For a family of CCIFS* $\{\mathbb{C}_k, k \in N\}$*, we have*

$$(1)\quad \bigcup_{k\in N}^{P} \mathbb{C}_k = \left(\left\langle \bigvee_{k\in N}\left[g_k^- e^{i\theta_{g_k}^-}, g_k^+ e^{i\theta_{g_k}^+}\right], \bigwedge_{k\in N}\left[h_k^- e^{i\theta_{h_k}^-}, h_k^+ e^{i\theta_{h_k}^+}\right]\right\rangle, \left\langle \bigvee_{k\in N} g_k e^{i\theta_{g_k}}, \bigwedge_{k\in N} h_k e^{i\theta_{h_k}}\right\rangle\right)$$

$$(2)\quad \bigcap_{k\in N}^{P} \mathbb{C}_k = \left(\left\langle \bigwedge_{k\in N}\left[g_k^- e^{i\theta_{g_k}^-}, g_k^+ e^{i\theta_{g_k}^+}\right], \bigvee_{k\in N}\left[h_k^- e^{i\theta_{h_k}^-}, h_k^+ e^{i\theta_{h_k}^+}\right]\right\rangle, \left\langle \bigwedge_{k\in N} g_k e^{i\theta_{g_k}}, \bigvee_{k\in N} h_k e^{i\theta_{h_k}}\right\rangle\right)$$

$$(3)\quad \bigcup_{k\in N}^{R} \mathbb{C}_k = \left(\left\langle \bigvee_{k\in N}\left[g_k^- e^{i\theta_{g_k}^-}, g_k^+ e^{i\theta_{g_k}^+}\right], \bigwedge_{k\in N}\left[h_k^- e^{i\theta_{h_k}^-}, h_k^+ e^{i\theta_{h_k}^+}\right]\right\rangle, \left\langle \bigwedge_{k\in N} g_k e^{i\theta_{g_k}}, \bigvee_{k\in N} h_k e^{i\theta_{h_k}}\right\rangle\right)$$

$$(4)\quad \bigcap_{k\in N}^{R} \mathbb{C}_k = \left(\left\langle \bigwedge_{k\in N}\left[g_k^- e^{i\theta_{g_k}^-}, g_k^+ e^{i\theta_{g_k}^+}\right], \bigvee_{k\in N}\left[h_k^- e^{i\theta_{h_k}^-}, h_k^+ e^{i\theta_{h_k}^+}\right]\right\rangle, \left\langle \bigvee_{k\in N} g_k e^{i\theta_{g_k}}, \bigwedge_{k\in N} h_k e^{i\theta_{h_k}}\right\rangle\right)$$

Definition 3.5. *For CCIFSs*

$$\mathbb{C}_k = \left(\left\langle \left[g_k^- e^{i\theta_{g_k}^-}, g_k^+ e^{i\theta_{g_k}^+}\right], \left[h_k^- e^{i\theta_{h_k}^-}, h_k^+ e^{i\theta_{h_k}^+}\right]\right\rangle, \left\langle g_k e^{i\theta_{g_k}}, h_k e^{i\theta_{h_k}}\right\rangle\right)$$

in $\mathcal{X}$, $(k=1,2)$ and for any real number $\lambda > 0$, we define some operations as follows:

$$(i)\quad \mathbb{C}_1 + \mathbb{C}_2$$
$$= \left(\left\langle\left[1-\prod_{k=1}^{2}(1-g_k^-)e^{i2\pi\left(1-\prod_{k=1}^{2}\left(1-\frac{\theta_{g_k}^-}{2\pi}\right)\right)},\right.\right.\right.$$
$$\left.\left. 1-\prod_{k=1}^{2}(1-g_k^+)e^{i2\pi\left(1-\prod_{k=1}^{2}\left(1-\frac{\theta_{g_k}^+}{2\pi}\right)\right)}\right], \left[\prod_{k=1}^{2} h_k^- e^{i2\pi\left(\prod_{k=1}^{2}\frac{\theta_{h_k}^-}{2\pi}\right)}, \prod_{k=1}^{2} h_k^+ e^{i2\pi\left(\prod_{k=1}^{2}\frac{\theta_{h_k}^+}{2\pi}\right)}\right]\right\rangle,$$
$$\left.\left\langle 1-\prod_{k=1}^{2}(1-g_k)e^{i2\pi\left(1-\prod_{k=1}^{2}\left(1-\frac{\theta_{g_k}}{2\pi}\right)\right)}, \prod_{k=1}^{2} h_k e^{i2\pi\left(\prod_{k=1}^{2}\frac{\theta_{h_k}}{2\pi}\right)}\right\rangle\right)$$

$$(ii)\quad \mathbb{C}_1\times\mathbb{C}_2=\Bigg(\Bigg\langle\Bigg[\prod_{k=1}^{2}g_k^- e^{i2\pi\left(\prod_{k=1}^{2}\frac{\theta_{g_k}^-}{2\pi}\right)},\prod_{k=1}^{2}g_k^+ e^{i2\pi\left(\prod_{k=1}^{2}\frac{\theta_{g_k}^+}{2\pi}\right)}\Bigg],$$

$$\Bigg[1-\prod_{k=1}^{2}(1-h_k^-)e^{i2\pi\left(1-\prod_{k=1}^{2}\left(1-\frac{\theta_{h_k}^-}{2\pi}\right)\right)},1-\prod_{k=1}^{2}(1-h_k^+)e^{i2\pi\left(1-\prod_{k=1}^{2}\left(1-\frac{\theta_{h_k}^+}{2\pi}\right)\right)}\Bigg]\Bigg\rangle,$$

$$\Bigg\langle\prod_{k=1}^{2}g_k e^{i2\pi\left(\prod_{k=1}^{2}\frac{\theta_{g_k}}{2\pi}\right)},1-\prod_{k=1}^{2}(1-h_k)e^{i2\pi\left(1-\prod_{k=1}^{2}\left(1-\frac{\theta_{h_k}}{2\pi}\right)\right)}\Bigg\rangle\Bigg)$$

$$(iii)\quad \lambda\mathbb{C}_1=\Bigg(\Bigg\langle\Bigg[1-(1-g_1^-)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_1}^-}{2\pi}\right)^\lambda\right)},1-(1-g_1^+)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_1}^+}{2\pi}\right)^\lambda\right)}\Bigg],$$

$$\Bigg[(h_1^-)^\lambda e^{i2\pi\left(\frac{\theta_{h_1}^-}{2\pi}\right)^\lambda},(h_1^+)^\lambda e^{i2\pi\left(\frac{\theta_{h_k}^+}{2\pi}\right)^\lambda}\Bigg]\Bigg\rangle,$$

$$\Bigg\langle 1-(1-g_1)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_1}}{2\pi}\right)^\lambda\right)},(h_1)^\lambda e^{i2\pi\left(\frac{\theta_{h_1}}{2\pi}\right)^\lambda}\Bigg\rangle\Bigg)$$

$$(iv)\quad (\mathbb{C}_1)^\lambda=\Bigg(\Bigg\langle\Bigg[(g_1^-)^\lambda e^{i2\pi\left(\frac{\theta_{g_1}^-}{2\pi}\right)^\lambda},(g_1^+)^\lambda e^{i2\pi\left(\frac{\theta_{g_1}^+}{2\pi}\right)^\lambda}\Bigg],$$

$$\Bigg[1-(1-h_1^-)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{h_1}^-}{2\pi}\right)^\lambda\right)},1-(1-h_1^+)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{h_1}^+}{2\pi}\right)^\lambda\right)}\Bigg]\Bigg\rangle,$$

$$\Bigg\langle(g_1)^\lambda e^{i2\pi\left(\frac{\theta_{g_1}}{2\pi}\right)^\lambda},1-(1-h_1)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{h_1}}{2\pi}\right)^\lambda\right)}\Bigg\rangle\Bigg)$$

Theorem 3.1. *If $\mathbb{C}_1,\mathbb{C}_2$ are two CCIFSs and a real number $\lambda>0$, then $\mathbb{C}_1+\mathbb{C}_2,\mathbb{C}_1\times\mathbb{C}_2,\lambda\mathbb{C}_1\&(\mathbb{C}_1)^\lambda$ are also CCIFS.*

Proof. $\mathbb{C}_3=\mathbb{C}_1+\mathbb{C}_2=\left(\left\langle\left[g_3^- e^{i\theta_{g_3}^-},g_3^+ e^{i\theta_{g_3}^+}\right],\left[h_3^- e^{i\theta_{h_3}^-},h_3^+ e^{i\theta_{h_3}^+}\right]\right\rangle,\left\langle g_3e^{i\theta_{g_3}},h_3e^{i\theta_{h_3}}\right\rangle\right)$. Since $\mathbb{C}_1=\left(\left\langle\left[g_1^- e^{i\theta_{g_1}^-},g_1^+ e^{i\theta_{g_1}^+}\right],\left[h_1^- e^{i\theta_{h_1}^-},h_1^+ e^{i\theta_{h_1}^+}\right]\right\rangle,\left\langle g_1e^{i\theta_{g_1}},h_1e^{i\theta_{h_1}}\right\rangle\right)$ and $\mathbb{C}_2=\left(\left\langle\left[g_2^- e^{i\theta_{g_2}^-},g_2^+ e^{i\theta_{g_2}^+}\right],\left[h_2^- e^{i\theta_{h_2}^-},h_2^+ e^{i\theta_{h_2}^+}\right]\right\rangle,\left\langle g_2e^{i\theta_{g_2}},h_2e^{i\theta_{h_2}}\right\rangle\right)$ are two CCIFSs, so by definition 3.1, we have $g_k^-,g_k^+,h_k^-,h_k^+,g_k,h_k\in[0,1]$ such that $0\le g_k^++h_k^+\le 1$, $0\le g_k+h_k\le 1$ and $\theta_{g_k}^-,\theta_{g_k}^+,\theta_{h_k}^-,\theta_{h_k}^+,\theta_{g_k},\theta_{h_k}\in[0,2\pi]$ such that $\theta_{g_k}^++\theta_{h_k}^+\le 2\pi$, $\theta_{g_k}+\theta_{h_k}\le 2\pi$; (k=1,2). As $0\le g_k^-,g_k^+\le 1$ which implies that $0\le$

$1-\prod_{k=1}^{2}(1-g_k^-), 1-\prod_{k=1}^{2}(1-g_k^+) \leq 1$. Hence as $0 \leq g_3^-, g_3^+ \leq 1$. On the other hand, $0 \leq h_k^-, h_k^+ \leq 1$ which implies that $0 \leq \prod_{k=1}^{2} h_k^-, \prod_{k=1}^{2} h_k^+ \leq 1$ and hence $0 \leq h_3^-, h_3^+ \leq 1$. Further, $g_k^+ + h_k^+ \leq 1$ for (k=1,2) which implies that $\prod_{k=1}^{2} h_k^+ \leq \prod_{k=1}^{2}(1-g_k^+)$ and hence $g_k^+ + h_k^+ = 1-\prod_{k=1}^{2}(1-g_k^+) + \prod_{k=1}^{2} h_k^+ \leq 1-\prod_{k=1}^{2}(1-g_k^+) + \prod_{k=1}^{2}(1-g_k^+) = 1$. Also $g_k^+ + h_k^+ \geq 0$ as $g_k^+ \geq 0$ and $h_k^+ \geq 0$. Hence $0 \leq g_k^+ + h_k^+ \leq 1$. Then the fuzzy membership and non-membership values are $g_k + h_k \leq 1$ for (k=1,2) which implies that $\prod_{k=1}^{2} h_k \leq \prod_{k=1}^{2}(1-g_k)$ and hence $g_k + h_k = 1-\prod_{k=1}^{2}(1-g_k) + \prod_{k=1}^{2} h_k \leq 1-\prod_{k=1}^{2}(1-g_k) + \prod_{k=1}^{2}(1-g_k) = 1$. Also $g_k + h_k \geq 0$ as $g_k \geq 0$ and $h_k \geq 0$. Hence $0 \leq g_k + h_k \leq 1$. Similarly, it can proved that $\theta_{g_3}^-, \theta_{g_3}^+, \theta_{h_3}^-, \theta_{h_3}^+, \theta_{g_3}, \theta_{h_3} \in [0, 2\pi]$ such that $0 \leq \theta_{g_3}^+ + \theta_{h_3}^+ \leq 2\pi$, $0 \leq \theta_{g_3} + \theta_{h_3} \leq 2\pi$. Hence $\mathbb{C}_3 = \mathbb{C}_1 + \mathbb{C}_2$ is a CCIFN. Similarly, we can proved for the others. □

Theorem 3.2. *Let $\mathbb{C}_1$ and $\mathbb{C}_2$ be two CCIFNs and the three real numbers is $\lambda, \lambda_1, \lambda_2 > 0$. Then,*

(i) $\mathbb{C}_1 + \mathbb{C}_2 = \mathbb{C}_2 + \mathbb{C}_1$.
(ii) $\mathbb{C}_1 \times \mathbb{C}_2 = \mathbb{C}_2 \times \mathbb{C}_1$.
(iii) $\lambda(\mathbb{C}_1 + \mathbb{C}_2) = \lambda\mathbb{C}_1 + \lambda\mathbb{C}_2$.
(iv) $(\mathbb{C}_1 \times \mathbb{C}_2)^\lambda = \mathbb{C}_1^\lambda \times \mathbb{C}_2^\lambda$.
(v) $\lambda_1\mathbb{C}_1 + \lambda_2\mathbb{C}_2 = (\lambda_1 + \lambda_2)\mathbb{C}_1$.
(vi) $\mathbb{C}_1^{\lambda_1} \times \mathbb{C}_1^{\lambda_2} = \mathbb{C}_1^{\lambda_1+\lambda_2}$.

Proof. Straight forward by using Definition 3.5. □

Theorem 3.3. *Let $\mathbb{C}, \mathbb{C}_1$ and $\mathbb{C}_2$ be three CCIFNs and any real number $\lambda > 0$. Then,*

(i) $(\mathbb{C}^c)^\lambda = (\lambda\mathbb{C})^c$.
(ii) $\lambda(\mathbb{C})^c = (\mathbb{C}^\lambda)^c$.
(iii) $\mathbb{C}_1 \cup \mathbb{C}_2 = \mathbb{C}_2 \cup \mathbb{C}_1$.
(iv) $\mathbb{C}_1 \cap \mathbb{C}_2 = \mathbb{C}_2 \cap \mathbb{C}_1$.
(v) $\lambda(\mathbb{C}_1 \cup \mathbb{C}_2) = \lambda\mathbb{C}_1 \cup \lambda\mathbb{C}_2$.
(vi) $(\mathbb{C}_1 \cup \mathbb{C}_2)^\lambda = \mathbb{C}_1^\lambda \cup \mathbb{C}_2^\lambda$.

Proof. Here we will prove the parts (i),(iii),(v) only, while others can be derived similarly.

(i) Since $\mathbb{C}$ is CCIFN.

$$(\mathbb{C}^c)^\lambda = \Bigg(\Bigg\langle\Bigg[(h^-)^\lambda e^{i2\pi\left(\frac{\theta_h^-}{2\pi}\right)^\lambda}, (h^+)^\lambda e^{i2\pi\left(\frac{\theta_h^+}{2\pi}\right)^\lambda}\Bigg],$$
$$\Bigg[1-(1-g^-)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_g^-}{2\pi}\right)^\lambda\right)}, 1-(1-g^+)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_g^+}{2\pi}\right)^\lambda\right)}\Bigg]\Bigg\rangle,$$
$$\Bigg\langle (h)^\lambda e^{i2\pi\left(\frac{\theta_h}{2\pi}\right)^\lambda}, 1-(1-g)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_g}{2\pi}\right)^\lambda\right)}\Bigg\rangle\Bigg)$$
$$= \Bigg(\Bigg\langle\Bigg[1-(1-g^-)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_g^-}{2\pi}\right)^\lambda\right)}, 1-(1-g^+)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_g^+}{2\pi}\right)^\lambda\right)}\Bigg],$$
$$\Bigg[(h^-)^\lambda e^{i2\pi\left(\frac{\theta_h^-}{2\pi}\right)^\lambda}, (h^+)^\lambda e^{i2\pi\left(\frac{\theta_h^+}{2\pi}\right)^\lambda}\Bigg]\Bigg\rangle,$$
$$\Bigg\langle 1-(1-g)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_g}{2\pi}\right)^\lambda\right)}, (h)^\lambda e^{i2\pi\left(\frac{\theta_h}{2\pi}\right)^\lambda}\Bigg\rangle\Bigg)^c = (\lambda\mathbb{C})^c$$

(iii) Since $\mathbb{C}_1, \mathbb{C}_2$ is CCIFNs.

$$\mathbb{C}_1 \cup \mathbb{C}_2 = \Bigg(\Bigg\langle \Big[\max\left\{g_1^-, g_2^-\right\}, \max\left\{g_1^+, g_2^+\right\} e^{i\left[\max\left\{\theta_{g_1}^-, \theta_{g_2}^-\right\}, \max\left\{\theta_{g_1}^+, \theta_{g_2}^+\right\}\right]}\Big],$$
$$\Big[\min\left\{h_1^-, h_2^-\right\}, \min\left\{h_1^+, h_2^+\right\} e^{i\left[\min\left\{\theta_{h_1}^-, \theta_{h_2}^-\right\}, \min\left\{\theta_{h_1}^+, \theta_{h_2}^+\right\}\right]}\Big]\Bigg\rangle,$$
$$\Big\langle \max\left\{g_1, g_2\right\} e^{i\left[\max\left\{\theta_{g_1}, \theta_{g_2}\right\}\right]}, \min\left\{h_1, h_2\right\} e^{i\left[\min\left\{\theta_{h_1}, \theta_{h_2}\right\}\right]}\Big\rangle\Bigg)$$
$$= \Bigg(\Bigg\langle \Big[\max\left\{g_2^-, g_1^-\right\}, \max\left\{g_2^+, g_1^+\right\} e^{i\left[\max\left\{\theta_{g_2}^-, \theta_{g_1}^-\right\}, \max\left\{\theta_{g_2}^+, \theta_{g_1}^+\right\}\right]}\Big],$$

$$\left[\min\left\{h_2^-,h_1^-\right\},\min\left\{h_2^+,h_1^+\right\}e^{i\left[\min\left\{\theta_{h_2}^-,\theta_{h_1}^-\right\},\min\left\{\theta_{h_2}^+,\theta_{h_1}^+\right\}\right]}\right]\Bigg\rangle,$$
$$\left\langle\max\left\{g_2,g_1\right\}e^{i\left[\max\left\{\theta_{g_2},\theta_{g_1}\right\}\right]},\min\left\{h_2,h_1\right\}e^{i\left[\min\left\{\theta_{h_2},\theta_{h_1}\right\}\right]}\right\rangle\Bigg)$$
$$=\mathbb{C}_2\cup\mathbb{C}_1$$

(v) Since $\mathbb{C}_1\cup\mathbb{C}_2$ are CCIFNs and any real number $\lambda>0$. So by using definition3.4, and 3.5, we have

$$\lambda(\mathbb{C}_1\cup\mathbb{C}_2)$$
$$=\lambda\Bigg(\Bigg\langle\left[\max\left\{g_1^-,g_2^-\right\},\max\left\{g_1^+,g_2^+\right\}e^{i\left[\max\left\{\theta_{g_1}^-,\theta_{g_2}^-\right\},\max\left\{\theta_{g_1}^+,\theta_{g_2}^+\right\}\right]}\right],$$
$$\left[\min\left\{h_1^-,h_2^-\right\},\min\left\{h_1^+,h_2^+\right\}e^{i\left[\min\left\{\theta_{h_1}^-,\theta_{h_2}^-\right\},\min\left\{\theta_{h_1}^+,\theta_{h_2}^+\right\}\right]}\right]\Bigg\rangle,$$
$$\left\langle\max\left\{g_1,g_2\right\}e^{i\left[\max\left\{\theta_{g_1},\theta_{g_2}\right\}\right]},\min\left\{h_1,h_2\right\}e^{i\left[\min\left\{\theta_{h_1},\theta_{h_2}\right\}\right]}\right\rangle\Bigg)$$
$$=\Bigg(\Bigg\langle\left[\max\left\{1-(1-g_1^-)^\lambda,1-(1-g_2^-)^\lambda\right\},\max\left\{1-(1-g_1^+)^\lambda,1-(1-g_2^+)^\lambda\right\}\right]$$
$$\times e^{i\left[\max\left\{2\pi\left(1-\left(1-\frac{\theta_{g_1}^-}{2\pi}\right)^\lambda\right),2\pi\left(1-\left(1-\frac{\theta_{g_2}^-}{2\pi}\right)^\lambda\right)\right\},\max\left\{2\pi\left(1-\left(1-\frac{\theta_{g_1}^+}{2\pi}\right)^\lambda\right),2\pi\left(1-\left(1-\frac{\theta_{g_2}^+}{2\pi}\right)^\lambda\right)\right\}\right]},$$
$$\Bigg[\min\left\{(h_1^-)^\lambda,(h_2^-)^\lambda\right\},\min\left\{(h_1^+)^\lambda,(h_2^+)^\lambda\right\}$$
$$e^{i\left[\min\left\{2\pi\left(\frac{\theta_{h_1}^-}{2\pi}\right)^\lambda,2\pi\left(\frac{\theta_{h_2}^-}{2\pi}\right)^\lambda\right\},\min\left\{2\pi\left(\frac{\theta_{h_1}^+}{2\pi}\right)^\lambda,2\pi\left(\frac{\theta_{h_2}^+}{2\pi}\right)^\lambda\right\}\right]}\Bigg]\Bigg\rangle,$$
$$\Bigg\langle\max\left\{1-(1-g_1)^\lambda,1-(1-g_2)^\lambda\right\}e^{i\left[\max\left\{2\pi\left(1-\left(1-\frac{\theta_{g_1}}{2\pi}\right)^\lambda\right),2\pi\left(1-\left(1-\frac{\theta_{g_2}}{2\pi}\right)^\lambda\right)\right\}\right]},$$
$$\min\left\{(h_1)^\lambda,(h_2)^\lambda\right\}e^{i\left[\min\left\{2\pi\left(\frac{\theta_{h_1}}{2\pi}\right)^\lambda,2\pi\left(\frac{\theta_{h_2}}{2\pi}\right)^\lambda\right\}\right]}\Bigg\rangle\Bigg)$$

$$= \left(\left\langle \left[1-(1-g_1^-)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_1}^-}{2\pi}\right)^\lambda\right)}, 1-(1-g_1^+)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_1}^+}{2\pi}\right)^\lambda\right)} \right], \right.\right.$$

$$\left. \left[(h_1^-)^\lambda e^{i2\pi\left(\frac{\theta_{h_1}^-}{2\pi}\right)^\lambda}, (h_1^+)^\lambda e^{i2\pi\left(\frac{\theta_{h_1}^+}{2\pi}\right)^\lambda} \right] \right\rangle,$$

$$\left. \left\langle 1-(1-g_1)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_1}}{2\pi}\right)^\lambda\right)}, (h_1)^\lambda e^{i2\pi\left(\frac{\theta_{h_1}}{2\pi}\right)^\lambda} \right\rangle \right)$$

$$\cup \left(\left\langle \left[1-(1-g_2^-)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_2}^-}{2\pi}\right)^\lambda\right)}, 1-(1-g_2^+)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_2}^+}{2\pi}\right)^\lambda\right)} \right], \right.\right.$$

$$\left. \left[(h_2^-)^\lambda e^{i2\pi\left(\frac{\theta_{h_2}^-}{2\pi}\right)^\lambda}, (h_2^+)^\lambda e^{i2\pi\left(\frac{\theta_{h_2}^+}{2\pi}\right)^\lambda} \right] \right\rangle,$$

$$\left. \left\langle 1-(1-g_2)^\lambda e^{i2\pi\left(1-\left(1-\frac{\theta_{g_2}}{2\pi}\right)^\lambda\right)}, (h_2)^\lambda e^{i2\pi\left(\frac{\theta_{h_2}}{2\pi}\right)^\lambda} \right\rangle \right)$$

$$= \lambda\mathbb{C}_1 \cup \lambda\mathbb{C}_2$$

□

Theorem 3.4. *Let* $\mathbb{C}_1, \mathbb{C}_2$ *be any two CCIFNs. Then,*

(i) $(\mathbb{C}_1 \cap \mathbb{C}_2)^c = \mathbb{C}_1^c \cup \mathbb{C}_2^c$;
(ii) $(\mathbb{C}_1 \cup \mathbb{C}_2)^c = \mathbb{C}_1^c \cap \mathbb{C}_2^c$;
(iii) $(\mathbb{C}_1 \times \mathbb{C}_2)^c = \mathbb{C}_1^c + \mathbb{C}_2^c$;
(iv) $(\mathbb{C}_1 + \mathbb{C}_2)^c = \mathbb{C}_1^c \times \mathbb{C}_2^c$.

Proof. Here we will demonstrate the parts (i) and (iii) only, while others can be derived similarly.
(i) Since $\mathbb{C}_1 \& \mathbb{C}_2$ are CCIFNs.

$$(\mathbb{C}_1 \cap \mathbb{C}_2)^c = \left(\left\langle \left[\max\left\{h_1^-, h_2^-\right\}, \max\left\{h_1^+, h_2^+\right\} e^{i\left[\max\left\{\theta_{h_1}^-, \theta_{h_2}^-\right\}, \max\left\{\theta_{h_1}^+, \theta_{h_2}^+\right\}\right]} \right], \right.\right.$$

$$\left. \left[\min\left\{g_1^-, g_2^-\right\}, \min\left\{g_1^+, g_2^+\right\} e^{i\left[\min\left\{\theta_{g_1}^-, \theta_{g_2}^-\right\}, \min\left\{\theta_{g_1}^+, \theta_{g_2}^+\right\}\right]} \right] \right\rangle,$$

$$\left\langle \max\left\{h_1, h_2\right\} e^{i\left[\max\left\{\theta_{h_1}, \theta_{h_2}\right\}\right]}, \min\left\{g_1, g_2\right\} e^{i\left[\min\left\{\theta_{g_1}, \theta_{g_2}\right\}\right]} \right\rangle \Bigg)$$

$$= \left(\left\langle \left[h_1^-, h_1^+\right] e^{i\left[\theta_{h_1}^-, \theta_{h_1}^+\right]}, \left[g_1^-, g_1^+\right] e^{i\left[\theta_{g_1}^-, \theta_{g_1}^+\right]} \right\rangle, \left\langle h_1 e^{i\theta_{h_1}}, g_1 e^{i\theta_{g_1}} \right\rangle \right)$$
$$\cup \left(\left\langle \left[h_2^-, h_2^+\right] e^{i\left[\theta_{h_2}^-, \theta_{h_2}^+\right]}, \left[g_2^-, g_2^+\right] e^{i\left[\theta_{g_2}^-, \theta_{g_2}^+\right]} \right\rangle, \left\langle h_2 e^{i\theta_{h_2}}, g_2 e^{i\theta_{g_2}} \right\rangle \right)$$
$$= \mathbb{C}_1^c \cup \mathbb{C}_2^c$$

(iii) Since $\mathbb{C}_1$ and $\mathbb{C}_2$ are CCIFNs.

$$(\mathbb{C}_1 \times \mathbb{C}_2)^c = \Bigg(\Bigg\langle \Bigg[1 - \prod_{k=1}^{2} (1 - h_k^-) e^{i2\pi\left(1 - \prod_{k=1}^{2}\left(1 - \frac{\theta_{h_k}^-}{2\pi}\right)\right)},$$
$$1 - \prod_{k=1}^{2} (1 - h_k^+) e^{i2\pi\left(1 - \prod_{k=1}^{2}\left(1 - \frac{\theta_{h_k}^+}{2\pi}\right)\right)} \Bigg],$$
$$\left[\prod_{k=1}^{2} g_k^- e^{i2\pi\left(\prod_{k=1}^{2} \frac{\theta_{g_k}^-}{2\pi}\right)}, \prod_{k=1}^{2} g_k^+ e^{i2\pi\left(\prod_{k=1}^{2} \frac{\theta_{g_k}^+}{2\pi}\right)} \right] \Bigg\rangle,$$
$$\left\langle 1 - \prod_{k=1}^{2} (1 - h_k) e^{i2\pi\left(1 - \prod_{k=1}^{2}\left(1 - \frac{\theta_{h_k}}{2\pi}\right)\right)}, \prod_{k=1}^{2} g_k e^{i2\pi\left(\prod_{k=1}^{2} \frac{\theta_{g_k}}{2\pi}\right)} \right\rangle \Bigg)$$
$$= \left(\left\langle \left[h_1^-, h_1^+\right] e^{i\left[\theta_{h_1}^-, \theta_{h_1}^+\right]}, \left[g_1^-, g_1^+\right] e^{i\left[\theta_{g_1}^-, \theta_{g_1}^+\right]} \right\rangle, \left\langle h_1 e^{i\theta_{h_1}}, g_1 e^{i\theta_{g_1}} \right\rangle \right)$$
$$+ \left(\left\langle \left[h_2^-, h_2^+\right] e^{i\left[\theta_{h_2}^-, \theta_{h_2}^+\right]}, \left[g_2^-, g_2^+\right] e^{i\left[\theta_{g_2}^-, \theta_{g_2}^+\right]} \right\rangle, \left\langle h_2 e^{i\theta_{h_2}}, g_2 e^{i\theta_{g_2}} \right\rangle \right)$$
$$= \mathbb{C}_1^c + \mathbb{C}_2^c.$$

□

4. VALUE EXTRACTION FROM CCIFS

As the entries in a CCIFS are combination of complex cubic membership values and complex cubic non-membership values, it is a challenge to arrive at a conclusion from CCIFS . Hence it is vital to convert these values into their equivalent numerical values in order to derive some useful information. We propose the following score function for this purpose.

Definition 4.1. *For each element* $\check{z} \in \mathcal{X}$ *the score function is defined as,*

$$\delta(\mathbb{C}) = \left| \frac{g^- + g^+ + h^- + h^+ - (g+h)}{2} e^{i\left[\frac{\theta_g^- + \theta_g^+ + \theta_h^- + \theta_h^+ - (\theta_g + \theta_h)}{4}\right]} \right|.$$

It is evident that $0 \leq \delta(\mathbb{C}) \leq 2$.

4.1. **Application of CCIFS in determining the best network in remote areas.** In this section, we present a problem of determining the best network in remote areas using CCIFS. An algorithm is developed for the same. The working of the algorithm is illustrated with an example.

4.2. **Statement of the problem.** Let $U = \{n_1, n_2, .., n_n\}$ denote the list of networks. Let $E = \{e_1, e_2, \ldots, e_m\}$ denote the parameters based on which the selection is to be finalized. Let a network expert analyze the list of network providers based on the given parameters. The entire data is presented in the form of CCIFS by the expert. Now, the problem is to convert the CCIFS's into significant set which determines the best network from the given list.

4.3. **The Method.** Let the expert provide the CCIFS's value as in Definition 3.1. By using Definition 4.1, convert each element into a single value. Add the values of each parameter to calculate the total value for each network providers. Arrange the values in increasing order. The network with highest value is suitable for the remote area.

4.4. **Algorithm.** Step 1: Identify the list of network providers and the list of parameters.
Step 2: Form the CCIFS for the expert using Definition 3.1.
Step 3: Calculate the score function using Definition 4.1.
Step 4: Calculate the total value for each network providers.
Step 5: Order the values and the highest value confirms the suitable network provider for remote areas.

4.5. **Case Study.** Here, we present the working model of the algorithm with an example. A network expert is in the process of selecting the best network provider in remote areas individually and independently based on a given set of parameters.
1. Let $U = \{n_1, n_2, \ldots, n_n\}$ be network providers and $E = \{e_1, e_2, \ldots, e_m\}$ be parameters related to network feasibility. Here e_1 = connectivity with the main

server, e_2 = expenses, e_3= measurement of data speed and e_4= accessible area 2. Let a network expert inspect the network providers based on the parameter set and provide their observation details in CCIFSs by applying Definition 3.1.

	e_1
n_1	$(\langle[0.2e^{i0.6\pi}, 0.5e^{i0.8\pi}], [0.3e^{i0.2\pi}, 0.4e^{i\pi}]\rangle, \langle 0.4e^{i0.5\pi}, 0.6e^{i0.9\pi}\rangle)$
n_2	$(\langle[0.4e^{i0.6\pi}, 0.7e^{i1.2\pi}], [0.1e^{i0.2\pi}, 0.2e^{i0.4\pi}]\rangle, \langle 0.3e^{i0.7\pi}, 0.4e^{i0.8\pi}\rangle)$
n_3	$(\langle[0.1e^{i0.4\pi}, 0.4e^{i0.6\pi}], [0.2e^{i0.2\pi}, 0.3e^{i\pi}]\rangle, \langle 0.7e^{i1.3\pi}, 0.2e^{i0.4\pi}\rangle)$
n_4	$(\langle[0.4e^{i0.6\pi}, 0.5e^{i\pi}], [0.1e^{i0.2\pi}, 0.2e^{i0.6\pi}]\rangle, \langle 0.2e^{i1.2\pi}, 0.4e^{i0.4\pi}\rangle)$

	e_2
n_1	$(\langle[0.3e^{i0.4\pi}, 0.6e^{i0.8\pi}], [0.1e^{i0.8\pi}, 0.3e^{i\pi}]\rangle, \langle 0.5e^{i0.8\pi}, 0.4e^{i\pi}\rangle)$
n_2	$(\langle[0.3e^{i0.4\pi}, 0.4e^{i1.2\pi}], [0.1e^{i0.2\pi}, 0.3e^{i0.6\pi}]\rangle, \langle 0.1e^{i1.6\pi}, 0.1e^{i0.2\pi}\rangle)$
n_3	$(\langle[0.2e^{i0.6\pi}, 0.7e^{i\pi}], [0.1e^{i0.4\pi}, 0.3e^{i0.8\pi}]\rangle, \langle 0.2e^{i1.1\pi}, 0.1e^{i0.2\pi}\rangle)$
n_4	$(\langle[0.3e^{i0.8\pi}, 0.4e^{i1.2\pi}], [0.1e^{i0.4\pi}, 0.2e^{i0.6\pi}]\rangle, \langle 0.6e^{i0.8\pi}, 0.2e^{i0.7\pi}\rangle)$

	e_3
n_1	$(\langle[0.2e^{i0.8\pi}, 0.3e^{i1.2\pi}], [0.5e^{i0.4\pi}, 0.6e^{i0.6\pi}]\rangle, \langle 0.4e^{i0.6\pi}, 0.1e^{i0.7\pi}\rangle)$
n_2	$(\langle[0.5e^{i\pi}, 0.6e^{i1.4\pi}], [0.2e^{i0.2\pi}, 0.3e^{i0.4\pi}]\rangle, \langle 0.1e^{i1.7\pi}, 0.1e^{i0.2\pi}\rangle)$
n_3	$(\langle[0.1e^{i0.2\pi}, 0.5e^{i0.8\pi}], [0.2e^{i0.4\pi}, 0.3e^{i0.6\pi}]\rangle, \langle 0.4e^{i0.9\pi}, 0.2e^{i0.8\pi}\rangle)$
n_4	$(\langle[0.4e^{i0.2\pi}, 0.6e^{i\pi}], [0.2e^{i0.4\pi}, 0.3e^{i0.6\pi}]\rangle, \langle 0.1e^{i0.8\pi}, 0.2e^{i0.5\pi}\rangle)$

	e_4
n_1	$(\langle[0.5e^{i0.6\pi}, 0.6e^{i\pi}], [0.1e^{i0.4\pi}, 0.3e^{i0.8\pi}]\rangle, \langle 0.1e^{i0.7\pi}, 0.1e^{i0.9\pi}\rangle)$
n_2	$(\langle[0.2e^{i0.2\pi}, 0.45e^{i0.6\pi}], [0.4e^{i0.4\pi}, 0.5e^{i1.2\pi}]\rangle, \langle 0.1e^{i1.7\pi}, 0.5e^{i0.3\pi}\rangle)$
n_3	$(\langle[0.2e^{i0.6\pi}, 0.8e^{i0.8\pi}], [0.1e^{i0.2\pi}, 0.2e^{i0.4\pi}]\rangle, \langle 0.2e^{i1.6\pi}, 0.7e^{i0.7\pi}\rangle)$
n_4	$(\langle[0.6e^{i0.6\pi}, 0.7e^{i\pi}], [0.2e^{i0.6\pi}, 0.3e^{i0.8\pi}]\rangle, \langle 0.3e^{i1.2\pi}, 0.4e^{i0.3\pi}\rangle)$

3. Score function is calculated by using Definition 4.1:

$$\delta(\mathbb{C}) = \begin{pmatrix} 0.2 & 0.19 & 0.54 & 0.64 \\ 0.34 & 0.44 & 0.69 & 0.47 \\ 0.04 & 0.49 & 0.25 & 0.2 \\ 0.29 & 0.09 & 0.6 & 0.54 \end{pmatrix}$$

4. Total score for each network provider is calculated and presented as below:

Tabular representation of network providers total score values

n_i	Score value	Rank
n_2	1.94	1
n_1	1.57	2
n_4	1.52	3
n_3	0.98	4

From the above table, we observe that the network provider n_2 is best suitable for the remote areas.

5. Conclusion

In this manuscript, we have presented the notion of CCIFS to deal with today's uncertainty. Also, we have proposed a decision-making problem to show the working model of the tool.

References

[1] L.A. Zadeh : *Fuzzy sets*, Information and control, **8** (1965), 338-353.

[2] K.T. Atanassov: *Intuitionistic fuzzy sets*, Fuzzy sets and systems, **20** (1989), 87-96.

[3] K. Atanassov, G. Gargov: *Interval-valued intuitionistic fuzzy sets*, Fuzzy sets and systems, **31** (1989), 343-349.

[4] V. Chinnadurai, S. Barvkavi: *Cubic soft matrices*, International journal for research inmathematics and statistics. **3**(4)(2017), 01-22.

[5] G. Kaur, H. Garg: *Generalized cubic intuitionistic fuzzy aggregation operators using t-norm operations and their applications to group decision-making process*, Arabian journal for science and engineering, **44** (2019), 2775-2794.

[6] D. Ramot, R. Milo, M. Fiedman, A. Kandel: *Complex fuzzy sets*, IEEE Transactions on Fuzzy Systems, **10**(2) (2002), 171-186.

[7] V. Chinnadurai, S. Thayalan, A. Bobin : *Complex cubic set and their properties*, Advances in mathematics: Scientific journal, **9**(4) (2020), 1561-1567.

[8] A. Alkouri, A. Salleh: *Complex intuitionistic fuzzy sets*, 2nd International Conference on Fundamental and Applied Sciences, **1482**(2012), 464-470.

[9] HARISH GARG AND DIMPLE RANI: *Complex interval-valued intuitionistic fuzzy sets and their aggregation operators*, Fundamenta informaticae, **164**(2019), 61-101.

DEPARTMENT OF MATHEMATICS
ANNAMALAI UNIVERSITY
CHIDAMBARAM-608002, TAMIL NADU, INDIA
Email address: chinnaduraiau@gmail.com

DEPARTMENT OF MATHEMATICS
ANNAMALAI UNIVERSITY
CHIDAMBARAM-608002, TAMIL NADU, INDIA
Email address: sssthayalan@gmail.com

DEPARTMENT OF MATHEMATICS
ANNAMALAI UNIVERSITY
CHIDAMBARAM-608002, TAMIL NADU, INDIA
Email address: bobinalbert@gmail.com