

RADIATION EFFECT ON MHD BOUNDARY LAYER FLOW DUE TO AN EXPONENTIALLY STRETCHING SHEET

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ABSTRACT. An exponentially stretching sheet with radiation impact was used to examine the flow of the MHD boundary layer. The governing equations are converted into ODE's solved numerically by MATLAB solver bvp5c. The impacts of non-dimensional variables on the flow area and skinfriction and the rate of heat transfer is studied and achieved. The closed surface heat transfer rate was observed to decrease with increased amounts of magnetic and radiation parameters and seen in graphs and tabular form.

1. INTRODUCTION

Owing to its different applications in modern assembly procedures, for example, wire drawing, heat moving, paper processing, glass fibre, plastic films, the metal and polymer expulsion the drawing of and the metal spinning, the limit layer on a permanent stretching sheet has become a major concern in the latter decades. Some researchers are investigated on MHD flow with different effects like a chemical reaction, radiation, viscous dissipation with stretching sheet viz, Afify [1], Magyari & Keller [2], Mukhopadhyay et al. [3], Bhargava et al. [4], El-Aziz [5], Cortell [6], Prasad et al. [7], Bidin and Nazar [8], Makinde [9] and Anuar Ishak [10]. In this concept, some of the researchers Reddy et al. [12 - 15] studied the different aspects.

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In this study, the results of the MHD boundary layer flow over an exponentially stretching sheet in the view of radiation effect e observed as a reference to the above examinations. Diagram illustrates the impact on the velocity and temperature profiles of the flow variables.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the continuous 2-D MHD flow of a viscous fluid that is incompressible through a porous medium and conducts electrically generated by a stretching plate, placed into a quiet ambient fluid with the uniform temperature T_∞ as presented in Figure 1.

Consider that a variable magnetic field $B(x)$ is implemented perpendicular to the sheet & that the induced magnetic is ignored, which is explained for MHD flow at a lesser Reynolds number. Under the typical estimates of the boundary layer, the flow & heat exchange with the radiation impacts are governed by the following continuity, momentum and temperature equations according to the usual boundary layer estimations.

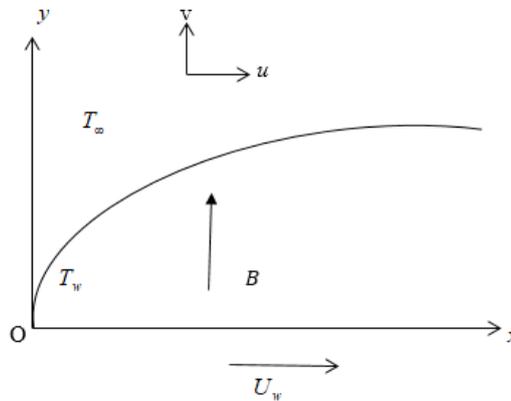


FIGURE 1. Gemperty of the model and coordinate system

$$(2.1) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$(2.2) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u$$

$$(2.3) \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y}$$

The required constraint is defined by:

$$\begin{aligned} v = 0, \quad u = U_w(x); \quad T = T_w(x), \quad \text{at } y \rightarrow 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad \text{as } y \rightarrow \infty. \end{aligned}$$

Now introducing the following parameters:

$$U_w(x) = ae^{x/L}$$

$$T_w(x) = T_\infty + T_0e^{x/2L},$$

where q_r is the Rosseland approximation such that:

$$(2.4) \quad q_r = -\frac{4}{3} \frac{\sigma^*}{k^*} \frac{\partial T^4}{\partial y}.$$

If the variation in temperature inside the flow is minimal sufficiently, by truncating T^4 in the Taylor's series expansion about T_∞ which later avoiding higher-order terms by taking the formula:

$$(2.5) \quad T^4 \cong T_\infty^3 (4T - 3T_\infty).$$

The equations (2.3) are regarded by the equations (2.4) and (2.5)

$$(2.6) \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \left(\alpha + \frac{16\sigma^*T_\infty^3}{\rho C_p} \right).$$

The stream functions fulfils the equation (2.1) which are define by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \dots$$

Next, by introduce the dimensionaless similarity variables and implementing,

$$\begin{aligned} \psi = \sqrt{2uLa} f(\eta) e^{x/2L}, \quad u = ae^{x/2L} f'(\eta), \quad v = -\sqrt{\frac{va}{2L}} e^{x/2L} (f(\eta) + \eta f'(\eta)) \\ T = T_\infty + T_0 e^{x/2L} \theta(\eta), \quad \eta = y \sqrt{(a/2uL)} e^{x/2L}. \end{aligned}$$

Equations (2.2) and (2.6) changed as in the form of

$$(2.7) \quad \begin{aligned} f' (2f' + M) = f'' + f''', \\ \left(1 + \frac{4}{3} K \right) \theta'' = \text{Pr} (f' \theta - f \theta'). \end{aligned}$$

The appropriate nondimensional boundary conditions changes to

$$(2.8) \quad \begin{aligned} f'(\eta) = 1, \quad f(\eta) = 0, \quad \theta(\eta) = 1, \quad \text{at} \quad \eta \rightarrow 0 \\ f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \text{as} \quad \eta \rightarrow \infty. \end{aligned}$$

The surface trying to contact the ambient fluid of constant density is the wall shear stress, thermal transfer, and mass transfer

$$(2.9) \quad \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad q_w(x) = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}.$$

The physical dimensions of this problem involve local skin friction, Nusselt number and local sherwood numbers, those are in the context of dimensionality.

$$\begin{aligned} C_f = \tau_w / \rho_f e^{2x/L} u_w^2 &\Rightarrow C_f = f''(0) / \sqrt{2 \operatorname{Re}_x}, \\ Nu_x = x q_x / k (T_w(x) - T_\infty) &\Rightarrow Nu_x = - \left(\sqrt{x \operatorname{Re}_x / L} \right) \theta'(0). \end{aligned}$$

3. RESULTS AND DISCUSSION

The ODE system (2.7) - (2.9) is solved using the numerical methods by making use of the MATLAB inbuilt solver bvp4c. There is a proper agreement with the existing and the literature, as seen in Table 1. Figures 2 illustrate the effects of Prandtl number $Pr = 1$, magnetic parameter $M = 1$, and radiation number $K = 1$ on the velocity profile $f(\eta)f'(\eta)$ and temperature profile $\theta(\eta)$.

This shows that the velocity profile $f(\eta)$ and $f'(\eta)$ are reciprocally proportional of one another. The velocity curves are shown in Figure 3 for numerous estimates of the magnetic parameter M . This shows that, as M increases, the rate of transport diminishes considerably. It is known that the Lorentz force produces greater resistance to the transportation phenomenon and the Lorentz force is changed with the change in M and Pr and R have no impact on the flow area of the equation (2.7). When M is increased, the velocity of the surface shear stress gradient enhances. The magnetic parameter M is therefore a magnetic parameter for controlling the shear stress of the surface. The temperature curves with other parameters for altered values of M, K , and Pr are unity. Figures 3 to 6 display the asymptotically complete conditions of the far-field boundaries to confirm the exactness of the numerical outcomes acquired. Figures 4 to 6 show that the thickness of the thermal boundary layer enhances with M & K rises, yet inverse patterns are observed to expand Pr values. This outcome in falling away of the nearby Nusselt number $(-\theta'(0))$ with enhancing M and K yet reverse patterns are viewed for

enhancing estimations of Pr . This is due to, when Pr builds, the warm diffusivity diminishes & hence the temperature is diffused far from the heated surface all the added gradually and in result increment the temperature gradient at the surface.

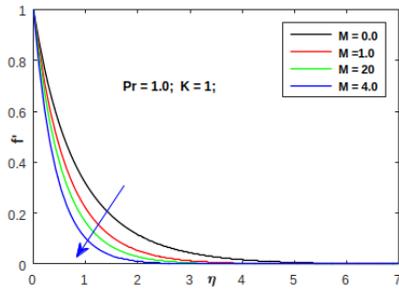


FIGURE 2. $f(\eta)$, $f'(\eta)$, $\theta(\eta)$ variation with η

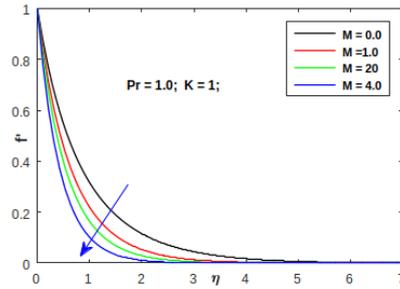


FIGURE 3. $f'(\eta)$ v/s M

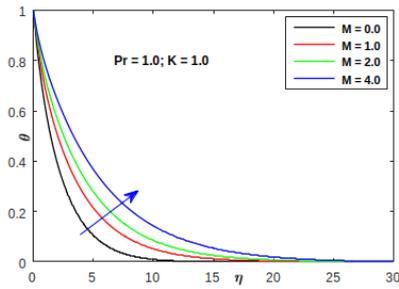


FIGURE 4. $\theta(\eta)$ vs M

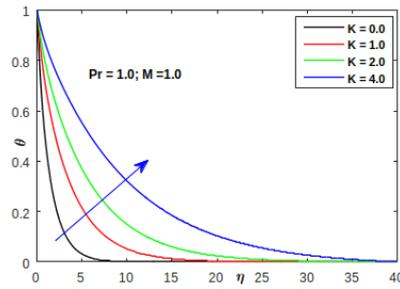


FIGURE 5. $\theta(\eta)$ vs K

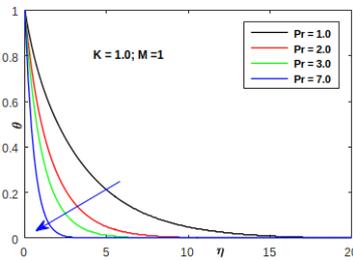


FIGURE 6. $\theta(\eta)$ vs Pr

TABLE 1. Estimates of $\theta'(0)$ for changed K , M , and Pr values.

S	M	Pr	[3]	[6]	[9]	[11]	Present Study
0	0	1	-0.954782	-0.954785	-0.9548	-0.9548	-0.954784
		2			-1.4714	-1.4715	-1.471462
		3	-1.869075	-1.869074	-1.8691	-1.8691	-1.869073
		5	-2.500135	-2.500132		-2.5001	-2.500111
		10	-3.660379	-3.660372		-3.6604	-3.660346
	1	1				-0.8611	-0.861097
1	0				-0.5315	-0.5312	-0.53117
	1					-0.4505	-0.450687

4. CONCLUSION

In this study, a radiation impact concerning the steady MHD boundary layer flow through an infinite stretching sheet was determined. The numerical values are in total agreement in the previously obtained values and it is found the surface shear stress enhances with M . As the Pr rises, the rate of heat transfer is also seen to rise. However, the rate of transfer of heat is found to go down with the magnetic and radiation parameters which are represented as M and K respectively.

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