Advances in Mathematics: Scientific Journal **10** (2021), no.1, 171–179
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.1.17

PAIRWISE SEQUENTIAL SPACE

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ABSTRACT. In this paper we define pairwise sequential space, and study basic properties. Also, we obtain some relations with other bitopological spaces.

1. INTRODUCTION

The concept of bitopological spaces was initiated by Kelly [5]. Abitopological space is an order triple $(\mathrm{X}, \tau_1, \tau_2)$ where X is a non-empty set and $\tau_1, \tau_2$are two topolog ies on X. Since then several mathematicians work on bitopological spaces. P. Fletcher, I. H. Hoyle & C. W. Patty (1969) [3], Y.W.Kim (1968) [7], A. Fora, H. Hdeib [9], A. Kilicman, Z. Salleh (2007) [6], and S. Ramkumar (2015) [11]. Several results were obtained in the above studies that generalize topological properties in bitopological spaces, and bitopological spaces turned to be an important field in general topology. Still pairwise sequential spaces are not investigated. In this paper we study pairwise sequential spaces and try to obtain various results concering their properties.

We will use the letters $P-$, $S-$ to denote the pairwise and semi respectively, e.g $P-$compact stands for pairwise compact and $S-$compact stands for semi compact. The product of $\tau_1$and τ_2 will be denoted by $\tau_1 \times \tau_2$, τ_i-closure of a set A will be denoted by cl_iA and $\tau_d$denotes the discrete topology.

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2020 *Mathematics Subject Classification.* 54E55, 54D55.

Key words and phrases. Bitopological space, pairwise sequential space, pairwise Frèchet space, pairwise k-space.

2. Pairwise Sequential Spaces

Definition 2.1. *[3] A cover* $\widetilde{U}$ *of the bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called* $\tau_1\tau_2-$ *open cover if* $\widetilde{U} \subset \tau_1 \cup \tau_2$*, if in addition* $\widetilde{U}$ *contains at least one non- empty member of* τ_1 *and at least one non- empty member of* τ_2*, it's called P- open cover.*

Definition 2.2. *[3] A bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called* $P-$*compact if every* $P-$*open cover of the space* $(\mathrm{X}, \tau_1, \tau_2)$ *has a finite subcover.*

Definition 2.3. *[3] A bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called S- compact if every* $\tau_1\tau_2-$*open cover of the space* $(\mathrm{X}, \tau_1, \tau_2)$ *has a finite subcover.*

Definition 2.4. *[4] A bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called* τ_1- *compact with respect to* τ_2*if for each* τ_1-*open cover of X there is a finite* τ_2-*subcover.*

Definition 2.5. *[1] A bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called B- compact if it is* τ_1- *compact with respect to* τ_2 *and* τ_2- *compact with respect to* τ_1*.*

Example 1. *Let*X=$\mathbb{R}$*,* $\mathcal{B}_1 = \{X, \{x\} : x \in X - \{0\}\ \}$*,* $\mathcal{B}_2 = \{X, \{x\} : x \in X - \{1\}\}$*. Let* τ_1 *and* τ_2 *be the topologies on X which are generated by the bases* $\mathcal{B}_1$ *and* $\mathcal{B}_2$ *,respectively. Then* (X, τ_1, τ_2) *is not* $P-$*compact, since the* $P-$*open cover* $\{\{x\} : x \in X\ \}$ *of X has no finite subcover, but it is B* $-$*compact, since any* τ_1-*open cover or any* τ_2-*open cover of X must contain X as a member and* $\{X\}$ *is a finte subcover of any* τ_1-*open cover or any* τ_2-*open cover of X. Also it is clear that* (X, τ_1, τ_2) *is not S- compact.*

Definition 2.6. *[5] A bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called pairwise Hausdorff if for any two distinct points* $x, y \in X$*, there exist disjoint open sets* $V_1 \in \tau_1$ *and* $V_2 \in \tau_2$ *with* $x \in V_1$ *and* $y \in V_2$*.*

Definition 2.7. *[5] A bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ τ_1*is said to be regular with respect to* τ_2*if for each point* $x \in X$ *and each* τ_1*–closed subset F s.t* $x \notin F$*, there are* τ_1*–open set* U *and* τ_2*–open set* V *s.t* $x \in U$*and* $F \subset V$ *and* $U \cap V = \emptyset$*.*

Definition 2.8. *[5] A bitopolgical space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called* $P-$*regular if it is* τ_1- *regular with respect to* τ_2 *and* τ_2- *regular with respect to* τ_1*.*

Proposition 2.1. *[12] If* $(\mathrm{X}, \tau_1, \tau_2)$ *is a bitopological space, then the following are equivalent:*

(1) τ_1- *regular with respect to* τ_2*.*

(2) *For each point $x \in X$ and τ_1– open set U containing x, there is a τ_1–open set V s.t $x \in V \subset cl_2\, V \ \subset U$.*

Definition 2.9. *[8] A bitopological space $(\mathrm{X}, \tau_1, \tau_2)$ is called pairwise countably compact if every countable $P-$open cover of X has a finite subcover.*

Definition 2.10. *[4] A bitopological space $(\mathrm{X}, \tau_1, \tau_2)$ is called S -countably compact if every countable $\tau_1\tau_2$- open cover of X has a finite subcover.*

Definition 2.11. *[4] A bitopological space $(\mathrm{X}, \tau_1, \tau_2)$ is called τ_1-countably compact with respect to $\tau_2$if and only if for each countable τ_1–open cover of X there is a finite τ_2–open subcover.*

Definition 2.12. *[4] A bitopological space $(\mathrm{X}, \tau_1, \tau_2)$ is called B -countably compact if it is τ_1–countably compact with respect to $\tau_2$and τ_2–countably compact with respect to τ_1.*

Definition 2.13. *Let $(\mathrm{X}, \tau_1, \tau_2)$ and $x \in \ \mathrm{X}$. A family β_x of τ_2–open sets ,each of which contains x, is called a τ_1-local base of x with respect to τ_2 if and only if for any τ_1–open set U of X containing x , there exists a $V \in \beta_x$ such that $x \in \ \mathrm{V}\subset\mathrm{U}$.*

Definition 2.14. *A bitopological space $(\mathrm{X}, \tau_1, \tau_2)$ is called $P-$first countable if and only if for each $x \in \ \mathrm{X}$,there exists a τ_1-local base of x with respect to τ_2 and a τ_2-local base of x with respect to τ_1.*

Definition 2.15. *Let $(\mathrm{X}, \tau_1, \tau_2)$ be a bitopological space then:*

a) $(\mathrm{X}, \tau_1, \tau_2)$ *is called τ_1-sequential with respect to $\tau_2$if and only if for every τ_1-non closed subset A of X there is a sequence (x_n) in A that converage to a point x in X\A in τ_2.*
b) $(\mathrm{X}, \tau_1, \tau_2)$ *is called τ_2-sequential with respect to $\tau_1$if and only if for every τ_2-non closed subset A of X there is a sequence (x_n) in A that converage to a point x in X\ A in τ_1.*
c) $(\mathrm{X}, \tau_1, \tau_2)$ *is called pairwise sequential (denoted by P-sequential) if and only if it is τ_1-sequential with respect to τ_2 and τ_2-sequential with respect to τ_1.*

Definition 2.16. *Let $(\mathrm{X}, \tau_1, \tau_2)$ be a bitopological space then:*

a) $(\mathrm{X}, \tau_1, \tau_2)$ *is called τ_1–$Fréchet$ space with respect to τ_2 if and only if for every subset A of X and $x \in cl_1(A)$, there is a sequence (x_n) of A and (x_n) converges to a point x in A in τ_2.*

b) $(\mathrm{X}, \tau_1, \tau_2)$ *is called* τ_2– $Fréchet$ *space with respect to* τ_1 *if and only if for every subset A of X and* $x \in cl_2(A)$*, there is a sequence* (x_n) *of A and* (x_n) *converges to a point* x *in A in* τ_1.

c) $(\mathrm{X}, \tau_1, \tau_2)$ *is called pairwise* $Fréchet$ *space (denoted by* $P - Fréchet$ *space) if and only if it is* τ_1–$Fréchet$ *space with respect to* τ_2 *and* τ_2–$Fréchet$ *space with respect to* τ_1.

Theorem 2.1. *Every pairwise first countable space is pairwise Fréchet space.*

Proof. Suppose $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise first countable space. Let *A* be a subset of X such that $x \in cl_1(A)$, then there exist a τ_2–countable local base $\widetilde{\beta} = \{\beta_n : n \in \mathbb{N}\}$ at x such that, $x_1 \in \beta_1, x_2 \in \beta_2 - \beta_1, \ldots$ each of which must intersect *A*. Choose $x_m \in \beta_n \cap A$, $\forall\, m \geq 1$, x_m is a τ_2– sequence in *A*, given any τ_2–neighborhood U of *x*, U must contains β_n for $n \in \mathbb{N}$. So, $x_n \in U\ \forall\, n \geq n_0$, and $x_n \to x$ in τ_2. Hence $(\mathrm{X}, \tau_1, \tau_2)$ is a τ_1– $Fréchet$ space with respect to τ_2. Similarly we can show that $(\mathrm{X}, \tau_1, \tau_2)$ is a τ_2– $Fréchet$ space with respect to τ_1. Therefore $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise $Fréchet$ space. ☐

Theorem 2.2. *Every pairwise Fréchet space is pairwise sequential space.*

Proof. Let *A* be a non τ_1–closed subset of X and $x \in cl_1(A)$, $x \notin A$. Since $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise $Fréchet$ space, then there exist a sequence (x_n) in A such that $x_n \to x \in \mathrm{X} \setminus A$ in τ_2. Hence $(\mathrm{X}, \tau_1, \tau_2)$ is τ_1-sequential with respect to τ_2, similarly we can show that it is τ_2-sequential with respect to τ_1. Therefore $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise sequential space. ☐

Definition 2.17. *Let* $(\mathrm{X}, \tau_1, \tau_2)$ *be a bitopological space then:*

a) $(\mathrm{X}, \tau_1, \tau_2)$ *is called* τ_1- $k-$*space with respect to* τ_2 *if whenever F is a* τ_1–*closed subset in X if and only if* $F \cap K$ *is a* τ_2–*closed in K, for every a* τ_1–*compact set K in X with respect to* τ_2.

b) $(\mathrm{X}, \tau_1, \tau_2)$ *is called* τ_2-k *space with respect to* τ_1 *if whenever F is a* τ_2– *closed subset in X if and only if* $F \cap K$ *is a* τ_1– *closed in K, for every a* τ_2– *compact set K in X with respect to* τ_1.

c) $(\mathrm{X}, \tau_1, \tau_2)$ *is called pairwise* $k-$*space (denoted by P-* $k-$*space) if and only if* $(\mathrm{X}, \tau_1, \tau_2)$ *is a* τ_1-$k-$*space with respect to* τ_2 *and a* τ_2-$k-$ *space with respect to* τ_1.

Theorem 2.3. *Every pairwise sequential space is pairwise* $k-$*space.*

Proof. Let A be a non τ_1–closed subset of X, since $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise sequential space, then there exist a sequence (x_n) in A such that $x_n \to x \in$ X\ A in τ_2. Then *K*= $\{x_n : n = 1, 2, \ldots\} \cup \{x\}$ is a τ_1– compact with respect to τ_2, A ∩ *K* is not $\tau_2$closed in *K*. Similarly for a non τ_2– closed subset of X. Therefore $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise -$k-$space. □

Corollary 2.1. *Every pairwise Fréchet space (hence pairwise first countable) is a pairwise $k-$space.*

Lemma 2.1. *Let* $(\mathrm{X}, \tau_1, \tau_2)$ *be a pairwise sequential space, if every subspace of X is pairwise sequential space. Then X is pairwise Fréchet space.*

Proof. Let *A* ⊆ X, suppose $x \in cl_1(A)$. If $x \in A$, then *A* is τ_1– closed. If $x \notin A$, then *A* is a non τ_1– closed subset of X. Consider a subspace $A \cup \{x\}$, so *A* is a non τ_1– closed subset of $A \cup \{x\}$, but by the assumption $A \cup \{x\}$ is pairwise sequential space, then there exist a sequence (x_n) in *A* that converges to y in *A* with respect to τ_2. Similarly if *A* ⊆ X and $x \in cl_2(A)$,then there exist a sequence (x_n) in *A* that converges to a point in *A* with respect to τ_1. Hence, X is pairwise $Fre\acute{c}het$ space. □

Definition 2.18. *Let* $(\mathrm{X}, \tau_1, \tau_2)$ *be a bitopological space then:*

a) *(X, τ_1, τ_2) has a τ_1–countable tightness with respect to τ_2 if whenever A ⊆ X and $x \in cl_1(A)$, there exist a countable subset B of A such that $x \in cl_2(B)$.*
b) $(\mathrm{X}, \tau_1, \tau_2)$ *has a τ_2–countable tightness with respect to τ_1 if whenever A ⊆ X and $x \in cl_2(A)$, there exist a countable subset B of A such that $x \in cl_1(B)$.*
c) $(\mathrm{X}, \tau_1, \tau_2)$ *has a pairwise countable tightness (denoted by P-countable tightness) if and only if* $(\mathrm{X}, \tau_1, \tau_2)$ *has a τ_1– countable tightness with respect to $\tau_2$and* $(\mathrm{X}, \tau_1, \tau_2)$ *has a τ_2– countable tightness with respect to τ_1.*

Theorem 2.4. *Every pairwise sequential space has a pairwise countable tightness.*

Proof. Let D be a subset of X such that $x \in cl_1(D)$. Let K= $\{cl_2(E) \mid E$ countable, $E \subseteq D\}$ clearly K⊆ $cl_2(D)$, K is a τ_2 sequentially closed (i.e every sequence in K converges in τ_2 to a point in K). Since X is a pairwise sequential space, then K is a τ_1–closed set. Here, $D \subseteq K$, $cl_1(D) \subseteq K$, then for any $x \in cl_1(D)$, there exist a countable subset E of D and $x \in cl_2(E)$. Therefore X has a countable tightness with respect to τ_2. Similarly X has a countable tightness with respect to τ_1. Hence X has a pairwise countable tightness. □

Definition 2.19. *[4] A function $f : (\mathrm{X}, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is P -continuous (P-open, P-closed, P-homomorphism respectively) if the function $f : (X, \tau_1) \to (Y, \sigma_1)$ and $f : (X, \tau_2) \to (Y, \sigma_2)$ are continuous (open, closed, homomorphism respectively) respectively.*

Definition 2.20. *[9] A function $f : (\mathrm{X}, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called pairwise perfect if f is pairwise continuous, pairwise closed, and for all $y \in Y$, the set $f^{-1}(y)$ is pairwise compact.*

In the next example it will be shown that the continuous image of pairwise sequential space need not be pairwise sequential space.

Example 2. *Let $(\mathrm{X}, \tau_d, \tau_d)$ and $(\mathrm{X}, \tau_\alpha, \tau_\beta)$ be a bitopological spaces. Here τ_d: discrete topology and $(\mathrm{X}, \tau_\alpha), (\mathrm{X}, \tau_\beta)$ are not sequential spaces, i.e $(\mathrm{X}, \tau_\alpha, \tau_\beta)$ is not a $P-$sequential space.The identity map I_x: $(\mathrm{X}, \tau_d) \to (\mathrm{X}, \tau_\alpha)$ is continuous surjection of the sequential space (X, τ_d) onto the non-sequential spaces $(\mathrm{X}, \tau_\alpha)$ and the identity map and I_x: $(\mathrm{X}, \tau_d) \to (\mathrm{X}, \tau_\beta)$ is continuous surjection of the sequential space (X, τ_d) onto the non-sequential spaces (X, τ_β).*

Theorem 2.5. *Let $(\mathrm{X}, \tau_1, \tau_2)$ be a $B-$countably compact space, and (Y, σ_1, σ_2) be a pairwise sequential space. Then the projection functions $\pi_1 : (X \times Y, \tau_1 \times \sigma_1) \to (Y, \sigma_1)$ and $\pi_2 : (X \times Y, \tau_2 \times \sigma_2) \to (Y, \sigma_2)$ are closed.*

Proof. Let F be a $\tau_1 \times \sigma_1$ closed subset of $X \times Y$. Let $y_1, y_2, \ldots$ be a sequence of point of $\pi_1(F)$ and $y \in \lim\ y_k$ in σ_1, choose $x_k \in X$ such that $(x_k, y_k) \in F$ in $\tau_1 \times \sigma_1$.If A=$\{x_1,\ x_2, \ldots\}$ is finite then there exist $x\ \in X$, $x_{r_k} = x$ for an infinite sequence $r_1 < r_2 < r_3 < ..$ of integers. So that $(x, y) \in \lim\ (x_r, y_r)$ in $\tau_1 \times \sigma_1$, this implies $(x, y) \in cl\,(F) = F$ in $\tau_1 \times \sigma_1$ which gives $y \in \pi_1(F)$ in σ_1. If A is infinite, then A has τ_2–cluster point *x* ,since X is pairwise countably compact this gives $(x, y) \in cl\,(F) = F$ in $\tau_1 \times \sigma_1$. We have $y \in \pi_1(F)$ in σ_1. Since X is pairwise sequential space then $\pi_1(F)$ is closed with respect to σ_1 (i.e $\pi_1 : (X \times Y, \tau_1 \times \sigma_1) \to (Y, \sigma_1)$ is closed). Similarly we can show that $\pi_2 : (X \times Y, \tau_2 \times \sigma_2) \to (Y, \sigma_2)$ is closed. □

The following theorem is easily proved.

Theorem 2.6. *If $f : (\mathrm{X}, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise closed and continuous mapping where $(\mathrm{X}, \tau_1, \tau_2)$ and (Y, σ_1, σ_2) are pairwise Hausdorff spaces, and $f^{-1}(y)$*

is pairwise countably compact, then for any pairwise countably compact subset $Z \subset Y$, we have $f^{-1}(Z)$ is pairwise countably compact.

Theorem 2.7. *The product of two pairwise countably compact spaces, one of which is pairwise sequential is pairwise countably compact.*

Proof. Let $\pi : X \times Y \to Y$ is pairwise closed continuous function then, $\pi^{-1}(y) = X \times \{y\} \cong X$. By the above theorem we get that $X \times Y$ is pairwise countably compact. □

Definition 2.21. *A bitopological space $(\mathrm{X}, \tau_1, \tau_2)$ is called pairwise sequentially compact (denoted by P-sequentially compact) if and only if both (X, τ_1) and (X, τ_2) are sequentially compact.*

Theorem 2.8. *Every pairwise sequentially compact space is pairwise countably compact.*

Proof. Let $(\mathrm{X}, \tau_1, \tau_2)$be a pairwise sequentially compact and let A be an infinite subset of X. Since A is infinite subset of X , then there exist an infinite sequence (x_n) , since X is pairwise sequentially compact, so (x_n) has τ_i– convergent subsequence (*i* =1,2) say (x_{n_k}) , $\forall\ n, k \in \mathbb{N}$ such that $(x_{n_k}) \to x \in X$. Therefore eachτ_i–neighborhood U containing *x* , contains infinite number of points of A , so *x* is a τ_i - cluster point of A. Hence $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise countably compact. □

Theorem 2.9. *Let $(\mathrm{X}, \tau_1, \tau_2)$ be a pairwise sequential space. Then $(\mathrm{X}, \tau_1, \tau_2)$ is pairwise sequentially compact if and only if it is pairwise countably compact.*

Proof. Pairwise sequentially compact always implies pairwise countably compact by Theorem 2.8. To prove the converse, let (x_n) be a sequence in X and let *A*= $\{x_1, x_2, x_3, \ldots\}$ be an infinite sequence in X. Since X is pairwise countably compact then A has a τ_1- cluster point *p* in X and a τ_2 - cluster point *q* in X. Thus *p* must be an element of $cl_1(A\backslash\{p\})$ and *q* must be an element of $cl_2(A\backslash\{q\})$. Therefore $A\backslash\{p\}$ and $A\backslash\{q\}$ are not closed. Since X is a pairwise sequential space then $A\backslash\{p\}$ contains a sequence converging to a point in the $(A\backslash\{p\})^c$ in $\tau_2$and $A\backslash\{q\}$ contains a sequence converging to a point in the $(A\backslash\{q\})^c$ in τ_1. Rearranging the terms of this sequence we obtain a convergence subsequence of (x_n) in τ_1 and τ_2. □

Definition 2.22. *[10] In the bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$*,* τ_1*is (countably) paracompact with respect to* τ_2 *if each (countable)* τ_1- *open cover of X has* τ_1*–open refinement which is* τ_2*–locally finite.*

Definition 2.23. *A bitopological space* $(\mathrm{X}, \tau_1, \tau_2)$ *is called pairwise paracompact if and only if it is* τ_1 *paracompact with respect to* τ_2*and* τ_2 *paracompact with respect to* τ_1.

Theorem 2.10. *A pairwise regular,* $B-$*countably compact and pairwise paracompact space is* $B-$*compact.*

Proof. If $(\mathrm{X}, \tau_1, \tau_2)$ is τ_1 paracompact with respect to τ_2. Then every τ_1–open cover of X has τ_1–open refinement which is τ_2–closed locally finite. Say $\mathcal{H}$, suppose $\mathcal{H}$ is not finite , so it has a countable subfamily $\{E_i | i \in N\}$. Since $\mathcal{H}$ is a τ_2–closed locally finite, it is closure persevering. Now for every subfamily $D \subseteq \mathcal{H}$, $\bigcup D$ is a τ_2–closed. Let $F_1 = \bigcup_{i=1}^{\infty} E_i$, then F_1 is τ_2–closed, and let $F_2 = \bigcup_{i=2}^{\infty} E_i$, then F_2 is τ_2–closed and $F_2 \subseteq F_1$. If we continue in this way we get a decreasing sequence (F_i) of closed subset in τ_2 of X. Since $\mathcal{H}$ is τ_2–closed locally finite, there is no point of X is contained in infinitely many elements of $\mathcal{H}$, so, no points of X is contained in all of F_i. Therefore $\cap F_i = \emptyset$. So $\{F_i | i \in N\}$ is a countable family of closed subset of X in τ_2 with no finite intersection property, but with empty intersection. So X is not $\tau_1$countably compact with respect to τ_2. Similarly in the same manner we show that X is not τ_2 countably compact with respect to τ_1. Hence X is not $B-$countably compact which is a contradiction. Therefore X must be $B-$compact. □

If we combine Theorem 2.1 and Theorem 2.9, we obtain the equivalence of the compactness and sequentially compactness in bitopological spaces.

Theorem 2.11. *Let X be a pairwise paracompact sequential space. Then X is* $B-$*compact if and only if it is pairwise sequentially compact.*

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