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INTUITIONISTIC K-PARTITIONED FUZZY GRAPH

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ABSTRACT. Graph partition is the reduction of a graph to a smaller graph by partitioning. In this paper, we use the concept of partitioning to divide an intuitionistic fuzzy graph into two, three etc. We have introduced a new graph named Intuitionistic k-partitioned fuzzy graph. An intuitionistic fuzzy graph with any number of nodes in the node set is partitioned into Intuitionistic k-partition fuzzy graph, where $k = 2, 3, 4$ and so on. The membership and non-membership values of the node set are summed and the node set is partitioned into k-subsets with the sum of each subset more or less equal to the other. This partitioning of the node set into subsets is allowed up to the maximum sum of membership and non-membership value of the nodes in the node set. We have discussed the Intuitionistic k-partitioning of the fuzzy graph with an example and have analysed the degree, size of the new graph through some theorems.

1. INTRODUCTION

To measure the uncertainty involved in the decision making by humans, L. Zadeh introduced a control logic in 1965 [20]. This logic is used to represent the input and output degrees of the estimated reasoning in numbers, to indicate the decisions made by the humans for imprecisions. In 1975, Fuzzy graph was first introduced by Rosenfeld [17], and analogy of several graph theoretical concepts and the related concepts to fuzzy graph were given by Yen and Bang while working on fuzzy relation and its applications [19]. In 1986 Krassimir T. Atanassov was the first to introduce the concept of intuitionistic fuzzy

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set [4], which is characterized as two functions expressing the belongingness and non-belongingness for the elements of the universe and the theory behind intuitionistic fuzzy graph. In 2006 R. Parvathi and M.G. Karunambigai [8] introduced intuitionistic fuzzy graph. Degree, order and size of intuitionistic fuzzy graph was worked out by A. Nagoor Gani and S. Shajitha Begam [5] in 2010. Strong intutionistic fuzzy graph is introduced by Muhammed Akram and Bijan Davvaz in 2012 [2]. In 2013 Muhammed Akram, Wiselawa and Dudek generliazed Intuitionstic fuzzy set to hypergraphs with applications [3]. Operations like lexicographic product, direct product and strong product are detailed by four authors namely Hossein, Sovan, Madhumangal Pal and Rajab Ali in 2015 [16]. Intuitionistic fuzzy graph structure [1] was introduced by Muhammed Akram in 2017.

T. Pathinathan and J. Jesintha Rosline introduced a new fuzzy graph named double layered fuzzy graph [14] in 2014 and its theoretical concepts were discussed. They also contributed in extending their concept to triple layered fuzzy graph [18]. In 2015 J. Jesintha Roseline and T. Pathinathan defined intuitionistic double layered fuzzy graph [10] and T. Pathinathan and M. Peter have extended to Hesitancy double layered and triple layered fuzzy graphs in 2018 [13]. In 2017 J. Jon Arockiaraj, J. Jesintha Rosline and B. Rejina have introduced intuitionistic double layered fuzzy planar graph with properties and examples [6]. In 2017 T. Pathinathan and M. Peter worked on the conditions of balanced concepts and introduced balanced double layered fuzzy graph [12]. Contributing explicitly they have also introduced Balanced intuitionistic double layered fuzzy graph in 2017 [15]. In 2019 T. Pathinathan and A. Kirupa introduced partitioning and a new fuzzy graph namely k-partition fuzzy graph and worked on its theoretical concepts with illustrations [11]. They have extended their work to balanced k-partitioned fuzzy graph and to star balanced k-partitioned fuzzy graphs [7].

2. Preliminaries

Definition 2.1. *[17] Let V be a nonempty subset. A fuzzy graph $\underset{\sim}{G}$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : E \to [0,1]$ such that σ is a fuzzy subset of V and μ is a fuzzy relation on σ such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V.*

Definition 2.2. *[9] Let V is a nonempty set. An intuitionistic fuzzy graph of $\underset{\sim}{G} : (V, E)$ is of the form $I\underset{\sim}{G} : (v_i, \mu, \gamma, e_j, \alpha, \vartheta)$, where $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu : V \times [0,1]$ and $\gamma : V \times [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu(v_i) \leq \gamma(v_i) \leq 1$ for every $v_i \in V$. Let the fuzzy relation $e_1, e_2, \ldots e_n \in E \subset V$, where $\alpha : V \times V \to [0,1]$ and $\vartheta : V \times V \to [0,1]$ such that $\alpha(v_i, v_j) \leq \min[\mu(v_i), \mu(v_j)]$ and $\vartheta(v_i, v_j) \leq \max[\gamma(v_i), \gamma(v_j)]$ for every $v_i, v_j \in E$.*

Definition 2.3. *[5] The degree of any node in intuitionistic fuzzy graph $I\underset{\sim}{G}$ is defined by $D(v) = \langle D_\mu(v), D_\gamma(v)\rangle$, where $D_{\mu(I\underset{\sim}{G})}(v) = \sum_i \alpha(u_i v)$ and $D_{\gamma(I\underset{\sim}{G})}(v) = \sum_i \vartheta(u_i v)$.*

Definition 2.4. *[5] The order of intuitionistic fuzzy graph $I\underset{\sim}{G}$ is defined as $O\left(I\underset{\sim}{G}\right) = \langle O_\mu\left(I\underset{\sim}{G}\right), O_\gamma\left(I\underset{\sim}{G}\right)\rangle$, where $O_\mu\left(I\underset{\sim}{G}\right) = \sum_{v_i \in V} \mu(v_i)$ and $O_\gamma\left(I\underset{\sim}{G}\right) = \sum_{v_i \in V} \gamma(v_i)$.*

Definition 2.5. *[5] The size of intuitionistic fuzzy graph $I\underset{\sim}{G}$ is defined as $S\left(I\underset{\sim}{G}\right) = \langle S_\alpha(\left(I\underset{\sim}{G}\right)), S_\vartheta\left(I\underset{\sim}{G}\right)\rangle$, where $S_\alpha\left(I\underset{\sim}{G}\right) = \sum_{u \neq v} \alpha(uv)$ and $S_\vartheta\left(I\underset{\sim}{G}\right) = \sum_{u \neq v} \vartheta(uv)$.*

Definition 2.6. *[5] Let $\underset{\sim}{G}(\sigma, \mu)$ be a fuzzy graph. We define a k-partition fuzzy graph of $\underset{\sim}{G}$, $\underset{\sim}{G}_{k_p} : \left(\sigma_{k_p(\underset{\sim}{G})}, \mu_{k_p(\underset{\sim}{G})}\right)$ as follows. The node set σ is partitioned into k disjoint subsets namely $\sigma_{X_1}, \sigma_{X_2}, \ldots, \sigma_{X_k}$ such that the sum of the membership of the nodes of the subsets is more or less equal to each other i.e., the sum of membership of nodes in σ_{X_i} satisfies the condition $\left|\sum \sigma_{X_i} - \sum \sigma_{X_j}\right| < \epsilon$, where $i, j = 1, 2, \ldots, k$ and $i \neq j$. We have to partition σ such that an edge in $\mu_{k_p(\underset{\sim}{G})}$ originates at σ_{X_i} and ends edge in σ_{X_j}. And*

$$\mu_{k_p(\underset{\sim}{G})}(x_i x_j) = \begin{cases} \mu_{\underset{\sim}{G}}(x_i x_j), & x_i \in \sigma_{x_i} \text{ and } x_j \in \sigma_{x_j} \quad \forall i \neq j; \\ 0, & \text{otherwise} \end{cases}$$

where $\mu \in [0,1]$. By definition $\mu_{k_p(\underset{\sim}{G})}(x_i x_j) \leq \sigma_{X_i}(x_i) \wedge \sigma_{X_j}(x_j)$. Here $\mu_{k_p(\underset{\sim}{G})}$ is a fuzzy relation on the subset $\sigma_{k_p(\underset{\sim}{G})}$.

3. Intuitionistic K-Partitioned Fuzzy Graph(IK-PFG)

We introduce the definition of intuitionistic k-partitioned fuzzy graph.

Definition 3.1. *Let $\underset{\sim}{IG}(\sigma, \mu, \gamma, e, \alpha, \vartheta)$ be a intuitionistic fuzzy graph and let σ be a nonempty set on $\underset{\sim}{IG}$. An intuitionistic k-partitioned fuzzy graph is of the form $\underset{\sim}{IG}_{k_p} : \left\langle \left(\sigma_{k_p}, \mu_{k_p}, \gamma_{k_p}\right), \left(e_{k_p}, \alpha_{k_p}, \vartheta_{k_p}\right)\right\rangle$, each elements of node set σ have degree of membership μ and non-membership γ satisfying a condition $0 \leq \mu_{\underset{\sim}{IG}}(v_i) \leq \gamma_{\underset{\sim}{IG}}(v_i) \leq 1$. The node set σ is partitioned into k disjoint subsets namely $\sigma_{Z_1}, \sigma_{Z_2}, \ldots, \sigma_{Z_k}$, such that the sum of the membership and non-membership values of nodes of the subsets is more or less equal to each other i.e., the sum of membership and non-membership of the nodes of the subset in σ_{Z_k}, satisfies the condition $\left|\sum \sigma_{Z_i} - \sum \sigma_{Z_j}\right| < \epsilon$, where $i, j = 1, 2, \ldots, k$ and $i \neq j$ and $0 \leq \mu_{k_p(\underset{\sim}{IG})}(v_i) \leq \gamma_{k_p(\underset{\sim}{IG})}(v_i) \leq 1$. We have to partition σ such that an edge e_i originates at σ_{Z_i} and ends in σ_{Z_j}. And*

$$\left\langle \alpha_{k_p(\underset{\sim}{IG})}(u_i v_j), \vartheta_{k_p(\underset{\sim}{IG})}(u_i v_j) \right\rangle = \begin{cases} \left\langle \mu_{k_p(\underset{\sim}{IG})}(u_i v_j), \gamma_{k_p(\underset{\sim}{IG})}(u_i v_j)\right\rangle, & u_i \in \sigma_{Z_i} \text{ and } v_j \in \sigma_{Z_j}\ \forall i \neq j; \\ 0, & \text{otherwise} \end{cases}$$

where

$$\alpha_{k_p(\underset{\sim}{IG})}(u_i v_j) \leq \min\left[\mu_{k_p(\underset{\sim}{IG})}(u_i), \mu_{k_p(\underset{\sim}{IG})}(v_j)\right]$$

and

$$\vartheta_{k_p(\underset{\sim}{IG})}(u_i v_j) \leq \min\left[\gamma_{k_p(\underset{\sim}{IG})}(u_i), \gamma_{k_p(\underset{\sim}{IG})}(v_j)\right],$$

$0 \leq \alpha_{k_p(\underset{\sim}{IG})}(u_i v_j) \leq \vartheta_{k_p(\underset{\sim}{IG})}(u_i v_j) \leq 1$. Here $\left\langle \alpha_{k_p(\underset{\sim}{IG})}(u_i v_j), \vartheta_{k_p(\underset{\sim}{IG})}(u_i v_j)\right\rangle$ is a fuzzy relation on the fuzzy subsets $\left\langle \mu_{k_p(\underset{\sim}{IG})}, \gamma_{k_p(\underset{\sim}{IG})}\right\rangle$.

Example 1. *Let us consider an intuitionistic fuzzy graph $\underset{\sim}{IG} : (V, E)$ with node set of 8 vertices. $\sigma(\underset{\sim}{IG}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ with the degrees of belongingness and non-belongingness. The membership and non-membership values of the node set are as follows*

$$\begin{array}{llll} v_1(0.4, 0.5), & v_2(0.6, 0.2), & v_3(0.2, 0.3), & v_4(0.3, 0.1), \\ v_5(0.5, 0.2), & v_6(0.1, 0.5), & v_7(0.2, 0.1), & v_8(0.1, 0.1). \end{array}$$

i.e., the node set σ inclusive of μ and γ is partitioned into subsets of 2 or 3 or 4 etc., each subsets having the sum of membership and non-membership values of the nodes more or less equal using the condition $\left|\sum \sigma_{Z_i} - \sum \sigma_{Z_j}\right| < \epsilon$.

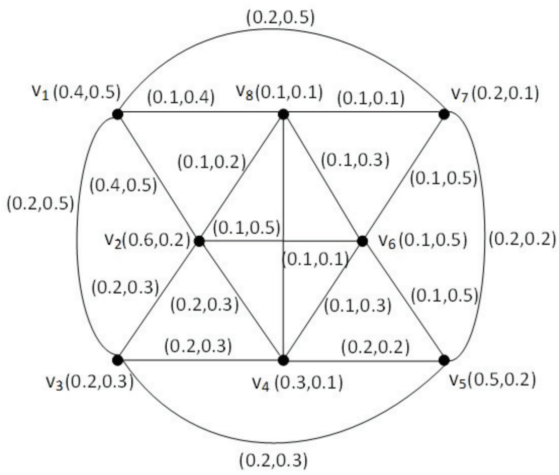

FIGURE 1. Intuitionistic Fuzzy Graph

Case 1: Partitioning the node set into 2 subsets

The node set σ is partitioned into 2 subsets $\sigma_{Z_1} = \{v_1, v_3, v_6, v_8\}$ and $\sigma_{Z_2} = \{v_2, v_4, v_5, v_7\}$

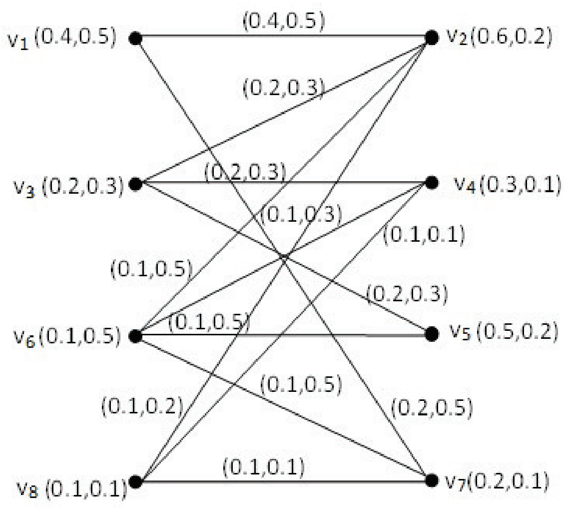

FIGURE 2. Intuitionistic 2-partitioned Fuzzy Graph

Case 2: Partitioning the node set into 3 subsets

The 3 partitioned subsets are $\sigma_{Z_1} = \{v_1, v_6\}$, $\sigma_{Z_2} = \{v_2, v_3, v_8\}$ and $\sigma_{Z_3} = \{v_4, v_5, v_7\}$.

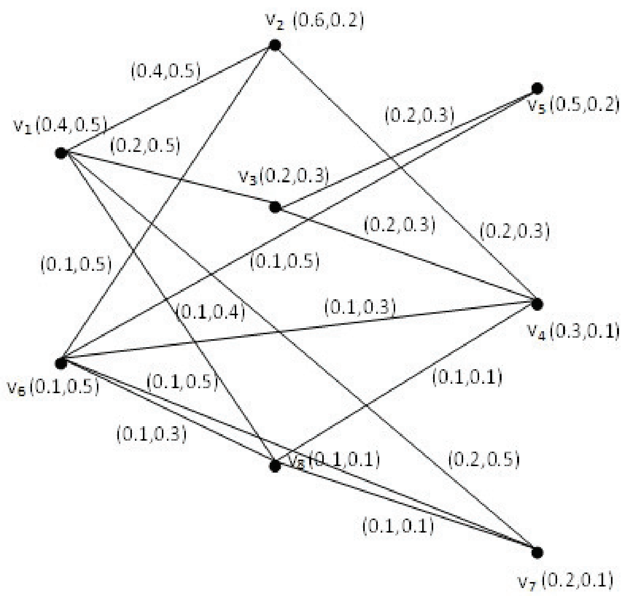

FIGURE 3. Intuitionistic 3-partitioned Fuzzy Graph

Case 3: Partitioning the node set into 4 subsets *The partitioned subsets are* $\sigma_{Z_1} = \{v_1, v_8\}$, $\sigma_{Z_2} = \{v_2, v_7\}$ *and* $\sigma_{Z_3} = \{v_4, v_5\}$, $\sigma_{Z_4} = \{v_3, v_6\}$.

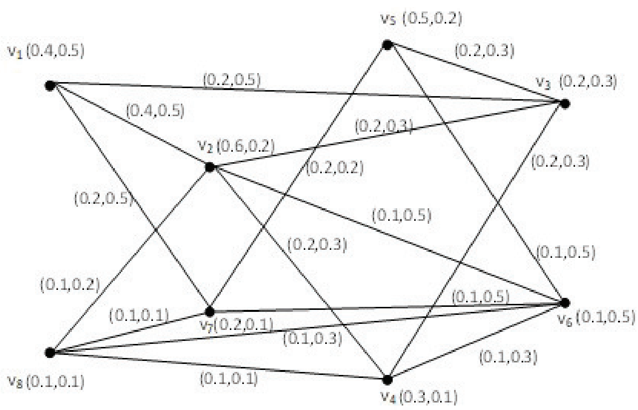

FIGURE 4. Intuitionistic 4-partitioned Fuzzy Graph

Note: Further the partitioning could not be proceeded, since the sum of membership and non-membership value of the subset exceeds to the maximum sum of the membership and non-membership value of the nodes in the node set.

Remark 3.1. *The number of edges will be equal in intuitionistic fuzzy graph and in partitioned intuitionistic fuzzy graph only if the edges are allowed among the subsets. Such partitioned fuzzy graph that allows the edges among the subsets could be named as strict intuitionistic k-partitioned fuzzy graph [15].*

4. Theoretical concepts

Theorem 4.1. *If $\left|\sum \alpha_{Z_i} - \sum \alpha_{Z_j}\right| < \epsilon$, then the order of intuitionistic fuzzy graph is equal to order of intuitionistic k-partitioned fuzzy graph i.e., $O(\underset{\sim}{IG}) = O(\underset{\sim}{IG}_{k_p})$, where $\underset{\sim}{IG}$ is intuitionistic fuzzy graph and $\underset{\sim}{IG}_{k_p}$ is intuitionistic k-partitioned fuzzy graph $k = 2, 3, \ldots$*

Proof. Given $\underset{\sim}{IG}$ is a intuitionistic fuzzy graph and $\underset{\sim}{IG}_{k_p}$ is a intuitionistic k-partitioned fuzzy graph. We know that by the definition of order of $\underset{\sim}{IG}$ is

$$O\left(\underset{\sim}{IG}\right) = \left\langle O_\mu\left(\underset{\sim}{IG}\right), O_\gamma\left(\underset{\sim}{IG}\right)\right\rangle = \left\langle \sum_{v_i\in V}\mu(v_i), \sum_{v_i\in V}\gamma(v_i)\right\rangle.$$

And the order of $\underset{\sim}{IG}_{k_p}$ is defined as $O\left(\underset{\sim}{IG}_{k_p}\right) = \left\langle O_\mu\left(\underset{\sim}{IG}_{k_p}\right), O_\gamma\left(\underset{\sim}{IG}_{k_p}\right)\right\rangle$, where $O_\mu\left(\underset{\sim}{IG}_{k_p}\right) = \sum_{v_i\in\sigma_{Z_j}} \mu_{\underset{\sim}{IG}_{k_p}(v_i)}$ and $O_\gamma\left(\underset{\sim}{IG}_{k_p}\right) = \sum_{v_i\in\sigma_{Z_j}} \gamma_{\underset{\sim}{IG}_{k_p}(v_i)}$ where $j = 2, 3, \ldots$.

Therefore we could find that

$$\sum_{v_i\in V}\mu(v_i) = \sum_{v_i\in\sigma_{Z_j}} \mu_{\underset{\sim}{IG}_{k_p}(v_i)} \text{ and } \sum_{v_i\in V}\gamma(v_i) = \sum_{v_i\in\sigma_{Z_j}} \gamma_{\underset{\sim}{IG}_{k_p}(v_i)},$$

where $i = 1, 2, \ldots, j = 1, 2, \ldots,$ and $k = 1, 2, \ldots$. Hence

$$\begin{aligned}
\left\langle \sum_{v_i\in V}\mu(v_i), \sum_{v_i\in V}\gamma(v_i)\right\rangle &= \left\langle \sum_{v_i\in\sigma_{Z_j}} \mu_{\underset{\sim}{IG}_{k_p}(v_i)}, \sum_{v_i\in\sigma_{Z_j}} \gamma_{\underset{\sim}{IG}_{k_p}(v_i)}\right\rangle \\
\Rightarrow \left\langle O_\mu\left(\underset{\sim}{IG}\right), O_\gamma\left(\underset{\sim}{IG}\right)\right\rangle &= \left\langle O_\mu\left(\underset{\sim}{IG}_{k_p}\right), O_\gamma\left(\underset{\sim}{IG}_{k_p}\right)\right\rangle \\
\Rightarrow O\left(\underset{\sim}{IG}\right) &= O\left(\underset{\sim}{IG}_{k_p}\right).
\end{aligned}$$

Hence the proof. □

Illustration. From figure 1, we have

$$O\left(\underset{\sim}{IG}\right) = \left\langle \sum_{v_i\in V}\mu(v_i), \sum_{v_i\in V}\gamma(v_i)\right\rangle = \langle 2.4, 2\rangle.$$

And by the definition of the order of intuitionistic k-partition fuzzy graph,

$$O\left(\underset{\sim}{IG}_{k_p}\right)=\left\langle \sum_{v_i\in\sigma Z_j}\mu_{\underset{\sim}{IG}_{k_p}(v_i)}, \sum_{v_i\in\sigma Z_j}\gamma_{\underset{\sim}{IG}_{k_p}(v_i)}\right\rangle$$

For $k=2$ from figure 2,

$$\begin{aligned}\sum_{v_i\in\sigma Z_j}\mu_{\underset{\sim}{IG}_{k_p}}(v_i) &= \left(\sum_{v_i\in\sigma Z_1}\mu_{\underset{\sim}{IG}_{2_p}}(v_i), \sum_{v_i\in\sigma Z_2}\mu_{\underset{\sim}{IG}_{2_p}}(v_i)\right)\\ &= (0.8+1.2)=2.4\\ \sum_{v_i\in\sigma Z_j}\gamma_{\underset{\sim}{IG}_{k_p}}(v_i) &= \left(\sum_{v_i\in\sigma Z_1}\gamma_{\underset{\sim}{IG}_{2_p}}(v_i), \sum_{v_i\in\sigma Z_2}\gamma_{\underset{\sim}{IG}_{2_p}}(v_i)\right)\\ &= (1.4+0.6)=2.\end{aligned}$$

Therefore $O\left(\underset{\sim}{IG}_{2_p}=\langle 2.4,2\rangle\right)$. Hence we have $O(\underset{\sim}{IG})=O(\underset{\sim}{IG}_{2_p})$. Similarly the theorem is true for $k=3,4,\ldots$

Theorem 4.2. *Size of* $\underset{\sim}{IG}$ $>$ *Size of* $\underset{\sim}{IG}_{k_p}$*, where* $\underset{\sim}{IG}_{k_p}$ *is a intuitionistic* k*-partitioned fuzzy graph.*

Proof. Let $\underset{\sim}{IG}_{k_p}$ is a intuitionistic k-partitioned fuzzy graph for $k=2,3,\ldots$ As we know by the definition, the size of intuitionistic fuzzy graph

$$S(\underset{\sim}{IG})=\left\langle S_\alpha(\underset{\sim}{IG}), S_\vartheta(\underset{\sim}{IG})\right\rangle.$$

From the definition of $\underset{\sim}{IG}$: $(v_1,\mu,\gamma,e_j,\alpha,\vartheta)$ and $\underset{\sim}{IG}_{k_p}$: $\langle(\sigma_{k_p},\mu_{k_p},\gamma_{k_p}),$ $(e_{k_p},\alpha_{k_p},\vartheta_{k_p})\rangle$ the size of intuitionistic k-partitioned fuzzy graph is defined as:

$$S\left(\underset{\sim}{IG}_{k_p}\right)=\left\langle S_\alpha\left(\underset{\sim}{IG}_{k_p}\right), S_\vartheta\left(\underset{\sim}{IG}_{k_p}\right)\right\rangle,$$

where

$$S_\alpha\left(\underset{\sim}{IG}_{k_p}\right)=\sum_{u_i\neq v_j\in\sigma Z_i}\alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j)$$

and

$$S_\vartheta\left(\underset{\sim}{IG}_{k_p}\right)=\sum_{u_i\neq v_j\in\sigma Z_i}\vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j),$$

$$\begin{aligned} S_\alpha(\underset{\sim}{IG}) &= \sum_{u\neq v}\alpha(uv) > \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j) = S_\alpha(\underset{\sim}{IG}_{k_p}) \\ S_\vartheta(\underset{\sim}{IG}) &= \sum_{u\neq v}\vartheta(uv) > \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j) = S_\vartheta(\underset{\sim}{IG}_{k_p}). \end{aligned}$$

Since $u_iv_j \notin \sigma_{Z_i}$ or $u_iv_j \notin \sigma_{Z_j}$ (by the definition of intuitionistic k-partitioned fuzzy graph). Therefore

$$\left\langle S_\alpha(\underset{\sim}{IG}), S_\vartheta(\underset{\sim}{IG})\right\rangle > \left\langle S_\alpha(\underset{\sim}{IG}_{k_p}), S_\vartheta(\underset{\sim}{IG}_{k_p})\right\rangle$$
$$S(\underset{\sim}{IG}) > S(\underset{\sim}{IG}_{k_p}).$$

Hence proved. □

Illustration. From figure 1, we have the size of $\underset{\sim}{IG}$ as $(\sum_{u\neq v}\alpha(uv), \sum_{u\neq v}\vartheta(uv)) = \langle 2.9, 6\rangle$. From figure 3, we have size of $\underset{\sim}{IG}_{3_p}$ as

$$\left\langle \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\alpha_{\underset{\sim}{IG}_{3_p}}(u_iv_j), \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\vartheta_{\underset{\sim}{IG}_{3_p}}(u_iv_j)\right\rangle = \langle 2.3, 4.8\rangle.$$

Therefore $S(\underset{\sim}{IG}) > S(\underset{\sim}{IG}_{3_p})$.

Theorem 4.3. *Let $\underset{\sim}{IG}_{k_p}$ be an intuitionistic k-partitioned fuzzy graph. Then*

$$S\left(\underset{\sim}{IG}\right) = S\left(\underset{\sim}{IG}_{k_p}\right) + \left\langle \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j), \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j)\right\rangle.$$

Proof. Let $\underset{\sim}{IG}$ be a intuitionistic fuzzy graph and let $\underset{\sim}{IG}_{k_p}$ be a intuitionistic k-partion fuzzy graph for $k = 2, 3, \ldots$. As we know the size of intuitionistic fuzzy graph $S\left(\underset{\sim}{IG}\right) = \left\langle S_\alpha\left(\underset{\sim}{IG}\right), S_\vartheta\left(\underset{\sim}{IG}\right)\right\rangle$. And we shall define the size of intuitionistic k-partitioned fuzzy graph as

$$S\left(\underset{\sim}{IG}_{k_p}\right) = \left\langle S_\alpha\left(\underset{\sim}{IG}_{k_p}\right), S_\vartheta\left(\underset{\sim}{IG}_{k_p}\right)\right\rangle.$$

If $u_iv_j \in \sigma_{Z_i}$ or $u_iv_j \in \sigma_{Z_j}$, then we have

$$S_\alpha\left(\underset{\sim}{IG}_{k_p}\right) + \sum_{u_i\neq v_j\in\sigma_{Z_i}}\alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j) = \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j) + \sum_{u_i\neq v_j\in\sigma_{Z_i}}\alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j)$$

and

$$S_\vartheta\left(\underset{\sim}{IG}_{k_p}\right) + \sum_{u_i\neq v_j\in\sigma_{Z_i}}\vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j) = \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}}\vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j) + \sum_{u_i\neq v_j\in\sigma_{Z_i}}\vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j),$$

$$S_\alpha(\underset{\sim}{IG}) = \sum_{u\neq v}\alpha(uv) = \sum_{u_i\neq v_j\in\sigma_{Z_i}} \alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j) + \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}} \alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j) = S_\alpha\left(\underset{\sim}{IG}_{k_p}\right),$$

$$S_\vartheta(\underset{\sim}{IG}) = \sum_{u\neq v}\vartheta(uv) = \sum_{u_i\neq v_j\in\sigma_{Z_i}} \vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j) + \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}} \vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j) = S_\vartheta\left(\underset{\sim}{IG}_{k_p}\right).$$

Therefore

$$\left\langle S_\alpha\left(\underset{\sim}{IG}\right), S_\vartheta\left(\underset{\sim}{IG}\right)\right\rangle = \left\langle S_\alpha\left(\underset{\sim}{IG}_{k_p}\right), S_\vartheta\left(\underset{\sim}{IG}_{k_p}\right)\right\rangle + \left\langle \sum_{u_iv_j\in\sigma_{Z_j}} \alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j), \sum_{u_iv_j\in\sigma_{Z_j}} \vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j)\right\rangle$$

$$S(\underset{\sim}{IG}) = S\left(\underset{\sim}{IG}_{k_p}\right) + \left\langle \sum_{u_iv_j\in\sigma_{Z_j}} \alpha_{\underset{\sim}{IG}_{k_p}}(u_iv_j), \sum_{u_iv_j\in\sigma_{Z_j}} \vartheta_{\underset{\sim}{IG}_{k_p}}(u_iv_j)\right\rangle$$

$\square$

Illustration. From figure 1, we have the size of $\underset{\sim}{IG}$ as

$$\left\langle \sum_{u\neq v}\alpha(uv), \sum_{u\neq v}\vartheta(uv)\right\rangle = \langle 2.9, 6\rangle.$$

From figure 3, we have the size of $\underset{\sim}{IG}_{3_p}$ as

$$\left\langle \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}} \alpha_{\underset{\sim}{IG}_{3_p}}(u_iv_j), \sum_{u_i\in\sigma_{Z_i},v_j\in\sigma_{Z_j}} \vartheta_{\underset{\sim}{IG}_{3_p}}(u_iv_j)\right\rangle = \langle 2.3, 4.8\rangle$$

$$\begin{aligned}
\sum_{u_i\neq v_j\in\sigma_{Z_i}} \alpha_{\underset{\sim}{IG}_{3_p}}(u_iv_j) &= 0.6\\
\sum_{u_i\neq v_j\in\sigma_{Z_i}} \vartheta_{\underset{\sim}{IG}_{3_p}}(u_iv_j) &= 1.2\\
S_\alpha\left(\underset{\sim}{IG}_{3_p}\right) &= 2.3 + 0.6\\
S_\vartheta\left(\underset{\sim}{IG}_{3_p}\right) &= 4.8 + 1.2.
\end{aligned}$$

Therefore $S\left(\underset{\sim}{IG}\right) = S\left(\underset{\sim}{IG}_{3_p}\right) + \langle 0.6, 1.2\rangle$.

5. Conclusion

An intuitionistic fuzzy graph is partitioned into intuitionistic partitioned fuzzy graph by partitioning the node set exclusively. The node set of the intuitionistic fuzzy graph is partitioned up to k-subsets, each subset with a set of more or less equal number of nodes. Intuitionistic 2-partitioned fuzzy graph, intuitionistic 3-partitioned fuzzy graph and intuitionistic 4-partitioned fuzzy graph for an intuitionistic fuzzy graph has been worked in this paper as an example; and its properties such as order and size of intuitionistic k-partitioned fuzzy graph has also been verified using theorems and illustrations. Further, the work could be carried out for strict intuitionistic k-partitioned fuzzy graph without limitation in edges among the partitioned subsets.

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