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# APPROXIMATE MULTIPLICATIVE INVERSE QUADRATIC MAPPINGS

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ABSTRACT. The intention of this study is to propose new multiplicative inverse quadratic functional equations and to validate the existence of their various stabiity results pertinent to Ulam stability theory in the setting of non-zero real numbers. An apt illustration is portrayed to prove that the stability result fails for a singular case. The significance of these equations are related with the well-known inverse square law arising in various fields such as universal law of gravity, radiation from a source, electric fields and forces, intensity of sound, intensity of light.

## 1. INTRODUCTION

The recent research on analysis is focussed on the subject of investigating classical stabilities of mathematical equations. The groundwork set to stability of equations is through the following illustrious problem posed in [35] regarding homomorphims in group theory. The problem is : “When an approximate solution and correct solution of an equation are close to each other?" In other words this problem can be considered mathematically as: If $L_1$ is a group and $L_2$ is a metric group equiped with a metric $d(\cdot,\cdot)$. Suppose $\rho > 0$ is a constant. Do there exist a positive constant $\eta$ such that if a homomorphim $s : L_1 \longrightarrow L_2$

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satisfies the inequality $d(s(xy), s(x)s(y)) < \eta$ and an approximate homomorphism $S : L_1 \longrightarrow L_2$ such that $d(s(x), S(x)) < \rho$ for all $x \in L_1$? If this problem has a solution, then the homomorphims is said to be stable in the sense of Ulam. An affirmative and brilliant answer is provided in [18] by considering Cauchy functional equation $s(x + y) = s(x) + s(y)$ instead of homomorphism and normed space and Banach space as domain and co-domain, respectively. The result obtained in [18] is termed as Ulam-Hyers stability or $\rho$-stability and the approach proposed to attain this result is called as direct method. Later, this problem is dealt by many researchers to broaden the results with various upper bounds such as sum of powers of norms $||x||^p + ||y||^p$, product of powers of norms $||x||^p||y||^q$ and a general control function $\phi(x, y)$ instead of the positive constant $\eta$ in [3, 16, 24, 25]. There are plenty of applications of groups and group homorphisms in various disciplines such as coding theory, languages and grammars in computer science, network theory, graph theory, Boolean algebra, etc. Hence the problem of finding the stability of homomorphisms has become a significiant research work in recent days. Further, the stability of numerous functional equations were dealt by many researchers to investigate their stabilities in various normed spaces, one can refer to [1, 2, 5, 8, 10–12, 14, 17, 19, 21–23].

Very recently, this research work is diverted to investigate stability results of different form of functional equations which involves functions in rational form. These type of equations are said to be of rational form of functional equations or multiplicative inverse functional equations. Interesting results connected with rational form of functional equations can be referred in [4, 6, 7, 9, 13, 20, 26, 27]. The applications of multiplicative inverse functional equations have been studied in variety of disciplines such as Electrical Engineering, Physics, Optics, Chemistry, scattering of light energy, decay of radar energy, mechanics, system design, spatial filtering in image enhancement. These interesting results are available in [28–34].

Motivated by the spectacular role of multiplicative inverse functional equations in other domains, we propose the following new multiplicative inverse quadratic functional equations which involves the arguments in rational form:

$$s\left(\frac{pq}{2p+q}\right) + s\left(\frac{pq}{2p-q}\right) = 2s(p) + 8s(q) \tag{1.1}$$

and

$$s\left(\frac{pq}{2p+q}\right)+s\left(\frac{pq}{p+2q}\right)=5s(p)+5s(q)+8\sqrt{s(p)s(q)}, \tag{1.2}$$

where $s : A \longrightarrow \mathbb{R}^{\star}$ is a mapping with $A = \{p, q \in \mathbb{R} : p \neq 0, q \neq 0, 2p + q \neq 0, 2p - q \neq 0, p + 2q \neq 0\}$ for all $p, q \in \mathbb{R}$. We employ the direct method to prove the stabilities of equations (1.1) and (1.2). We also provide an apt example when a singular case arises. One can easily verify that the multiplicative inverse function $s(p) = \frac{1}{p^2}$ is a solution of equations (1.1) and (1.2).

## 2. Approximate solution near to the exact solution of equation (1.1)

In this section, we prove that an approximate solution exists near to the exact solution of equation (1.1). To achieve this, we apply the direct method introduced in [18]. Firstly, we establish the stability of equation (1.1) involving a general control function $\zeta(p, q)$ as an upper bound and then we investigate various other stabilities involving different upper bounds. Let us assume that $A = \{p, q \in \mathbb{R} : p \neq 0, q \neq 0, 2p + q \neq 0, 2p - q \neq 0, p + 2q \neq 0\}$ for all $p, q \in \mathbb{R}$. In order to accomplish the desired result in a simple approach, let us define the difference operator$\Delta_s : A \times A \longrightarrow \mathbb{R}$ as

$$\Delta_s(p,q)=s\left(\frac{pq}{2p+q}\right)+s\left(\frac{pq}{2p-q}\right)-2s(p)-8s(q),$$

for all $p, q \in A$. In the following results, let $s : A \longrightarrow \mathbb{R}$ be a mapping.

**Theorem 2.1.** *Let $\nu = \pm 1$ be fixed. Suppose the mapping $s$ satisfies*

$$|\Delta_s(p,q)| \leq \zeta(p,q), \tag{2.1}$$

*for all $p, q \in A$, where $\zeta : A \times A \longrightarrow [0, \infty)$ is a function with the condition*

$$\sum_{r=0}^{\infty}\frac{1}{9^{\nu r}}\zeta\left(\frac{p}{3^{\nu(r+1)}},\frac{q}{3^{\nu(r+1)}}\right)<+\infty, \tag{2.2}$$

*for all $p, q \in A$. Then a unique multiplicative inverse quadratic mapping $S : A \longrightarrow \mathbb{R}$ exists and satisfies* (1.2) *with the condition*

$$|s(p)-S(p)| \leq \sum_{r=0}^{\infty}\frac{1}{9^{\nu r}}\zeta\left(\frac{p}{3^{\nu(r+1)}},\frac{p}{3^{\nu(r+1)}}\right), \tag{2.3}$$

*for all $p \in A$.*

*Proof.* Let us first prove this theorem when $\nu = 1$. Considering $(p, q)$ as $(p, p)$ in (2.1), we obtain

$$\left|\frac{1}{9^{\nu}} s\left(\frac{p}{3^{\nu}}\right) - s(p)\right| \leq |9|^{\frac{|\nu-1|}{2}} \zeta\left(\frac{p}{3^{\frac{\nu+1}{2}}}, \frac{p}{3^{\frac{\nu+1}{2}}}\right), \tag{2.4}$$

for all $p \in A$. Next, replacing $p$ by $\frac{p}{3^{k\nu}}$ in (2.4) and then multiplying by $\left|\frac{1}{9}\right|^{k\nu}$, we obtain

$$\left|\frac{1}{9^{k\nu}} s\left(\frac{p}{3^{k\nu}}\right) - \frac{1}{9^{(k+1)\nu}} s\left(\frac{p}{3^{(k+1)\nu}}\right)\right| \leq \frac{1}{9^{k\nu}} \zeta\left(\frac{p}{3^{(k+1)\nu}}, \frac{p}{3^{(k+1)\nu}}\right). \tag{2.5}$$

Now, letting $k \to \infty$ in (2.5) and using (2.2), we noticed that $\{\frac{1}{9^{k\nu}} s(\frac{p}{3^{(k+1)\nu}})\}$ is a Cauchy sequence. Since $\mathbb{R}$ is complete, the sequence $\{\frac{1}{9^{k\nu}} s(\frac{p}{3^{(k+1)\nu}})\}$ converges to a function $S : A \longrightarrow \mathbb{R}$ defined by

$$S(p) = \lim_{n\to\infty} \frac{1}{9^{k\nu}} s(3^{(k+1)\nu} p). \tag{2.6}$$

Now, we claim that $S$ satisfies (1.1). Plugging $(p, q)$ into $\left(\frac{p}{3^{k\nu}}, \frac{q}{3^{k\nu}}\right)$ in (2.1) and then in the resultant, dividing by $9^{k\nu}$ on its both sides, we obtain

$$\left|\frac{1}{9^{k\nu}} \Delta_s(p, q)\left(\frac{p}{3^{k\nu}}, \frac{q}{3^{k\nu}}\right)\right| \leq \frac{1}{9^{k\nu}} \zeta\left(\frac{p}{3^{k\nu}}, \frac{q}{3^{k\nu}}\right), \tag{2.7}$$

for all $p, q \in A$ and for all positive integer $k$. Now, using (2.2), (2.6) in (2.7), we find that $S$ satisfies (1.1) for all $p, q \in A$. For each $p \in A$ and each integer $k$, we have

$$\begin{aligned}\left|\frac{1}{9^{k\nu}} s\left(\frac{p}{3^{k\nu}}\right) - s(p)\right| &\leq \sum_{\ell=0}^{k-1} \left|\frac{1}{9^{\ell\nu}} s\left(\frac{p}{3^{(\ell+1)\nu}}\right) - \frac{1}{9^{(\ell-1)\nu}} s\left(\frac{p}{3^{\ell\nu}}\right)\right| \\ &\leq \sum_{\ell=0}^{k-1} \frac{1}{9^{\ell\nu}} \zeta\left(\frac{p}{3^{(\ell+1)\nu}}, \frac{p}{3^{(\ell+1)\nu}}\right).\end{aligned}$$

Applying (2.6) and letting $k \to \infty$, we obtain (2.3). Next is to prove the uniquness of $S$. For this, let us consider $S' : A \longrightarrow \mathbb{R}$ be another approximate multiplicative inverse quadratic mapping satisfying (1.1) and (2.3). Then, we have

$$S'\left(\frac{p}{3^{k\nu}}\right) = 9^{k\nu} S'(p) \qquad \text{and} \qquad S\left(\frac{p}{3^{k\nu}}\right) = 9^{k\nu} S(p),$$

for all $p \in A$. By the application of (2.3), we obtain

$$\begin{aligned}
|S'(p) - S(p)| &= 9^{-k\nu} \left|S'(3^{-k\nu}p) - S(3^{-k\nu}p)\right| \\
&\leq 9^{-k\nu} \left|S'(3^{-k\nu}p) - s(3^{-k\nu}p)\right| + 9^{-k\nu} \left|s(3^{-k\nu}p) - S(3^{-k\nu}pr)\right| \\
&\leq 2\sum_{r=0}^{\infty} \frac{1}{9^{(k+r)\nu}}\zeta\left(\frac{p}{3^{(k+r+1)\nu}}, \frac{p}{3^{(k+r+1)\nu}}\right) \\
&\leq 2\sum_{r=k}^{\infty} \frac{1}{9^{r\nu}}\zeta\left(\frac{p}{3^{(r+1)\nu}}, \frac{p}{3^{(4+1)\nu}}\right), \qquad (2.8)
\end{aligned}$$

for all $p \in A$. It can be noticed that $S$ is unique by allowing $k$ tends to $\infty$ in (2.8). This completes the proof for $\nu = 1$. The proof for the case $\nu = -1$ is similar to the above arguments. □

The following stability results are direct outcomes of the above Theorem 2.1. Hence the proof of the results are dropped in this study. The results are obtained by taking upper bounds as a positive constant $\mu$, sum of powers of moduli $|p|^\alpha + |q|^\alpha$ and product of powers of moduli $|p|^a|q|^b$, respectively, where $\alpha \neq -2$, $a+b \neq -2 p, q \in A$.

The proof of the following corollary is obtained by considering the case $\nu = -1$ in Theorem 2.1 and taking $\zeta(p,q) = \eta$.

**Corollary 2.1.** *Suppose there exists a positive constant $\mu$ (independent of $p$ and $q$) such that the inequality*

$$|\Delta_s(p,q)| \leq \frac{\mu}{2},$$

*holds for all $p, q \in A$. Then a unique multiplicative inverse quadratic mapping $S : A \longrightarrow \mathbb{R}$ satisfying* (1.1) *exists and satisfies*

$$|s(p) - S(p)| \leq \mu,$$

*for all $p \in A$.*

**Corollary 2.2.** *For any fixed $\mu_1 \geq 0$ and $\alpha \neq -2$, if the mapping $s$ satisfies*

$$|\Delta_s(p,q)| \leq \mu_1(|p|^\alpha + |q|^\alpha),$$

*for all $p, q \in A$. Then there exists a unique multiplicative inverse quadratic mapping $S : A \longrightarrow \mathbb{R}$ satisfying* (1.1) *and*

$$|s(p) - S(p)| \leq \frac{2^{\nu+1}\mu_1}{|2^{\nu(\alpha+1)} - 1|} |p|^\alpha,$$

*and for all* $p \in A$.

**Corollary 2.3.** *Suppose the constants* $a, b$ *exist such that* $\alpha = a + b \neq -2$ *and* $\mu_2 \geq 0$ *such that for all* $p, q \in A$,

$$|\Delta_s(p,q)| \leq \mu_2 |p|^a |q|^b .$$

*Then, there exists a unique multiplicative inverse quadratic mapping* $S : A \longrightarrow \mathbb{R}$ *satisfying* (1.1) *and the inequality*

$$|s(p) - S(p)| \leq \frac{2^\nu \mu_2}{|2^{\nu(\alpha+1)} - 1|} |p|^\alpha ,$$

*and for all* $p \in A$.

Using the excellent refutation demonstrated in [15], we prove that the equation (1.1) fails to be stable for a singular case $\alpha = -2$ in Corollary 2.2. The following function is used to disprove the stability result.

Let a function $\mu : A \longrightarrow \mathbb{R}$ be defined as

$$\mu(p) = \begin{cases} \frac{\beta}{p^2}, & \text{for } p \in (1, \infty) \\ \beta, & \text{elsewhere} \end{cases} . \tag{2.9}$$

Let $s : A \longrightarrow \mathbb{R}$ be defined by

$$s(p) = \sum_{k=0}^{\infty} 9^{-k} \mu(3^{-k} p),$$

for all $p \in A$. Then the function $s$ turns into a suitable example to illustrate that (1.1) is unstable for $\alpha = -2$ in Corollary 2.2 in the following theorem.

**Theorem 2.2.** *Assume that the function* $s : A \longrightarrow \mathbb{R}$ *described in* (2.9) *satisfies the inequality*

$$|\Delta_s(p,q)| \leq 54\beta \left(|p|^{-2} + |q|^{-2}\right) , \tag{2.10}$$

*for all* $p, q \in A$. *Then a multiplicative inverse quadratic mapping* $S : A \longrightarrow \mathbb{R}$ *and a constant* $C > 0$ *do not exist such that*

$$|s(p) - S(p)| \leq C |p|^{-2} \tag{2.11}$$

*for all* $p \in A$.

*Proof.* First of all, let us show that the mapping $s$ satisfies (2.10). Applying the definition of $s$, we notice that $|s(p)| = \left|\sum_{k=0}^{\infty} 9^{-k}\mu(3^{-k}p)\right| \leq \sum_{k=0}^{\infty} \frac{\beta}{3^k} = \frac{3\beta}{2}$, which implies that $s$ has an upper bound $\frac{3\beta}{2}$ on $\mathbb{R}$. If $|p|^{-2} + |q|^{-2} \geq 1$, then the expression on the left hand side of (2.10) is less than $54\beta$. On the other hand, assume that $0 < |p|^{-2} + |q|^{-2} < 1$. Hence, there exists a $j \in \mathbb{N}^+$ such that

$$\frac{1}{9^{j+1}} \leq |p|^{-2} + |q|^{-2} < \frac{1}{9^j}. \tag{2.12}$$

As a result, the relation (2.12) produces $9^j\left(|p|^{-2} + |q|^{-2}\right) < 1$, or equivalently; $9^j p^{-2} < 1, 9^j q^{-2} < 1$. So, $\frac{p}{3^j} > 1, \frac{q}{3^j} > 1$. From the last inequalities, we observe that $\frac{p}{3^{j-1}} > 3 > 1, \frac{q}{3^{j-1}} > 3 > 1$ and as a consequence, we have

$$\frac{1}{3^{j-1}}(p) > 1, \frac{1}{3^{j-1}}(q) > 1, \frac{1}{3^{j-1}}\left(\frac{pq}{2p+q}\right) > 1, \frac{1}{3^{j-1}}\left(\frac{pq}{2p-q}\right) > 1.$$

Thus, for every $k = 0, 1, 2, \ldots, j-1$, we acquire

$$\frac{1}{3^k}(p) > 1, \frac{1}{3^k}(q) > 1, \frac{1}{3^k}\left(\frac{pq}{2p+q}\right) > 1, \frac{1}{3^k}\left(\frac{pq}{2p-q}\right) > 1$$

and $\Delta_s(3^{-k}p, 3^{-k}q) = 0$ for $k = 0, 1, 2, \ldots, j-1$. By the application of (2.9) and utilizing the definition of $s$, we obtain

$$\begin{aligned}
|\Delta_s(p,q)| &\leq \sum_{k=j}^{\infty} \frac{\beta}{3^k} + \sum_{k=j}^{\infty} \frac{\beta}{3^k} + 2\sum_{k=j}^{\infty} \frac{\beta}{3^k} + 8\sum_{k=j}^{\infty} \frac{\beta}{3^k} \\
&\leq 12\beta \sum_{k=j}^{\infty} \frac{1}{3^k} \leq \frac{54\beta}{3^{k+1}} \leq 54\beta\left(|p|^{-2} + |q|^{-2}\right),
\end{aligned}$$

for all $p, q \in A$, which indicates that the inequality (2.10) holds. Now, we assert that equation (1.1) is not stable for $\alpha = -2$ in Corollary 2.2. For this, let us consider that a multiplicative inverse quadratic mapping $S : A \longrightarrow \mathbb{R}$ exists and satisfies (2.11). Then, we arrive at

$$|S(p)| \leq (C+1)|p|^{-2}. \tag{2.13}$$

But, still we can choose a positive integer $m$ with $m\beta > C + 1$. If $p \in (1, 3^{m-1})$ then $3^{-k}p \in (1, \infty)$ for all $k = 0, 1, 2, \ldots, m-1$ and thus

$$|S(p)| = \sum_{k=0}^{\infty} \frac{\mu(3^{-k}p)}{9^k} \geq \sum_{k=0}^{m-1} \frac{\frac{9^k\beta}{p^2}}{9^k} = \frac{m\beta}{p^2} > (C+1)p^{-2},$$

which contradicts (2.13). Hence we conclude that the equation (1.2) is unstable for $\alpha = -2$ in Corollary 2.2. $\square$

**Remark 2.1.** *The results concerning the stabilities of equation* $(1.2)$ *are omitted as they can be obtained by similar arguments employed to obtain the stabilities of equation* $(1.1)$.

## 3. Explication of equations (1.1) and (1.2) through inverse square law

In physics many laws observe the inverse square law, one such law is the governing intensity. In particular, an inverse square law states that the intensity is equal to the inverse of the square of the distance from the source. If $X$ is the intensity of sound or light or electric field and $d$ is the distance from the centre, then the inverse square law can be stated as $X \propto \frac{1}{d^2}$. The inverse square law arises in various fields such as universal law of gravity, radiation from a source, electric fields and forces, intensity of sound, intensity of light, etc.

Motivated by the inverse square law in various disciplines, suppose if $p$ is the distance, then the intensity $s(p)$ is a multiplicative inverse quadratic function of $d$. The sum of the intensities with distances $\frac{pq}{2p+q}$ and $\frac{pq}{2p-q}$ is equal to the sum of $2$ times of intensity with distance $p$ and $8$ times of intensity with distance $q$. This can be studied through equation (1.1). Similarly, the sum of the intensities with distances $\frac{pq}{2p+q}$ and $\frac{pq}{p+2q}$ is equal to the sum of $5$ times of intensity with distance $p$, $5$ times of intensity with distance $q$ and $8$ times square root of intensities with distances $p$ and $q$. This phenomena can be dealt through equation (1.2).

## 4. Conclusion

We conclude this investigation with the following remarks. So far there are many research papers published on functional equations with integral expression of arguments. But this paper contains, the stability results concerning functional equations with rational expression of arguments. The intention of considering rational expression of arguments is that many rational functions play significant roles in medicine, economics, biology, physics, chemistry. We conclude that the stability results hold good for the equations dealt in this study in the sense of Ulam stability theory. We have presented an appropriate example to disprove the stability result of (1.1) for a singular case. The interpretations of equations (1.1) and (1.2) are elucidated via inverse square law arising in different fields.

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