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ALGORITHM FOR ENCODING N-ARY BLOCK CODES BY USING THE HYPER FUNCTION

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ABSTRACT. Block codes are one of the indispensable families of error-correcting codes that are in control of encoding data in blocks. The encoding system completed by joining redundancy segments supplies to place errors produced in the communication flows. Multiple endeavors have been fulfilled to combine digital communication technology with logical algebras within algorithms allow converting algebras into block codes to be broadcasted through various communication channels. In this paper, we introduce an algorithm depends on forming a unique-identified hyper KU-algebra (X) of n-ary block code (G), and encoding this code in the form of n-ary block code(G_X) by using hyper KU-function.

1. INTRODUCTION

F.Marty [4] presented hyperstructures theory (multialgebras) in 1934. Abundant applications of hyperstructures theory have been manipulated in various sciences (theoretical and applied). S. M. Mostafa et al. suggested the approach of hyperstructures to KU- algebras and presented a hyper KU-algebra concept as an overview of a KU-algebra. Additionally, they investigated other associated properties [11]. Hashemi et al. investigated the connections between hyper equality algebras and hyper K (BE, MV)-algebras [5].Borzooei et al. applied hyperstructures theory to hoop-algebras and introduced the concept of (quasi)

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hyper hoop-algebra which is considered a generalization of hoop-algebra. They also presented the concept of (weak) filters on hyper hoop-algebras [6]. Xin et al. showed a hyper BL-algebras that is the generalization of BL-algebras. they contributed some properties of hyper BL-algebras and related examples. Besides, they suggested weak deductive systems, weak filters, and hyper BL-algebras [13]. S. M. Mostafa et al. introduced the notions of soft PU-algebras, A - soft new-ideals,(α, A) - soft new-ideals and discuss various operations of these concepts [12].

Coding theory encompasses the discovery of numerous coding schemes to grow errors number that can be corrected within the process of data transmission. The unique part of the processing performed to information to be transferred through a channel is error control coding. Coding theory concerns with investigating error-control codes which allow increasing the trustworthiness of data transmission through noisy communication channels. The process inaugurates with information source and the encoding process transforms the source into a sequence of alphabets denoted by a message [8]. Any structure present in code production during the implementation of these processes will be greatly beneficial. During producing a message by any source of data, it can be illustrated as a stream of symbols d_1d_2,...... from a fixed set G which is signified by the source alphabet [10]. In the following, we high spot on some applications of algebraic structures in coding theory. Cristina Flaut investigated the associations between binary block-codes and Hilbert algebras. Moreover, she suggested extra properties linked to Hilbert algebras [3]. Atamewoue tsafack surdive et al. declared the concept of hyper BCK-function with some properties related and produced binary codes by the hyper BCK- function through an algorithm allows constructing a hyper BCK-algebra from binary block code [2]. Samy m.mostafa et al. presented the concept of KU-function with relevant properties and formed binary code from KU- function [9]. Arsham Borumand Saeid et al. presented an algorithm to generate BCK-algebra from n-ary block code [1]. In this paper, we present an algorithm to create a unique- identified hyperstructure (hyper KU-algebras X) for every n-ary block code G and encode this code in n-ary block code (G_X) by using the hyper KU-function.

2. Preliminaries

In this section, we present some of the earliest results related to hyper KU-algebras and applications of logical algebraic structures in coding theory that will be used in our work.

Definition 2.1. *[11] Let X be a non-empty set supplied with " $\circ$ " as a hyper operation on X such that $\circ : X \times X \to P^*(X)$ and a constant " θ". X is called a hyper KU-algebra, If (X, $\circ$, θ) fulfills the next axioms: $\forall$ x, y, z $\in$ X,*

(HKU1) (y o z) o (x o z)$\ll$ (x o y) ,

(HKU2) x o θ=θ ,

(HKU3) θ o x=x ,

(HKU4) if x $\ll$ y and y $\ll$ x imply x=y.

x $\ll$ y is illustrated with $\theta \in$ y o x , and A$\ll$ B is well-defined with $\forall$ a $\in$ A, $\exists$ b $\in$ B s.t.

a $\ll$ b $\forall$ A, B $\subseteq$ X. The operation $\ll$ is called the hyper order in X . Besides, we are sometimes indicate x $\circ$ y instead x $\circ$ {y}, {x} $\circ$ y, or {x} $\circ$ {y}.

Proposition 2.1. *[11] Let (X, $\circ$, θ) is a hyper KU-algebra, then we have the following axioms: $\forall$ x, y, z $\in$ X, and for any non-empty subsets A,B $\subseteq$ H,*

(1) $A \subseteq B \to A \ll B$,

(2) $\theta \circ \theta = \theta$,

(3) $\theta \ll x$,

(4) $z \ll z$,

(5) $x \circ z \ll z$,

(6) $A \circ \theta = \theta$,

(7) $\theta \circ A = A$,

(8) $(\theta \circ \theta) \circ x = x$,

(9) $x \ll y \Rightarrow y \circ z \ll x \circ z$,

(10) $z \circ (y \circ x) = y \circ (z \circ x)$,

(11) $\theta \ll A \Rightarrow \theta \in A$,

(12) $y \in (c \circ x) \Rightarrow y \ll x$.

Definition 2.2. *[7] A mapping $A^{\sim}$: $A \to X$ is named with a KU-function on A.*

Definition 2.3. *[7] A cut function of $A^{\sim}$, for $q \in X$ is defined as a mapping $A_q^{\sim}$: $A \to \{0,1\}$ such that $\forall x \in A$, $A_q^{\sim}(x) = 1 \Leftrightarrow A^{\sim}(x) * q = 0$.*

Here, we show some results about generating KU-algebra from a given binary block code.

Theorem 2.1. *[7] Suppose a matrix M_v is a lower triangular with $m_{ii} = 1$, $\forall i \in \{1,2,\dots,n\}$, and $1x_{ik\dots in}$; $ik,\dots,in \in \{ 0,1 \}$ is in V. Then, Through the following notations*

$$\begin{cases} (1)\ \theta * x = x,\ x * x = \theta, & \forall x \in X, \\ (2)\ x * y = \theta \text{ if } y \leq x, & x, y \in X, \\ (3)\ x * y = y, & otherwise, \end{cases}$$

we have a set A with n elements, KU-algebra X, and KU-function $f : A \to X$ such that f fixes V.

Theorem 2.2. *[7] Suppose V is a binary block code with number of codewords n of length m, $n \neq m$, or a block-code with n codewords of length n such that the codeword $1x_{ik},\dots,x_{in}$; $x_{ik},\dots,x_{in} \in \{0,1\}$ is not in V, or a block-code with n codewords of length n such that the matrix M_v is not lower triangular. Through the previous notations, we have a number $q \geq \max\{m,n\}$, a set A with m elements and a KU-function f: $A \to C_q$ such that the acquired block code V_{Cn} holds the block code V with 1s as a first digit in its codewords.*

3. Results

This section introduces the algorithm of encoding n-ary block codes by using hyper KU-function.

3.1. First Stage: Generating a hyper KU-algebra of the n-ary block code. .

We generate a matrix $Q \in q_t(j_n)$ as hyper KU-algebra from a given n-ary block code.

(1) Consider the finite set $J'_n = \{1,2,\dots,n\text{-}1\}$.By descending ordered after lexicographic order the n-ary codewords $G = \{d_1,d_2,\dots,d_s\}$ of length p, $p \geq n\text{-}2$, Let $d_i = d_{i1},d_{i2},\dots,d_{ip}$, $d_{ij} \in J'_n$, $j \in \{1,2,\dots,p\}$, with d_{ij} ascending ordered such that $d_{idik} \leq k$, $i \in \{1,2,\dots,s\}$,$k \in \{1,2,\dots,\min\{n\text{-}1,p\}$, , $d_{ij} = 1$ in the rest.

(2) we build a matrix Q = $(\gamma_{uv})_{u,v \in \{0,1,2,\ldots,t-1\}}$, Q $\in q_t(J_n)$; t = s + p + 1, Q is the associated matrix with the n-ary code G = { $d_1, d_2, \ldots, d_s$}. Therefore, we suppose that t = s + p + 1 and we define the following:

$$(3.1)\qquad \begin{cases} (1)\gamma_{u0} = 0, \gamma_{0v} = v; u, v \in \{0,1,2,\ldots,t-1\}, \\ (2)\gamma_{uv} = 0 if u \geq v, \\ (3) for s \geq u \geq 1, s \geq v \geq 1, u < v, \text{ we consider } \gamma_{uv} = 1, \\ (4) for s \geq u \geq 1, t-1 \geq v > s, \text{ we consider} \\ \qquad \gamma_{uv} = d_{i\,(s+j)}\, ; s + j < i, \\ (5) for s < u, \text{ we consider } \gamma_{uv} = 1 if u < v. \end{cases}$$

(3) If we suppose Q $\in q_t(J_n)$ is the associated matrix to the n-ary block code G={$d_1, d_2, \ldots, d_s$} defined on J'_n and J_t = {0,1,2,. . . ,t-1} is a non-empty set, by using the previous notations (3.1), we defined on J_t the multiplication operation i $\circ$ j = γ_{ij}.

Theorem 3.1. *(J_t, $\circ$, 0) is a hyper KU-algebra with J_t = {0,1,2,. . . ,t-1} by using the previous notations (3.1).*

Proof. Conditions (HKU2), (HKU3) and (HKU4) from the definition of hyper KU-algebra above are satisfied. Next, we prove the first condition (HKU1):

$$(\mathbf{j} \circ \mathbf{k}) \circ (\mathbf{i} \circ \mathbf{k}) \ll (\mathbf{i} \circ \mathbf{j}), \forall\, i, j, k \in \{0,1,2,\ldots,t\text{-}1\}.$$

1st case: **j = 0, k $\neq$ 0.** (0 $\circ$ k) $\circ$ (i $\circ$ k) $\ll$ (i $\circ$ 0), since 0 $\circ$ k = {0}, i $\circ$ 0 = {i}, then we have the following condition {0} $\circ$ (i $\circ$ k) $\ll$ {i} that is required to be proved.

For i = 0, it is clear.

For k = 0, we get the following {0} $\circ$ (i $\circ$ 0) $\ll$ i $\Rightarrow$ {i} $\circ$ {0} $\ll$ {i}, then we get {0} $\ll$ {i}.

For k $\neq$ 0, i $\geq$ t-s, k $\ll$ {1,2,. . . ,p}, we have $w_{i \circ k} \ll$ {i}, so that {0} $\circ$ (i $\circ$ k) $\ll$ {i}.

For k $\neq$ 0, i $\geq$ t-s, k $\geq$ p+1, we have {0} $\circ$ (i $\circ$ k) $\ll$ {i}, if i $\circ$ k = 1, since {0} $\circ$ 1 $\ll$ p+1 $\ll$ n - 1 $\ll$ {i}, then we have {0} $\circ$ (i $\circ$ k) = {0} $\circ$ 1 $\ll$ {i} $\Rightarrow$ {1} $\ll$ {i}, if i $\circ$ k = 0, we have {0} $\circ$ 0 = {0} $\ll$ {i}.

For i < t - s, k $\leq$ p+1, since we have i $\circ$ k = 1, then {0} $\circ$ (i $\circ$ k) = {0} $\circ$ 1 = {1} $\ll$ {i}.

For $i < t - s, k > p + 1$, since we have $i \circ k = 0$, then $\{0\} \circ (i \circ k) = \{0\} \circ 0 = \{0\} \ll \{i\}$.

2nd case: $k = 0, j \neq 0$. The following condition $\{0\} \circ (i \circ k) \ll \{i\}$ is satisfied since we have $\{i \circ k\} \ll \{i\}$.

3rd case: $k \neq 0, j \neq 0$. Now, we prove the following (HK1) $(j \circ k) \circ (i \circ k) \ll (i \circ j)$,

For $i = 0$, it is clear.

Suppose $i \neq 0$, for $i \geq t - s, j, k < t - s, j < k$, we have $n - 1 \geq i \circ k \geq j \circ k$. so, we have $(j \circ k) \circ (i \circ k) = \{1\}$, and $i \circ j = 1$, it implies that$(j \circ k) \circ (i \circ k) = 1 \ll (i \circ j) = 1$. So, the relation is proved.

For $i \geq t - s, j, k < t - s, k < j$. we have $(j \circ k) \circ (i \circ k) = 0$. it implies that $(j \circ k) \circ (i \circ k) = \{0\} \ll (i \circ j) = 1$. So, the relation is proved.

For $i \geq t - s, j, k \geq t - s, j < k$. we have $j \circ k = 1$ & $i \circ k = 1$. it implies that $(j \circ k) \circ (i \circ k) = 1 \circ 1 = \{0\} \ll (i \circ j)$. Also, We have, $j \circ k = 0, i \circ k = 1$ and $i \circ j = 1$. It implies that$(j \circ k) \circ (i \circ k) = 0 \circ 1 = \{1\} \ll i \circ j = 1$. or, you have $j \circ k = 0$ & $\circ k = 0$. It implies that $(j \circ k) \circ (i \circ k) = 0 \circ 0 = \{0\} \ll i \circ j$. the relation is proved.

For $i \geq t - s, j, k \geq t - s, k < j$, we have $j \circ k = 1$ & $i \circ k = 1$. it implies that $(j \circ k) \circ (i \circ k) = 1 \circ 1 = \{0\} \ll (i \circ j)$. Also, We have $j \circ k = 0$ & $i \circ k = 1$ and $i \circ j = 0$. It implies that$(j \circ k) \circ (i \circ k) = 0 \circ 1 = \{1\} \ll i \circ j = 0$. or, you have $j \circ k = 0$ & $i \circ k = 0$. It implies that$(j \circ k) \circ (i \circ k) = 0 \circ 0 = \{0\} \ll i \circ j$. so,the relation is proved.

For $i \geq t - s, k < t - s < j$, if $i \circ k = 0$, then the relation is proved. Also, if $i \circ k = 1$,then $(j \circ k) \circ (i \circ k) = (j \circ k) \circ 1 = \beta \circ 1 \ll (i \circ j)$.it implies $\beta \circ 1=0$ for $\beta \geq 1$ since $k < j$, then $(j \circ k) \circ (i \circ k)=0 \ll (i \circ j)$. for $i \geq t\text{-}s, j < t - s < k$. we have $i \circ k = 1$.if $j \circ k = 1$, then the relation is proved, and If $j \circ k = 0$,we have $(j \circ k) \circ (i \circ k) = 0 \circ 1 = \{1\} \ll i \circ j$, since $i \circ j \geq 1$.

For $i < t - s, j, k < t - s, j < k$, we have $j \circ k = 1$ & $i \circ k = 1$. it implies that $(j \circ k) \circ (i \circ k) = 1 \circ 1 = \{0\} \ll i \circ j$.

For $i < t - s, j, k < t - s, k \leq j$, we have the following:

$$(j \circ k) \circ (i \circ k) = 1 \circ 1 = \{0\} \ll i \circ j=0.$$

Or, $i \circ k = 0$. each of them implies that the relation is proved.

For $i < t\text{-}s, j,k < t\text{-}s, j < k$, we have $j \circ k = 0$ since $j < k$ and $i \circ j = 1, i \circ k = 0$. By substituting in the relation $(j \circ k) \circ (i \circ k) = 0 \circ 0 = \{0\} \ll (i \circ j) = 1$ implies $\{0\} \ll 1$.

For $i < t\text{-}s$, $k < t\text{-}s \leq j$, We have $(j \circ k) \circ (i \circ k) = 1 \circ 1 = \{0\} \ll (i \circ j) = 1$. if $i \circ j = 0$, then the relation is proved.

For $i < t\text{-}s$, $j, k \geq t\text{-}s$, $j < k$,we have $i \circ k = 0$, so that the relation is proved.

For $i < t\text{-}s$,$j, k \geq t\text{-}s$, $j > k$, we have $j \circ k = 1$, $i \circ k = 0$ and $i \circ j = 0$, then we have $(j \circ k) \circ (i \circ k) = 1 \circ 0 = \{0\} \ll (i \circ j) = 0$. The relation is proved. □

3.2. **Second Stage: Encoded n-ary block codes by using hyper KU-function.**

Now, we illustrate constructing n-ary block code with codewords of length p for every hyper KU-function.

Suppose that we have the following: Finite hyper KU-algebra $(X,\circ,\theta)$ with number of elements (n), finite non-empty set (J) and $J_n=\{0,1,2,\ldots,n\text{-}1\}$ as a finite set, where $X=\{t_0,t_1,\ldots,t_{n-1}\}$, $J=\{a_0,a_1,\ldots,a_{s-1}\}$, $s \leq n$. The map $g: J \to X$ is a hyper KU-function and the generalized cut function of g is $g_{t_j} : J \to J_n$, $t_j \in X$; $g_{t_j}(a_i) = k \Leftrightarrow t_j \circ g(a_i) = t_k$, $\forall\, t_j, t_k \in X$, $a_i \in J$, $i, j, k \in \{0,1,2,\ldots,n\text{-}1\}$. Since the objective is generating n-ary block code with codewords of length p for every hyper KU-function $g : J \to X$. Therefore, we suppose $\forall\, t \in X$, the generalized cut function g_t: $J \to J_n$. For every generalized cut function, we construct the following codeword d_t, with digits belong to the set J_n as the following : $d_t = d_0\, d_1 \ldots, d_{s-1}$, $d_i = j$, $j \in J_n \Leftrightarrow g_t(a_i) = j$; $t \circ g(a_i) = t_k$.

3.3. **Elucidations.**

(1) $(X, \circ, 0)$ is a hyper KU-algebra since we acquired a hyper KU-algebra (J_t , $\circ$, 0) with $J_t = \{0,1,2,\ldots,t\text{-}1\}$ over Theorem 3.1, and we consider $X = \{a_0 = 0, a_1 = 1, \ldots, a_{t-1}\}$, with the relation $a_i \circ a_j = a_k \Leftrightarrow i \circ j = k$, for $i, j, k \in \{0,1,2,\ldots,t\text{-}1\}$.

(2) $(J_t, \circ, 0)$ is a unique identified hyper KU-algebras since it obtained over Theorem 3.1, by using Q, that was a unique identified by $G=\{d_1,d_2,\ldots,d_s\}$.

(3) Let $g : J_q \to X$, $g(i) = a_i$, $J_q = \{0,1,2,\ldots,q\text{-}1\}$, $i \in J_q$. we grant n-ary block code (G_X) that generated by hyper KU-function and encodes n-ary block code $G = \{d_1,d_2,\ldots,d_s\}$ inside.

(4) Consider G is n-ary block code. From Theorem 3.1 and elucidations (1),(2) and (3), we can get a hyper KU-algebras X, such that n-ary block code that generated by hyper KU-function(G_X) encodes G inside.

(5) Consider G is a block code with s codewords of length p. By using the preceding elucidations, consider X is the associated hyper KU-algebra

and $G_X = \{d_0, d_1, \ldots, d_t\}$ is the associated n-ary block code generated by hyper KU-function that encodes the code $G = \{d_1, d_2, \ldots, d_s\}$ inside.

(6) Let $d_x = \{x_1, x_2, \ldots, x_q\}$, $d_y = \{y_1, y_2, .., y_q\} \in G_X$. We define an order relation $\leq_c$ on G_X as follow $d_x \leq_c d_y \Leftrightarrow y_i \leq x_i, \forall\, i \in \{1, 2, \ldots, q\}$.

(7) On X we define the following:

$$x \circ y = \begin{cases} (1)\{y\} & if x = 0, \\ (2) Z & otherwise. \end{cases}$$

where it gets a hyper KU-algebra structure.

3.4. Pseudo - Code algorithm for generating associated matrix Q (hyper KU-algebra) of n-ary code G and encoded matrix G_x of G by using hyper KU-function.

```
Input: array G = {d_1,........d_s}
Initialization:
Set s ← len(G) , p← len(d), t(←) s + p + 1, Q← 2D matrix of size (t × t),
  range ← J[x,y], G_x ← (t × range)
Q[t][t] = 0
% the creation of associated matrix Q
  for i  → t do
     Q[0][i] =i
     Q[i][0] =0
  for i = 1 → p + 1 do
     for j = i + 1 → p + 1 do
          Q[i][j] =1
  for i = p + 1 → t do
     for j = i + 1 → t do
          Q[i][j] =1
convert input array of codes to c← 2D matrix of size (s × p)
     for i = 1 → p + 1do
         for j = s + 1 → t do
               Q [i] [j] =c[i-1] [(i+3)-p]
% apply Hyper KU-function
  for i = range[x] → range[y] do
       for j = 1 → t do
             Q[i][j] =G_x[i][j]
Output: Encoded Matrix G_x
```

4. EXAMPLES

Example 1. *Generate a hyper KU-algebra from the following n-ary block code and encode this code by using the hyper KU-function: G={ 4555, 3345, 2334, 2223}, $n = 6$, $p = 4$, $s = 4$, $t = 9$; $t = s + p + 1$, $J_6 = \{0, 1, 2, 3, 4, 5\}$.*

First, we form the matrix Q of the n-ary block code G as follow:

$$Q = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 0 & 1 & 1 & 1 & 4 & 5 & 5 & 5 \\ 0 & 0 & 0 & 1 & 1 & 3 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Second, If $X = \{a_0 = \theta, a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 4, a_5 = 5, a_6 = 6, a_7 = 7, a_8 = 8\}$, we have the corresponding hyper KU-algebra $(X, \circ, \theta)$ as follows, X equals to

$\circ$	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
θ	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1	θ	θ	a_1	a_1	a_1	a_4	a_5	a_5	a_5
a_2	θ	θ	θ	a_1	a_1	a_3	a_3	a_4	a_5
a_3	θ	θ	θ	θ	a_1	a_2	a_3	a_3	a_4
a_4	θ	θ	θ	θ	θ	a_2	a_2	a_2	a_3
a_5	θ	θ	θ	θ	θ	θ	a_1	a_1	a_1
a_6	θ	θ	θ	θ	θ	θ	θ	a_1	a_1
a_7	θ	θ	θ	θ	θ	θ	θ	θ	a_1
a_8	θ	θ	θ	θ	θ	θ	θ	θ	θ

Because of $X = \{ a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \}$, let $J = \{a_5, a_6, a_7, a_8\}$, the function $g{:}J \to X$ given by

$$\begin{bmatrix} a_5 & a_6 & a_7 & a_8 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

We have the following codewords belong to the block code: In case of $t = 0$, we have

$d_0 = 5678$, $t = a_1$, we have $d_{a1} = 4555$, $t = a_2$, we have $d_{a2} = 3345$, $t = a_3$, we have $d_{a3} = 2334$, $t = a_4$, we have $d_{a4} = 2223$, $t = a_5$, we have $d_{a5} = 0111$, $t = a_6$, we have $d_{a6} = 0011$, $t = a_7$, we have $d_{a7} = 0001$, and $t = a_8$, we have $d_{a8} = 0000$. then, we have the following n-ary block code $G_X = \{5678,4555,3345,2334, 2223, 0111,0011,0001,0000\}$, as shown in Figure 1, such that n-ary block code G_X encodes the given n-ary block code G.

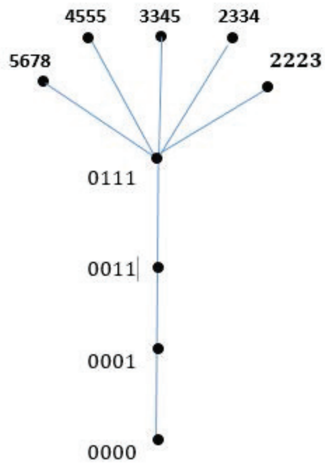

FIGURE 1. Graph of G_x

Example 2. *Generate a hyper KU-algebra from the following n-ary block code and encode this code by using the hyper KU-function: G=$\{2445, 2344, 1233, 1122\}$, $n = 6$, $p = 4$, $s = 4$, $t = 9$; $t = s + p + 1$, $J_6 = \{0, 1, 2, 3, 4, 5\}$.*

First, we form the matrix Q of the n-ary block code G as follow:

$$Q = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 0 & 1 & 1 & 1 & 2 & 4 & 4 & 5 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 & 4 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Second, If $X = \{a_0 = \theta, a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 4, a_5 = 5, a_6 = 6, a_7 = 7, a_8 = 8\}$, we have the corresponding hyper KU-algebra $(X, \circ, \theta)$ as follows in the next table X.

$\circ$	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
θ	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1	θ	θ	a_1	a_1	a_1	a_2	a_4	a_4	a_5
a_2	θ	θ	θ	a_1	a_1	a_2	a_3	a_4	a_4
a_3	θ	θ	θ	θ	a_1	a_1	a_2	a_3	a_3
a_4	θ	θ	θ	θ	θ	a_1	a_1	a_2	a_2
a_5	θ	θ	θ	θ	θ	θ	a_1	a_1	a_1
a_6	θ	θ	θ	θ	θ	θ	θ	a_1	a_1
a_7	θ	θ	θ	θ	θ	θ	θ	θ	a_1
a_8	θ	θ	θ	θ	θ	θ	θ	θ	θ

Because of $X = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$, let $J = \{a_5, a_6, a_7, a_8\}$, the function $g:J \to X$ given by

$$\begin{bmatrix} a_5 & a_6 & a_7 & a_8 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

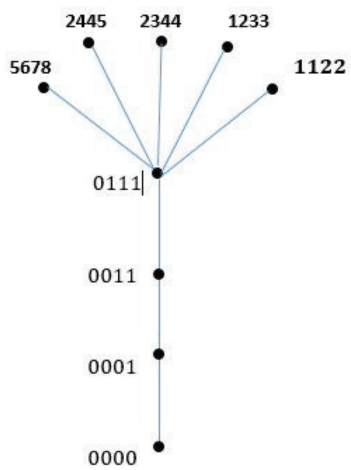

FIGURE 2. Graph of G_x

We have the following codewords belong to the block code : In case of $t = 0$, *we have* $d_0 = 5678$, $t = a_1$, *we have* $d_{a1} = 2445$, $t = a_2$, *we have* $d_{a2} = 2344$, $t = a_3$, *we have* $d_{a3} = 1233$, $t = a_4$, *we have* $d_{a4} = 1122$, $t{=}a_5$, *we have* $d_{a5} = 0111$, $t = a_6$, *we have* $d_{a6} = 0011$, $t = a_7$, *we have* $d_{a7} = 0001$, *and* $t = a_8$, *we have* $d_{a8} = 0000$. *then, we have the following n-ary block code* $G_X = \{5678, 2445, 2344, 1233, 1122, 0111, 0011, 0001, 0000\}$, *as shown in Figure 2, such that n-ary block code* G_X*encodes the given n-ary block code G.*

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