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EFFECT OF AN APPLIED MAGNETIC FIELD AND GRAVITY MODULATION ON THE TIME DEPENDENT HYDRO-MAGNETIC INSTABILITY

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ABSTRACT. In this present study a linear hydro-magnetic instability of time-dependent convection is designed and analyzed by using extended Stuart-Davis technique. The time variations are applied by fluctuating the fluid layer in the direction perpendicular to the flow and also the gravity modulation is introduced as sine and exponential function of time is considered to be one of the important effect. The extended Stuart-Davis technique is applied in tackling the time-dependency. To understand the effect of applied magnetic field and gravity modulation on the convection is analyzed with respect to different values of Chandrasekhar's number. The results shows that the magnetic field is having stabilizing impact in case of sinusoidal variation gravity field on the contrast it as destabilizing impact in case of exponential variation of gravity for short time but in long run it is having stabilizing effect.

1. INTRODUCTION:

The impact of applied magnetic field on the fluid flow through porous media has captured lot of attention within the researchers in the recent days because of its applications in science and engineering. There are adequate works published

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during past few decades with respect to the hydrodynamic and hydro-magnetic convection and heat transfer through porous media in the rectangular geometries. The instability problem in case of time-dependent basic state is quite complicated and has given less consideration. Gravity modulation arises when a fluid or a porous layer is subjected to perpendicular oscillations. The applied magnetic field interaction with convection is considered to be one of the most important problems of hydro-magnetic stability.

There are many researchers in the past few decades who have worked on the convection in porous medium(see Stern (1975) and Morton (1957)). Terrones and Chen (1993) worked on Gravity modulation and studied a theoretical investigation of the effects of gravity as function of time on the convective instability of a horizontally unbounded double-diffusive fluid layers with cross diffusion. Riahi (1995) had investigated finite amplitude thermal convection with spatially moderated boundary temperatures. Srimani and Vishalakshi (1999) have studied linear and non-linear porous convection under gravity modulation by applying Stuart-Davis (1970) technique. Dulal Pal and Shivakumara (2006) investigated the convective instability of fluid through porous medium. Gayathri at. al.(2015) analyzed the onset of electro-thermo-convection of dielectric fluid saturated in porous medium in presence modulated electric field. Recently, the through flow and gravity modulation effects on heat transfer was studied by Kiran(2016).

All the works cited above are deliberated either study of individual effect of the magnetic field or gravity variation and most of these studies are on linear convection but the non-linear convection is not given much of attention and also attempted to study effect of an applied magnetic field and gravity modulations on the convective instability of fluid flow through porous layer. Analytical solutions are obtained by using modified Stuart-Davis technique and the results of linear and non-linear theories are presented.

2. Mathematical Formulation

The physical configuration consists of an electrically conducting fluid layer confined between two horizontal plates of infinite length and kept at constant dissimilar temperatures. The layer is subject to a applied uniform vertical magnetic field $\vec{H} = (0, 0, H)$ with plates are at distance d apart. Further, the lower plate and upper plates are at temperature T_h and T_c respectively with $T_h > T_c$. The gravity modulation is introduced by oscillating the system vertically, so that $\vec{g} = g_0 (1 - g(t)) \hat{k}$. The governing equations of the flow in dimensionless form are given by:

$$-\left(\frac{\partial}{\partial t} + \nabla^2\right) \vec{q} + (1 - g(t)) R\theta\hat{k} - \nabla p = (\vec{q} \cdot \nabla) \vec{q} - Q\frac{\partial \vec{H}}{\partial z}, \tag{2.1}$$

$$-\frac{\partial \vec{H}}{\partial t} = \frac{\partial w}{\partial z} + \nabla^2 \vec{H}, \tag{2.2}$$

$$\left(-\frac{\partial}{\partial t} + \nabla^2\right) \theta - w\overline{T}_z = (\vec{q} \cdot \nabla) \theta - \left(\overline{w\theta}\right)_z, \tag{2.3}$$

$$\nabla \cdot \vec{q} = 0, \tag{2.4}$$

$$\rho = (1 - \alpha(T - T_0)). \tag{2.5}$$

The dimensionless parameters associated with the governing equations (2.1) to (2.5) are; Modified Rayleigh number: $R = \frac{\alpha\beta g_0 d^4}{\pi^4 \kappa\nu}\frac{P_m}{P}$, Prandtl number: $P = \frac{\nu}{\kappa}$, Viscosity parameter: $P_m = \frac{\nu}{\nu_m}$, Magnetic Prandtl number: $P_* = \frac{P}{P_m}$, modified Chandrasekhar number: $Q = \frac{\mu^* H_0^2 d^2}{\pi^2 \rho_0 \nu \nu_m}$.

In the present study we are restricted to $x - z$ plane under the limiting case where the Prandtl number is infinite and we get $\phi_*^{-1} = \frac{P_m}{P} \to 0$ and Eliminating pressure from equation(2.1), the non-dimensional form of above equations becomes

$$\nabla^4 w + R (1 - g(t)) \theta_{xx} = -Q\frac{\partial}{\partial z}\left(\nabla^2 H\right),$$

$$\left(-\frac{\partial}{\partial t} + \nabla^2\right) \theta - w\overline{T}_z = u\theta_x + w\theta_z - \left(\overline{w\theta}\right)_z.$$

From equation (2.3), by considering the horizontal average of the temperature T with $\theta = T - T_0$, we can write

$$\left(-\frac{\partial}{\partial t} + \frac{\partial^2}{\partial z^2}\right) \overline{T} = \left(\overline{w\theta}\right)_z, \tag{2.6}$$

$$u_x + w_z = 0, \tag{2.7}$$

(2.8) $$\nabla^2 H = -\frac{\partial w}{\partial z}.$$

Equations (2.6) to (2.8) are solved subject to the boundary conditions:

$$w = u_z = \theta = H = w_{zz} = 0 \quad \text{at} \quad z = 0, \pi;$$

$$\overline{T}(0,t) = \pi, \quad \overline{T}(\pi,t) = 0.$$

The corresponding initial conditions are given by:

$$(w,\theta) = (W_0, \Theta_0)\cos\alpha x \sin z, \;\; u = U_0 \sin\alpha z \cos z, \;\; H = H_0 \cos\alpha x \cos z,$$

with $W_0 = \dfrac{\alpha^2 R(1-g(0))}{(\alpha^2+1)^2+Q}\Theta_0$, $U_0 = -\dfrac{W_0}{\alpha}$ and $H_0 = \dfrac{W_0}{1+\alpha^2}$.

3. Solution Procedure

To obtain the desired results, we consider the linearized problem with the basic state as $\vec{q} = (0,0)$, $\overline{T} = \overline{T}_0(z)$, $\theta = 0$ and $H = H_0$. Using these in governing equations and solving for the basic state temperature, we get

(3.1) $$\overline{T}_0(z) = \pi - z.$$

The linearized form of the governing equations is given by

(3.2) $$\left[\nabla^4 - Q\frac{\partial^2}{\partial z^2}\right] W - \alpha^2 R\,(1-g(t))\,\Theta = 0,$$

(3.3) $$\left[-\frac{\partial}{\partial t} + \frac{\partial^2}{\partial z^2} - \alpha^2\right]\Theta + W = 0,$$

(3.4) $$\nabla^2 H + \frac{\partial W}{\partial z} = 0.$$

The above set of equations (3.1) to (3.4) have α-dependence and hence field variable are expressed in the form $e^{i\alpha x}$ and $\overline{T} = -1$. The solution of the above system of equations (3.2) to (3.4) is in the form

(3.5) $$(W,\Theta) = (F(t), G(t))\, e^{a_0 t}\sin z,$$

(3.6) $$H = L(t) e^{a_0 t}\cos z.$$

Using this in equations (3.2) to (3.4) along with the condition mentioned above, we get

(3.7) $$\left[\left(\alpha^2+1\right)^2 + Q\right] F - \alpha^2 R\,(1-g(t))\, G = 0,$$

$$F - \left[-\frac{d}{dt} + a_0 + \alpha^2 + 1 \right] G = 0, \tag{3.8}$$

$$F - \left(\alpha^2 + 1 \right) L = 0. \tag{3.9}$$

The solutions of the equations (3.7) to (3.9) are:

$$G(t) = G(0) e^{-\left[\frac{\alpha^2 R}{\left(\alpha^2+1\right)^2 + Q} \right] \int_0^t g(s) ds},$$

$$F(t) = \left[\frac{\alpha^2 R \left(1 - g(t)\right)}{\left(\alpha^2+1\right)^2 + Q} \right] G(t),$$

$$L(t) = \frac{F(t)}{\alpha^2 + 1}.$$

Here $a_0 = \dfrac{\alpha^2 \left(R - R_L\right)}{\left(\alpha^2+1\right)^2 + Q}$ and $R_L = \dfrac{\left(\left(\alpha^2+1\right)^2 + Q \right) \left(\alpha^2 + 1\right)}{\alpha^2}$.

The investigation of the nonlinear theory with time-dependent basic state is quite complicated and hence to avoid the difficulties associated with the problem, the infinite Prandtl number is considered and hence $P_*^{-1} \to 0$. Since, an exponentially growing disturbance quickly invalidates linearization, an unknown function $A(t)$, which is exponential in nature. This enables us to give the formal expansions of the variables in terms of $A(t)$ as

$$\frac{dA}{dt} = a_0 A - a_2 A^3 + \cdots . \tag{3.10}$$

According to equation (3.10), the coefficients a_i, are so chosen such that the expansions non-uniformly valid in time, are suppressed. Expanding of all physical variable $\Gamma = (\Theta, W, H, \alpha U)$ in the form given below

$$\begin{aligned} \Gamma\left(x, z, t\right) = \quad & \left[A^2(t)\Gamma_0(z,t) + ...\right] + \left[A(t)\Gamma_1(z,t) + A^3(t)\Gamma_3(z,t)\right] \cos \alpha x \\ & + \left[A^2(t)\Gamma_2(z,t) + \cdots\right] \cos 2\alpha x + \left[A^3(t)\Gamma_4(z,t) + \cdots\right] \cos 3\alpha x \cdots . \end{aligned}$$

The initial conditions responsible for most unstable mode are given by

$$\begin{aligned} \Theta_1\left(z, 0\right) &= \Theta_0 \sin z, \quad W_1\left(z,0\right) = W_0 \sin z, \\ H_1(z,0) &= L_0 \cos z, \quad U_1(z,0) = U_0 \cos z, \end{aligned}$$

and for $i > 0$,

$$\Theta_i\left(z,0\right) = 0, \;\; W_i\left(z,0\right) = 0, \;\; H_i(z,0) = 0, \;\; U_i(z,0) = 0.$$

Further, the above expressions3.10) to (2.6) are substituted into (3.2) to (3.4). The solutions corresponding to different order are computed. The order zeroth order equations are:

$$\left(-\frac{\partial}{\partial t}+D^2\right)T_0=0.$$

Solution of zeroth order equation is given by

$$T_0=-\left(z-\pi\right).$$

The first-order set of equations of correspond to the linear theory and the equations are:

$$\left[\left(D^2-\alpha^2\right)^2-QD^2\right]W_1-\alpha^2R\left(1-g(t)\right)\Theta_1=0,$$

$$\left(-\frac{\partial}{\partial t}+D^2-\alpha^2\right)\Theta_1+W_1=0,$$

$$\left(D^2-\alpha^2\right)H_1+DW_1=0.$$

The solution satisfying the boundary conditions (3.4) to (3.5), the initial conditions (3.6) and (3.2) to (3.4) are:

$$W_1=W_0\frac{\alpha^2R\left(1-g(t)\right)\sin z}{\alpha^2+1)^2+Q}e^{-\frac{\alpha^2R}{\alpha^2+1)^2+Q}\int_0^t g(s)ds},$$

$$\Theta_1=\Theta_0e^{-\frac{\alpha^2R}{(\alpha^2+1)^2+Q}\int_0^t g(s)ds},$$

$$H_1=\frac{L_0W_1}{\alpha^2+1}.$$

Next, we compute the first order solution of the linear theory and involve terms of $\mathbf{O}\left(\mathbf{A^2}\cos\mathbf{2}\alpha\mathbf{x}\right)$. Further, from equation (3.2), the retaining second-order terms which correct the average temperature are of $\mathbf{O}(\mathbf{A^2})$:

$$\left(-\frac{\partial}{\partial t}+D^2-2a_0\right)\Theta_2=0.5F_1G_1\sin 2z, \tag{3.11}$$

$$F_1=\frac{\alpha^2R\left(1-g(t)\right)}{\alpha^2+1)^2+Q},$$

and

$$G_1=\Theta_0e^{-\frac{\alpha^2R}{(\alpha^2+1)^2+Q}\int_0^t g(s)ds}.$$

The solution of (3.11) subject to the boundary and initial conditions mentioned in above section is given by:

$$\Theta_2=H_2(t)e^{-\frac{2\alpha^2R}{(\alpha^2+1)^2+Q}\int_0^t g(s)ds}\sin 2z.$$

The corresponding magnetic field effects upto second order than satisfies the following differential equation,

$$\frac{dH_2}{dt} + \left(4 + 2a_0 - \frac{2\alpha^2 R}{(\alpha^2+1)^2 + Q} g(t)\right) H_2 = -\frac{\alpha^2 R \Theta_0^2 (1 - g(t)}{2((\alpha^2+1)^2 + Q)}. \tag{3.12}$$

The solution of above differential equation(3.12) is given by

$$H_2 = -\frac{\alpha^2 R \Theta_0^2 J^{-1}(t)}{2((\alpha^2+1)^2 + Q)} \int_0^t (1 - g(s))J(s)ds. \tag{3.13}$$

Here J is an integrating factor of equation (3.13) and it is given by

$$J = e^{\left[(4+2a_0)t - \frac{2\alpha^2 R}{(\alpha^2+1)^2+Q} \int_0^t g(s)ds\right]}.$$

4. Results and discussions

The impact of magnetic field along with gravity as function function of time on the convection is studied using modified Stuart-Davis method and the results are presented. In figures 1, 2 and 3 the effect of Chandrasekhar's number on amplitude functions $F(t)$, $G(t)$ and $H(t)$ versus t for fixed values of R is analyzed. In these figures the gravity modulation is considered as sine function with $g(t) = 0.2 \sin t$. It is witnessed that the magnetic field has a strong influence on the profiles and Sinusoidal modulation effect is observed to have more impact on $F(t)$ and $H(t)$ rather on $G(t)$. It is evident that on increasing the values of Q the field variable $F(t)$ and $H(t)$ decreases. Where as $G(t)$ is almost unchanged even after increasing the values Q. Thus here we can note that magnetic field effects along with sinusoidal modulation of gravity field is having stabilizing impact on the flow.

In figures 4, 5 and 6 the effect of the magnetic field on $F(t)$, $G(t)$ and $H(t)$ versus t with some fixed values of R is analyzed. In these figures the gravity modulation is considered as an exponential function with $g(t) = 0.2e^{-t}$. Exponential modulation effect is observed to have same impact on $F(t)$, $H(t)$ and $G(t)$. It is evident that on increasing the values of Q, field variable $F(t)$ and $H(t)$ decreases. Thus, here we can note that the magnetic field effects along with exponential modulation of gravity field is having stabilizing impact on the flow.

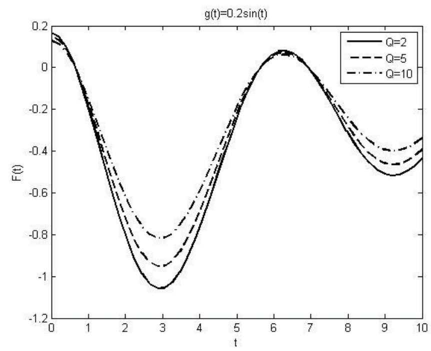

FIGURE 1. Amplitude of velocity function $F(t)$ versus t

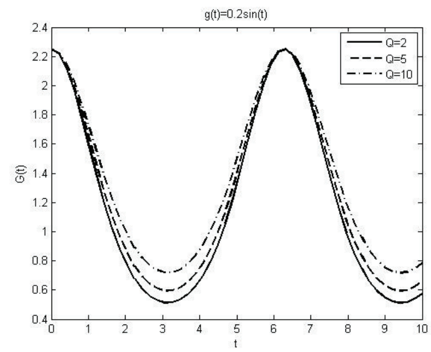

FIGURE 2. Amplitude of temperature field $G(t)$ versus t

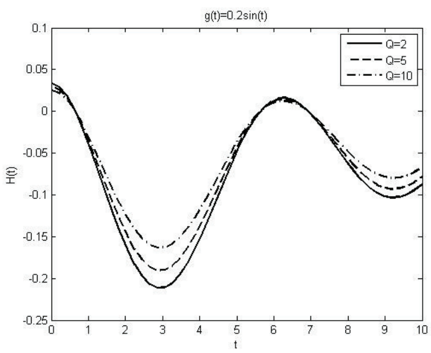

FIGURE 3. Amplitude of magnetic field $H(t)$ versus t

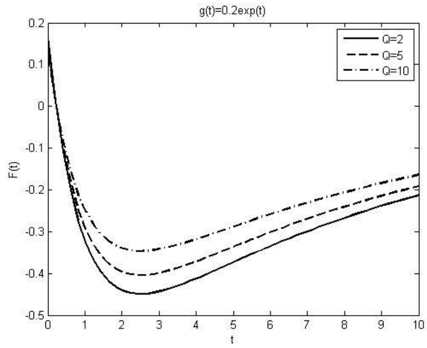

FIGURE 4. Amplitude of velocity function $F(t)$ versus t

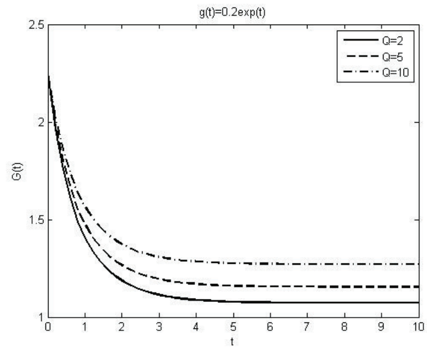

FIGURE 5. Amplitude of temperature field $F(t)$ versus t

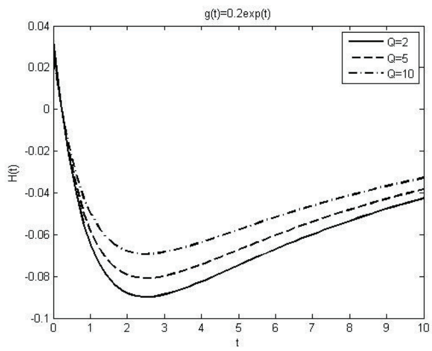

FIGURE 6. Amplitude of magnetic field $F(t)$ versus t

5. CONCLUSIONS:

The non-linear time dependent flow through a porous layer with gravity modulation is studied and solution of governing equations are obtained by using modified Stuart-Davis method. The results are deliberated in previous section and from the above results it is concluded that, if a less restricted model is considered, then the oscillation might not have been synchronous. It is also noteworthy that the magnetic field is having stabilizing effect in case of gravity modulation is sinusoidal function. On the other hand the combined effect of magnetic field with gravity modulation as exponential functions leads destabilize the flow for short time. Hence we conclude that by choosing the suitable gravity modulation function and magnetic field intensity it is possible to control the flow.

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