ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **10** (2021), no.1, 557–570
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.1.55 Spec. Iss. on ICIRPS-2020

MAGNETO-THERMOELASTIC PROBLEM WITH EDDY CURRENT LOSS OF A THERMOSENSITIVE CONDUCTIVE PLATE

L.C. Bawankar[1] and G.D. Kedar

ABSTRACT. In this paper a two dimensional magneto-thermoelastic problem of a thermosensitive finite conducting plate with eddy current loss is considered. It is assumed that the plate is influenced by a time-varying external magnetic field and that the heating is caused by Joule heat. The fundamental equations for magnetic field, heat conduction and elastic fields are formulated. Temperature dependent material properties and heat source as eddy current loss is considered in the heat conduction equation. Kirchhoff's variable transformation is employed to convert nonlinear to linear heat conduction equation. Integral transform technique is used to solve the magnetic field and temperature distribution. The stresses in a plane state are determined by using Airy's stress function. The numerical analysis is carried out and the results are graphically displayed.

1. INTRODUCTION

The much attention has received in the study of magneto-thermoelasticity from recent years because of its practical uses of electro-magnetism in the era of continuum mechanics. Magneto-thermoelasticity is the combined study of

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 74A10, 74A15, 78A25, 78A30, 78A40.
Key words and phrases. Magneto-thermoelasticity, Thermosensitive, Kirchoff's Variable, Eddy current, Airy's stress function.
Submitted: 18.11.2020; *Accepted:* 25.12.2020; *Published:* 22.01.2021.

the temperature field, magnetic field, and elastic field. Study of magneto- thermoelasticity plays an important role in diverse topics, such as earthquakes, seismology, geophysical problems, certain topics in optics, and acoustics. Maxwell's electromagnetic field has been related to the motion of elastic solids and influence with the wave propagation from the earth material. The skin effect occurs in the plate by opposing eddy current created by the time-varying magnetic field. The eddy current loss is created because of eddy current. Due to the loss of eddy current, the power degenerated in the form of heat raises the temperature of the conducting plate. Many authors [1–3] studied magneto-thermoelastic problems under the effect of time varying magnetic field in a conducting medium and explained different problems of magneto-elasticity and magneto-thermoelasticity. [4,5] discussed the quasistatic and dynamic behaviour of magneto-thermoelastic stresses caused by the transient magnetic field in the infinite conducting plate and by the excitement of the ramp function with the sine function profile. [6] studied the effect of a sinusoidally time-varying magnetic field in a rectangular cylinder due to increased frequency of the magnetic field.

In [7] authors had given a brief explanation about the heat conduction problems for the thermosensitive bodies and use Kirchoff's variable transformation to convert nonlinear into the linear form of problem. [8] discussed the effect of thermosensitive material properties on temperature and stresses in a functionally graded rectangular plate.

The idea of this paper is to study the theoretical development of time-varying external magnetic field on two-dimensional thermosensitive finite conducting plate. The heat source is taken as eddy current loss and temperature-dependent material properties are considered. The external magnetic field varies exponentially with time. The induced eddy current due to time-varying magnetic field are obtained on the theory of the quasi-stationary current. The Kirchhoff's variable transformation is used to convert nonlinear to linear form of heat equation. The magnetic field, temperature change are simplified by using integral transform technique. The stresses in plane strain conditions are obtained by using Airy's stress function. The magnetic field, eddy current loss, temperature field and stresses variations are discussed analytically and presented graphically.

2. Formulation of Problem

2.1. **Electromagnetic Field.** A two dimensional problem of finite conducting plate with length a and thickness b, subject to time-dependent magnetic field $H_0\phi(\tau)$ which is uniformly distributed along the x and y direction. The magnetic field be $\mathbf{H} = (0, 0, H_z)$, and the electric field $\mathbf{E} = (E_x, E_y, 0)$ in the conducting plate. Disregarding the displacement current due to the effect of quasi-stationary current.

The governing equations of the electromagnetic field and the constitutive relations are given by [5] :

$$curl\,\mathbf{E} = -\dot{\mathbf{B}} \tag{2.1}$$

$$curl\,\mathbf{H} = \mathbf{J} \tag{2.2}$$

$$\mathbf{J} = \sigma\mathbf{E} \tag{2.3}$$

$$\mathbf{B} = \mu_m\mathbf{H}, \tag{2.4}$$

where $\mathbf{E}, \mathbf{B}$ and $\mathbf{H}$ represent electric field, magnetic flux and magnetic field in a conducting plate. $\mathbf{J}, \sigma$ and μ_m are current density, electric conductivity and the magnetic permeability.

Solving eqs. (2.1)–(2.4) the equation for magnetic field is given by

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = \mu_m\sigma\frac{\partial H_z}{\partial t}. \tag{2.5}$$

The boundary conditions and initial condition are

$$H_z\left(x, y, t\right) = H_0\phi(\tau) \text{ at } x = 0, a, \tag{2.6}$$

$$H_z\left(x, y, t\right) = H_0\phi(\tau) \text{ at } y = 0, b, \tag{2.7}$$

$$H_z\left(x, y, t\right) = 0 \text{ at } t = 0. \tag{2.8}$$

Components of current density J_x and J_y are obtained as

$$J_x\left(x, y, t\right) = \frac{\partial H_z}{\partial y}, \tag{2.9}$$

$$J_y\left(x, y, t\right) = -\frac{\partial H_z}{\partial x}. \tag{2.10}$$

Eddy current loss is defined as

$$w\left(x,y,t\right)=\frac{J_x^2+J_y^2}{\sigma}. \tag{2.11}$$

A change of variable is consider for homogeneous boundary conditions defined for H_z as follows

$$H_z\left(x,y,t\right)=h_z\left(x,y,t\right)+H_0\phi(\tau). \tag{2.12}$$

2.2. **Temperature Field.** We choose a two-dimensional transient heat conduction equation with temperature dependent material properties in a conducting plate with internal heat generation as eddy current loss. The fundamental equation for temperature change $T(x,y,t)$ is given by

$$\frac{\partial}{\partial x}\left[K\left(T\right)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[K\left(T\right)\frac{\partial T}{\partial y}\right]+w(x,y,t)=\ C\left(T\right)\rho\frac{\partial T}{\partial t}. \tag{2.13}$$

The boundary conditions and initial condition are

$$T=0\ \text{ at } x=0,a \text{ and } y=0,b, \tag{2.14}$$

$$T=F(x,y)\ at\ t=0, \tag{2.15}$$

where $w(x,y,t)$ is the eddy current loss and ρ is mass density. $K(T),C(T)$ are the thermal conductivity and specific heat capacity dependent on temperature.

2.3. **Elastic Field.** The components of stresses in the form of stress function χ are given by [6]

$$\sigma_{xx}=\frac{\partial^2\chi}{\partial y^2}+\frac{\mu_m H_z^2}{2}, \tag{2.16}$$

$$\sigma_{yy}=\frac{\partial^2\chi}{\partial x^2}+\frac{\mu_m H_z^2}{2}, \tag{2.17}$$

$$\sigma_{xy}=-\frac{\partial^2\chi}{\partial x\partial y}. \tag{2.18}$$

The boundary conditions are

$$\sigma_{xx}=0,\sigma_{xy}=0\ \text{ on }\ x=0,a;$$
$$\sigma_{yy}=0,\sigma_{xy}=0\ \text{ on }\ y=0,b.$$

By using compatibility equation, equilibrium equation and hooks law [9], the equation of thermal stress function χ called as Airy's stress function is given by

$$\nabla^4 \chi = -\frac{(1-2v)\mu_m}{2(1-v)} \nabla^2 H_z^2 - \frac{\alpha(T)E(T)}{1-v}\nabla^2 T, \tag{2.19}$$

where $v, \alpha(T)$ and $E(T)$ denote the Poisson ratio, coefficient of linear thermal expansion and Young modulus.

Dimensionless quantities: For the convenience we use dimensionless quantities as follows

(2.20)

$$\overline{x} = \frac{x}{a}, \overline{y} = \frac{y}{a}, \overline{b} = \frac{b}{a}, \left(\overline{H}_z, \overline{h}_z\right) = \frac{(H_z, h_z)}{H_0}, \tau = \frac{t}{\mu_m \sigma a^2},$$
$$\left(\overline{J}_x, \overline{J}_y\right) = \frac{(aJ_x, aJ_y)}{H_0}, \overline{w} = \frac{\sigma a^2}{H_0^4} w, \overline{T} = \frac{\rho}{\mu_m H_0^2} T, \overline{F}(x,y) = \frac{\rho}{\mu_m H_0^2} F(x,y),$$
$$(\overline{\sigma}_{xx}\, \overline{\sigma}_{yy}, \overline{\sigma}_{xy}) = \frac{2}{\mu_m H_0^2} \times (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}), \overline{\chi} = \frac{2}{\mu_m H_0^2}\chi.$$

3. Solution

3.1. **Solution of Electromagnetic Field.** Using eq. (2.20) in eqs. (2.5)–(2.8) the fundamental equation for magnetic field is given by(neglecting bar for convenience)

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = \frac{\partial H_z}{\partial t}. \tag{3.1}$$

The boundary conditions and initial condition are

$$H_z(x,y,\tau) = \phi(\tau) \text{ at } x = 0, 1; \tag{3.2}$$

$$H_z(x,y,\tau) = \phi(\tau) \text{ at } y = 0, b; \tag{3.3}$$

$$H_z(x,y,\tau) = 0 \text{ at } \tau = 0. \tag{3.4}$$

Using dimensionless quantities in eq. (2.12) we get

$$H_z(x,y,t) = h_z(x,y,t) + \phi(\tau). \tag{3.5}$$

Using eq. (3.5) in eqs. (3.1)–(3.4) reduces to

$$\frac{\partial^2 h_z}{\partial x^2}+\frac{\partial^2 h_z}{\partial y^2}=\frac{\partial h_z}{\partial \tau}+\phi'(\tau). \tag{3.6}$$

The initial condition and boundary conditions are

$$\begin{aligned} h_z\,(x,y,\tau) &= -\phi(\tau) \text{ at } \tau=0;\\ h_z\,(x,y,\tau) &= 0 \text{ at } x=0,1;\\ h_z\,(x,y,\tau) &= 0 \text{ at } y=0,b. \end{aligned} \tag{3.7}$$

Solution of a magnetic field is obtained by using finite Fourier double sine transform with respect to coordinates x and y is defined as [10]

$$\overline{\overline{h_z}}\,(\beta_m,\nu_n,\tau)=\int_0^1\int_0^b k\,(\beta_m,x)k\,(\nu_n,y)h_z\,(x,y,\tau)dxdy, \tag{3.8}$$

where $k\,(\beta_m,x)=\sqrt{2}\sin\left(\beta_m.x\right), k\,(\nu_n,y)=\frac{\sqrt{2}}{\sqrt{b}}\sin\left(\nu_n.y\right), \beta_m=m\pi, \nu_n=\frac{n\pi}{b}$.

Taking Fourier transform of eqs. (3.6) and (3.7) by using eq. (3.8) we get

$$\frac{\partial\overline{\overline{h_z}}}{\partial\tau}+({\beta_m}^2+{\nu_n}^2)\overline{\overline{h_z}}=\phi'(\tau)\frac{2}{\sqrt{b}}\frac{(1-cos\beta_m)(1-cos\nu_n b)}{\beta_m\nu_n}.$$

The solution of internal magnetic field h_z is obtained as

$$\overline{\overline{h_z}}=e^{-({\beta_m}^2+{\nu_n}^2)\tau}\frac{2}{\sqrt{b}}\frac{(1-cos\beta_m)(1-cos\nu_n b)}{\beta_m\nu_n}\left[\int_0^\tau e^{({\beta_m}^2+{\nu_n}^2)\tau}\phi'(\tau)d\tau+\phi(\tau)\right]. \tag{3.9}$$

By taking double Fourier inversion transform of equation eq. (3.9) becomes

$$h_z=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{mn}(\tau)\sin\beta_m x\sin\nu_n y, \tag{3.10}$$

where $B_{mn}(\tau)$ is given by

$$\begin{aligned} B_{mn}(\tau)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{4}{b}\frac{(1-cos\beta_m)(1-cos\nu_n b)}{(\beta_m\nu_n)}e^{-(\beta_m^2+{\nu_n}^2)\tau}\\ \cdot\left[\int_0^\tau e^{({\beta_m}^2+{\nu_n}^2)\tau}\phi'(\tau)d\tau+\phi(\tau)\right]. \end{aligned}$$

The magnetic field is obtained by substituting eq. (3.10) in eq. (3.5) we get

$$H_z(x,y,\tau)=\phi(\tau)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{mn}(\tau)\sin(\beta_m x)\sin(\nu_n y).$$

Dimensionless form of current density components J_x and J_y are obtained from eqs. (2.9) and (2.10) as

$$J_x\left(x,y,\tau\right)=\frac{\partial H_z}{\partial y}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\nu_n B_{mn}(\tau)\sin\beta_m x\cos\nu_n y, \tag{3.11}$$

$$J_y\left(x,y,\tau\right)=-\frac{\partial H_z}{\partial x}=-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta_m B_{mn}(\tau)\cos\beta_m x\sin\nu_n y. \tag{3.12}$$

Equation (2.11) in dimensionless form is given by

$$w\left(x,y,\tau\right)=J_x^2+J_y^2. \tag{3.13}$$

Substituting eqs. (3.11) and (3.12) in eq. (3.13), eddy current loss is expressed as

$$w\left(x,y,\tau\right)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\aleph_{mnkl}\left(\tau\right)\left(\nu_n^2\sin^2\beta_m x\cos^2\nu_n y+\beta_m^2\cos^2\beta_m x\sin^2\nu_n y\right),$$

where $\aleph_{mnkl}\left(\tau\right)=B_{mn}^2(\tau)$.

3.2. **Solution of temperature Field.** The dimensionless form of heat conduction equation from eq. (2.13) is (dropping bar for convenience)

$$\frac{C(T)}{C_1}\frac{\partial T}{\partial\tau}=\frac{\partial}{\partial x}\left[K\left(T\right)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[K\left(T\right)\frac{\partial T}{\partial y}\right]+\frac{w(x,y,\tau)}{C_2}. \tag{3.14}$$

Dimensionless form of eqs. (2.14) and (2.15)

$$\begin{aligned}&T=0 \text{ at } \ x=0,1 \text{ and } y=0,b;\\&T=F(x,y) \text{ at } \tau=0.\end{aligned} \tag{3.15}$$

We assume $K(T)=K_0\ e^T$, $C(T)=C_0\ e^T$, K_0,C_0 are the reference values. Introducing the kirchhoff's variable by following [7],

$$\Theta\left(T\right)=\int_0^T K\left(T\right)dT.$$

Using Kirchoff's variable in eqs. (3.14) and (3.15) we obtain

$$\left[\frac{\partial^2\Theta}{\partial x^2}+\frac{\partial^2\Theta}{\partial y^2}\right]+\frac{w(x,y,\tau)}{C_3}=\frac{1}{C_4}\frac{\partial\Theta}{\partial\tau}, \tag{3.16}$$

$$\begin{aligned}&\Theta=0 \text{ at} x=0,1 \text{and } y=0,b,\\&\Theta=F(x,y) \text{at } \tau=0,\end{aligned} \tag{3.17}$$

$$C_1 = \frac{\mu_m \sigma}{\rho}, \quad C_2 = \frac{\mu_m^2 \sigma}{\rho^2} = \frac{C_1^2}{\sigma}, \quad C_3 = \lambda_0 C_2 \quad \text{and} \quad C_4 = \frac{\lambda_0 C_1}{C_0}.$$

Solution of heat conduction equation with eddy current loss can be solved by using integral transform technique for x and y variable and defined as

$$\overline{\overline{\Theta}}\,(\eta_i, \zeta_j, \tau) = \int_0^1 \int_0^b k\,(\zeta_l, x) k\,(\eta_k, y) \Theta\,(x, y, \tau) dy dx, \tag{3.18}$$

where $k\,(\zeta_l, x) = \sqrt{2} \sin(\zeta_l . x)$, $k\,(\eta_k, y) = \sqrt{\frac{2}{b}} \sin\,(\eta_k . y)$, and where $\zeta_l = n\pi, \eta_k = \frac{n\pi}{b}$.

Using eq. (3.18) in eqs. (3.16) and (3.17) we get

$$\frac{\partial \overline{\overline{\Theta}}}{\partial \tau} + C_1({\zeta_l}^2 + {\eta_k}^2)\overline{\overline{\Theta}} = \frac{C_4}{C_3} \overline{\overline{w}}(\zeta_l, \eta_k, \tau) \tag{3.19}$$

The solution of eq. (3.19) is obtained as

$$\begin{aligned} \overline{\overline{\Theta}} = e^{-C_4({\zeta_l}^2 + {\eta_k}^2)\tau} \Bigg[\int_0^\tau \frac{C_4}{C_3} \overline{\overline{w}} e^{C_4({\zeta_l}^2 + {\eta_k}^2)\tau} d\tau \\ + \frac{2}{\sqrt{b}} \int_0^1 \int_0^b \overline{F}(x, y) \sin(\zeta_l x) \sin(\eta_k y) dy dx \Bigg]. \end{aligned} \tag{3.20}$$

Taking double fourier inversion transform, eq. (3.20) becomes

$$\Theta = \sum_{l=1}^{\infty} \sum_{k=1}^{\infty} C_{lk}(\tau) \sin(\zeta_l x) \sin(\eta_k y),$$

where

$$\begin{aligned} C_{lk}(\tau) = e^{-C_4({\zeta_l}^2 + {\eta_k}^2)\tau} \Bigg[\int_0^\tau \frac{C_4}{C_3} \overline{\overline{w}} e^{C_4({\zeta_l}^2 + {\eta_k}^2)\tau} d\tau \\ + \frac{2}{\sqrt{b}} \int_0^1 \int_0^b \overline{F}(x, y) \sin(\zeta_l x) \sin(\eta_k y) dy dx \Bigg]. \\ \overline{\overline{w}}(\zeta_l, \eta_k, \tau) = \int_0^1 \int_0^b \sin \zeta_l x \sin \eta_k y w(x, y, \tau) dy dx. \end{aligned}$$

Inverse transformation from Θ to T becomes

$$\Theta = \lambda_0 (e^T - 1). \tag{3.21}$$

Solving eq. (3.21) and neglecting the order more than one we obtain

$$T = \sum_{l=1}^{\infty} \sum_{k=1}^{\infty} \frac{C_{lk}(\tau)}{\lambda_0} \sin(\zeta_l x) \sin(\eta_k y).$$

3.3. **Solution of Elastic Field.** Using eq. (2.20) in eqs. (2.16)–(2.19) and section 2.3 stress components and governing equation of stress function is expressed in the form as (neglecting bar over quantities):

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2} + H_z^2, \tag{3.22}$$

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2} + H_z^2, \tag{3.23}$$

$$\sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y}, \tag{3.24}$$

$$\nabla^4 \chi = -C_5 \nabla^2 H_z^2 - C_6 e^{2T} \nabla^2 T, \tag{3.25}$$

where

$$\nabla^4 = \nabla^2 \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4},$$

and

$$C_5 = \frac{1-2\nu}{1-\nu}, \quad C_6 = \frac{-2\alpha_0 E_0}{(1-\nu)\rho}.$$

The stress function χ solution is a combination of complementary solution χ_c and the particular solution χ_p. Solution of eq. (3.25) χ is obtained as

$$\begin{aligned}\chi = \sum_{\lambda=1}^{\infty} d_{11}\, x\, \cos \lambda x \sinh \lambda y + d_{12}\, y \sin \lambda x \cosh \lambda y + d_{13} \sin \lambda x \sinh \lambda y \\ + d_{14} \sin \lambda x \cosh \lambda y - C_5\, \phi(\tau) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{B_{mn}(\tau) \sin(\beta_m\, x) \sin(\nu_n y)}{(\beta_m^2 + \nu_n^2)^2} \\ - C_6 \sum_{l=1}^{\infty} \sum_{k=1}^{\infty} \frac{C_{lk}(\tau)\, e^{2T} \sin(\zeta_l\, x) \sin(\eta_k y)}{(\zeta_l^2 + \eta_k^2)^2}.\end{aligned}$$

Substituting stress function in eqs. (3.22)–(3.24), we get

$$\begin{aligned}\sigma_{xx} = \sum_{\lambda=1}^{\infty} & d_{11}\ (\lambda^2)x\ \cos\lambda x \sinh\lambda y \\ & + d_{12} \sin\lambda x\ (\lambda^2 y\ \cosh\lambda y + 2\ \lambda \sinh\lambda y) \\ & + d_{13}\ \lambda^2 \sin\lambda x \sinh\lambda y + d_{14}\lambda^2 \cos\lambda x \sinh\lambda y \\ & + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{C_5\ \nu_n^2}{(\beta_m^2+\nu_n^2)^2} + 2\right]\phi(\tau)\ B_{mn}(\tau)\sin(\beta_m\ x)\sin(\nu_n y) \\ & + 16\ C_6\ \sum_{l=1}^{\infty}\sum_{k=1}^{\infty}\frac{\eta_k^2\ C_{lk}^2(\tau)\ \sin^2(\zeta_l\ x)\sin^2(\eta_k y)}{(\zeta_l^2+\eta_k^2)^2},\end{aligned} \tag{3.26}$$

$$\begin{aligned}\sigma_{yy} = \sum_{\lambda=1}^{\infty} & -d_{11}(\lambda^2 x\ \cos\lambda x - 2\ \ \lambda \sinh\lambda x)\cosh\lambda y \sinh\lambda y \\ & - d_{12}\ \lambda^2 \sin\lambda x\ \cosh\lambda y - d_{13}\ \lambda^2 \sin\lambda x \sinh\lambda y \\ & - d_{14}\lambda^2 \cos\lambda x \sinh\lambda y \\ & + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{C_5\ \beta_m^2}{(\beta_m^2+\nu_n^2)^2} + 2\right]\phi(\tau)\ B_{mn}(\tau)\sin(\beta_m\ x)\sin(\nu_n y) \\ & + 16\ C_6\ \sum_{l=1}^{\infty}\sum_{k=1}^{\infty}\frac{\zeta_l^2\ C_{lk}^2(\tau)\ \sin^2(\zeta_l\ x)\sin^2(\eta_k y)}{(\zeta_l^2+\eta_k^2)^2},\end{aligned} \tag{3.27}$$

$$\begin{aligned}\sigma_{xy} = \sum_{\lambda=1}^{\infty} & -d_{11}\ \lambda\ (-\lambda x\ \sin\lambda x + \ \cos\lambda x)\sinh\lambda y \\ & - d_{12}\ \lambda \cos\lambda x(\lambda y\ \sinh\lambda y + \ \cosh\lambda y) \\ & - d_{13}\ \lambda^2 \cos\lambda x \cosh\lambda y + d_{14}\lambda^2 \sin\lambda x \cosh\lambda y \\ & + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{C_5\ \beta_m\ \nu_n}{(\beta_m^2+\nu_n^2)^2}\phi(\tau)\ B_{mn}(\tau)\sin(\beta_m\ x)\sin(\nu_n y) \\ & + 3\ C_6\ \sum_{l=1}^{\infty}\sum_{k=1}^{\infty}\frac{\zeta_l\ \eta_k\ C_{lk}^2(\tau)\ \sin(\zeta_l\ x)\sin(\eta_k y)\cos(\zeta_l\ x)\cos(\eta_k y)}{(\zeta_l^2+\eta_k^2)^2},\end{aligned} \tag{3.28}$$

where d_{11}, d_{12}, d_{13} and d_{14} are to be calculated by using dimensionless stress free boundary conditions we get

$$\begin{aligned} &\sigma_{xx} = 0, \sigma_{xy} = 0 \ \text{ on } \ x = 0, 1; \\ &\sigma_{yy} = 0, \sigma_{xy} = 0 \ \text{ on } \ y = 0, b. \end{aligned} \tag{3.29}$$

Using eq. (3.29) in eqs. (3.26)–(3.28) we obtained the simultaneous equations and solved the unknown coefficients by using MatLab software.

4. Numerical results and discussion

The numerical calculation has been carried out for the temperature field, magnetic field, and elastic field. The effect of eddy current loss on a thermosensitive temperature field and time-varying magnetic field in the plate is observed. We choose aluminum material for the conducting plate with dimensions $(\bar{b} = \frac{b}{a}), a = 1, b = 2$ and the function $\phi(\tau)$ is defined as

$$\phi(\tau) = \sin(\omega\tau),$$

where ω is the non-dimensional frequency of the magnetic field.

Dimensions of physical parameters are

$H_0 = \frac{1}{4\pi\times10^{-7}}$ A/m, $\mu_m = 4\pi \times 10^{-7}$ H/m,
$\sigma = 2.5 \times 10^7$ S/m, $\rho = 2.7 \times 10^3\ kg/m^3$,
C = 0.9×10^3 J/kgK, K = 230 W/mK, $\nu = 0.33$,
$E = 70\ GPa$, $\alpha = 24 \times 10^{-6}$ 1/K

We consider a numerical example that gives the variation in a magnetic field, eddy current loss, temperature change, and elastic field. For all graphs the value $x = 0.1$ is chosen. The fig. 1 is the plot of magnetic field versus τ for different value of time $\omega = 3, 6, 8, 10$. The graph is initially increasing and then decreases gradually with the passage of time. Due to increase in frequency of magnetic field the nature and speed of waves changes. The fig. 2 shows variation of eddy current loss with time for different values of ω. Significant variation with different frequencies is observed.

Figures 3 and 4 represent temperature variation for different values of x and τ. Significant changes of temperature is observed with space variables and different values of time. Temperature increases initially and then decreases gradually. In figs. 5 and 6 represent variation of stresses σ_{xx} and σ_{yy} for different values of τ. Elastic changes observed significantly with time. After graphical analysis it is observed that significant effect of thermosensitive properties observed on a finite conducting plate which is not discussed earlier in uncoupled problem with eddy current loss.

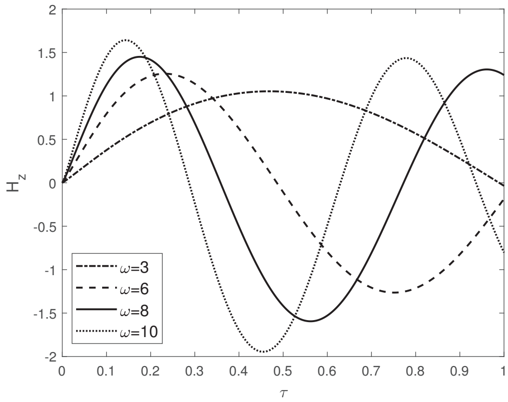

FIGURE 1. Variation of magnetic field(H_z) with time τ.

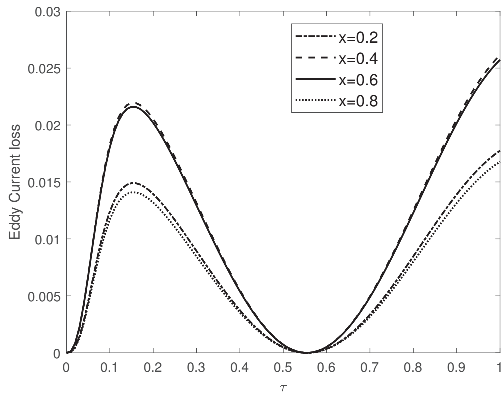

FIGURE 2. Variation of eddy current loss(w) with time τ.

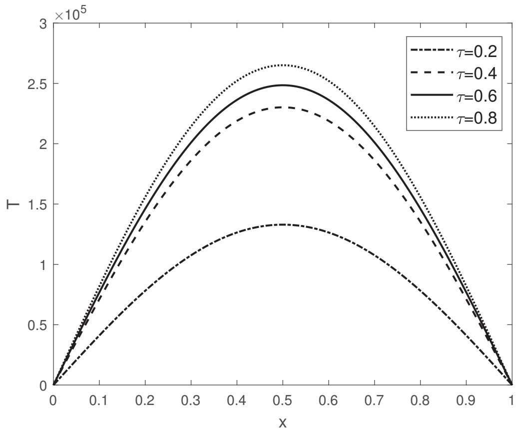

FIGURE 3. Variation of temperature(T) with space variable x.

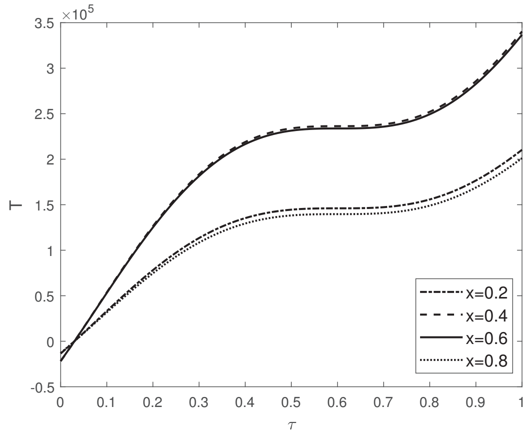

FIGURE 4. Variation of temperature(T) with time τ.

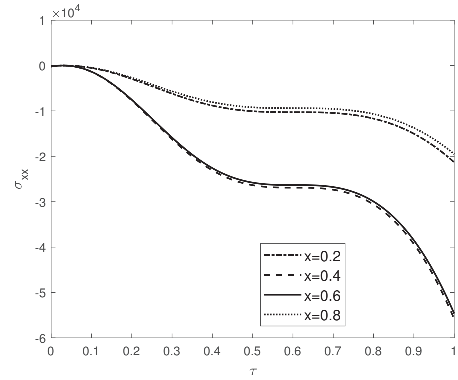

FIGURE 5. Variation of stress(σ_{xx}) with time τ.

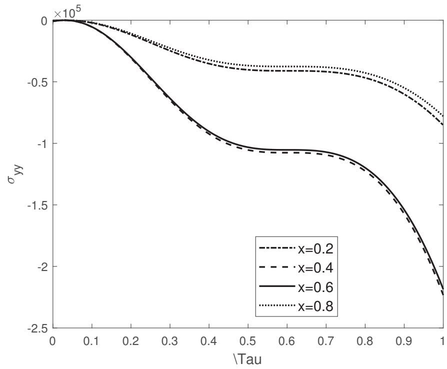

FIGURE 6. Variation of stress(σ_{yy}) with time τ.

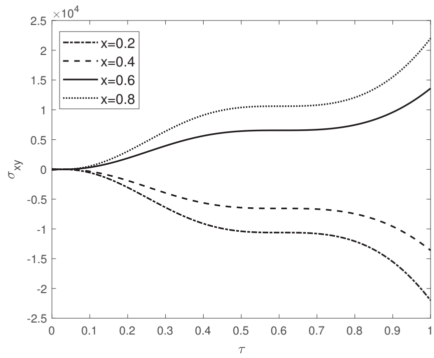

FIGURE 7. Variation of stress(σ_{xy}) with time τ.

5. CONCLUSION

In the study of magneto-thermoelastic problem with eddy current loss in a thermosensitive conducting rectangular plate is considered. Sinusoidally time varying external magnetic field is applied to a rectangular plate. The internal heat generation is due to eddy current loss and effect of temperature dependent material properties is observed. The solution of magnetic field, eddy current loss, temperature and elastic field is derived numerically by using integral transform technique. Because of skin effect in the eddy current, heat generation due to eddy current loss is observed in a rectangular plate except at the boundary due to increase in frequency of time varying magnetic field. Temperature rises initially and then gradually decreases. The compressive stress occurs with mechanical stress free boundary conditions. The effect of thermosensitive properties as well as time varying magnetic field with the increase in frequency is observed graphically. The theoretical development of this model will prove useful to the researchers working in the area of magneto-thermoelasticity.

REFERENCES

[1] G.M. PANTELYAT, M. FELIACHI : *magneto-thermo-elastic-plastic simulation of inductive heating of metals*, EPJ Appl. Phys. **2**(17) (2002), 29–33.

[2] S. KALISKI, W. NOWACKI : *Combined elastic and electromagnetic waves produced by thermal shock*, Bull. Acad. Polon. Sci., Ser. Sci. Techn., **2**(10) (1962), 159–168.

[3] G. PARIA: *Magneto-Elasticity and Magneto-Thermo-Elasticity*, Advances in Applied Mechanics, Elsevier, (1966), 73–112.

[4] M. HIGUCHI, R. KAWAMURA, Y. TANIGAWA, H. FUJIEDA: *Dynamic and quasi-static behaviors of magneto-thermo-elastic stresses in a conducting infinite plate subjected to an arbitrary variation of magnetic field*, Acta Mechanica, **191**(3-4) (2006), 135–154.

[5] M. HIGUCHI, R. KAWAMURA, Y. TANIGAWA, H. FUJIEDA : *Magnetothermoelastic Stresses Induced by a Transient Magnetic Field in an Infinite Conducting Plate*, Journal of Mechanics Of Material And Structures, **2**(1) (2007), 113–130.

[6] R. KAWAMURA, K. SANO, Y. OOTAO, Y. TANIGAWA, K. IKEDA, H. KINOSHITA: *Magneto-thermo-elastic problem of a conducting rectangular cylinder subjected to sinusoidal change in time of magnetic field*, Acta Mechanica, **214**(1-2) (2010), 145–158.

[7] V.S. POPOVICH, I.M. MAKHORKIN: *On the solution of heat-conduction problems for thermosensitive bodies*, Journal of Mathematical Sciences, **88**(3) (1998), 352–359.

[8] V.R. MANTHENA, N.K. LAMBA, G.D. KEDAR AND K.C. DESHMUKH: *Effects of Stress Resultants on Thermal Stresses in a Functionally Graded Rectangular Plate Due to Temperature Dependent Material Properties*, International Journal of Thermodynamics (IJoT), **19**(4) (2016), 235–242.

[9] N. NODA: *Thermal Stresses in Materials with Temperature Dependent Properties*, Thermal Shock and Thermal Fatigue Behavior of Advanced Ceramics, Springer Netherlands, (2010), 15–26.

[10] M.N. OZISIK : *Boundary Value Problems of Heat Conduction*, International textbook company Scranton, Pennsylvania , (1968), 84–109.

DEPARTMENT OF MATHEMATICS, RTM NAGPUR UNIVERSITY, NAGPUR, SANJIVANI COLLEGE OF ENGINEERING, KOPARGAON- 423 603 (MAHARASHTRA), INDIA.

Email address: `latikasawarkar@gmail.com`

DEPARTMENT OF MATHEMATICS, RTM NAGPUR UNIVERSITY, NAGPUR-440 033 (MAHARASHTRA), INDIA.

Email address: `g.d.kedar@rediffmail.com`