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# ON MULTIVARIATE SPATIO-TEMPORAL TIME SERIES MODELS AND CAUSALITY TEST USING COVID-19 DATA

Madjda Amrani and Halim Zeghdoudi[1]

ABSTRACT. The purpose of this article is to illustrate the development of patient-specific GSTAR models using spatio-temporal time series data of number of confirmed cases of covid-19 in four countries to test spatial causality between them.

## 1. INTRODUCTION

In the spatio-temporal time series, the definition of the spatial weight matrix is the key for our study on causality in the sense of the barn. In spatio-temporal models such as STARMA, STAR and STMA the right choice of the spatial weight matrix is characterized in light of the range and limitations of the study in different ways. the weight matrices are exogenous, they are defined a priori by the modeler taking into account his knowledge of the relationships and interactions between the spatial units.

Two types of matrix were chosen: the uniform weight matrix and the distance weight matrix, then we define the notion of spatial causality in relation to the two types of main concept matrix used for this study it is the causality of barn

[1]*corresponding author*

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Uniform weight matrix is most often used as spatial weight. The disadvantages of a uniform spatial weight matrix are the following not taking into account spatial dynamics and heterogeneity and the trainee fails to grasp spatial heterogeneity (Kelejian and Prucha 2010, [9]) and porating instrumental variables in the first step (for more details see Boudjellaba and Dufour 1992 [3], Cliff and Ord 1969, [4]). COVID-19 is a new type of coronavirus that is infecting people around the world. Coronaviruses are very common, and typically cause coughs and colds. In rare cases, coronaviruses can cause severe infections like SARS.

In December 2019, there was a novel coronavirus called SARS-CoV-2. Novel means it has never been seen in humans. In people who get infected by the virus, there is a range of illness, and the illness that you get from the virus is called COVID-19 for Corona Virus Disease-2019. COVID-19 poses a serious threat to health, and the situation is changing every day. The risk varies within the same community, and from one community to another.

The rest of this paper is organized as follows. Section 2 describes the theoretical model, including the spatio-temporal time series model (STARMA, STAR and GSTAR). Furthermore, Section 3 deals with the causal structure with a GSTAR model. Finally, the last section presents the conclusion.

## 2. Theoretical Model

2.1. **STARMA model.** An extension of the well-known ARMA model to deal with space–time dependency is the STARMA($p_{,\lambda_1,\ldots,\lambda_p}$ , $q_{,,\delta_1,\ldots,\delta_p}$ ) model proposed by Cliff and Ord (1975), Dufour(1989, 1990) Dufour and Renault (1998)and Pfeifer and Deutsch (1980a, b), see [5–8, 10, 11] which is expressed as :

$$z_t = \sum_{k=1}^{p}\sum_{j=0}^{\lambda_k} \phi_{k,,j} w_j z_{t-k} - \sum_{k=1}^{q}\sum_{j=0}^{\delta_k} \theta_{k,j} w_j \varepsilon_{t-k} + \varepsilon_t,$$

where

- $p$ and q are the lags of autoregressive and moving average components, respectively;
- $\lambda_k$ is the degree of spatial dependency within the kth autoregressive lag component;
- $\delta_k$ is the degree of spatial dependency within the kth moving average lag component;

- $\phi_{k,j}$ are the parameters of the autoregressive components;
- $\theta_{k,j}$ are the parameters of the moving average component.

Two particular models are derived from $STARMA(p_\lambda, q_\delta)$ models: $STAR(p_\lambda)$ models with only spatiotemporal autoregressive components, and $STMA(q_\delta)$ models with only spatio-temporal moving average components. The $STARMA(p_\lambda, q_\delta)$ model has two special subclasses. When models that contain no autoregressive term $p = 0$ are considered as Space Time Moving Average $STMA(q_\delta)$ model. When $q = 0$ the class is considered as Space Time Auto Regressive $STAR(p_\lambda)$ model.

2.2. **STAR Model.** Space Time Autoregressive (STAR) is one of space time modeling. That's model includes combining elements of time and location dependencies. the STAR model of autoregressive order $p$ and spatial orders $\lambda_1, \ldots, \lambda_p$ $(STAR(p_{\lambda_1,\ldots,\lambda_p}))$ is defined as:

$$z_t = \sum_{s=1}^{p} \sum_{k=1}^{\lambda_s} \Phi_{sk} w^{(k)} z_{t-s} + \varepsilon_t,\ t = 0, \pm 1, \pm 2, \ldots.$$

For example, if a number of location is 3 then $STAR(1_1)$ show as:

$$\begin{aligned}
z_t &= \phi_{10} z_{t-1} + \phi_{11} w^{(1)} z_{t-1} + \varepsilon_t \\
z_t &= \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix} = \begin{pmatrix} \phi_{10} & 0 & 0 \\ 0 & \phi_{10} & 0 \\ 0 & 0 & \phi_{10} \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \\ z_{3,t-1} \end{pmatrix} \\
&\quad + \begin{pmatrix} \phi_{11} & 0 & 0 \\ 0 & \phi_{11} & 0 \\ 0 & 0 & \phi_{11} \end{pmatrix} \begin{pmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \\ z_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{x,t} \\ \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix} \\
\Phi_{sk} &= \text{diag}(\phi_{sk}^{(1)}, \ldots, \phi_{sk}^{(n)}).
\end{aligned}$$

2.3. **GSTAR model.** The GSTAR model is specific form of VAR (Vector Autoregressive) model. It reveals linear dependencies of space and time. The main difference is on the spatial dependent, that in GSTAR model, it is expressed by weight matrix. Let $\{z_t\ t = 0, \pm 1, \pm 2, \ldots\}$ be a multivariate time series of $N$ components. In matrix notation, the GSTAR model of autoregressive order $p$ and spatial orders $\lambda_1, \lambda_2, \ldots, \lambda_p$, GSTAR $\left(p_{\lambda_1,\lambda_2,\ldots,\lambda_p}\right)$ could be written as (see

Borovkova et al.(2008), [1]),

$$z_t = \sum_{s=1}^{p_s} \left[ \Phi_{s0} + \sum_{k=1}^{\lambda_s} \Phi_{sk} w^{(k)} z_{t-s} \right] + \varepsilon_t, \qquad t = 0, \pm 1, \pm 2, \dots,$$

where, $\Phi_{s0} = \mathrm{diag}(\phi_{s0}^{(1)}, \dots, \phi_{s0}^{(N)})$ and $\Phi_{sk} = \mathrm{diag}(\phi_{sk}^{(1)}, \dots, \phi_{sk}^{(N)})$; weights are choosen to satisfy $w_{ii}^k = 0$ and $\sum_{i \neq j} w_{ij}^k = 1$; $\varepsilon_t$ is residual model that satisfies identically, independent, distributed with mean and covariance $\sum$.

For example, GSTAR model with time and spatial order one for three locations is as follows: $GSTAR(1_1)$ with $N = 3$

$$z_t = \Phi_{10} z_{t-1} + \Phi_{11} w^{(1)} z_{t-1} + \varepsilon_t$$

$$z_t = \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix} = \begin{pmatrix} \phi_{10} & 0 & 0 \\ 0 & \phi_{20} & 0 \\ 0 & 0 & \phi_{30} \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \\ z_{3,t-1} \end{pmatrix} + \begin{pmatrix} \phi_{11} & 0 & 0 \\ 0 & \phi_{21} & 0 \\ 0 & 0 & \phi_{31} \end{pmatrix}$$

$$\cdot \begin{pmatrix} 0 & \omega_{12} & \omega_{13} \\ \omega_{21} & 0 & \omega_{23} \\ \omega_{31} & \omega_{32} & 0 \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \\ z_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}.$$

Borovkova et al.(2002), [2] stated that several matrices of spatial weights or $w$ are usually used in GSTAR model, i.e. uniform weight, weight based on inverse of distance between locations, weight based on normalization of cross correlation inference, and weight based on normalization of partial cross correlation inference. In general, the number of parameters in VAR is greater than in GSTAR model.

2.4. **Spatial Weight Matrix.** A weight matrix is used to quantify neighbourhood relationships between points, i.e., to assign a weight to each neighbour. Structure depends on the concept of neighbourhood chosen. So far, the choice of spatial weights is subjective, depending on the researcher. There are several ways to select the weights to use: with uniform, binary and non-uniform weight, based on the distance matrix and cross-correlation inference, where location weighting can be performed by normalizing the cross-correlation amplitudes between locations during the corresponding process.

**Definition 2.1** (Spatial Weight Matrix)**.** *Let $n$ be the number of spatial units. The spatial weight matrix, $w$, a $n \times n$ positive symmetric and non-stochastic matrix with element $w_{ij}$ at location $i, j$. The values of $w_{ij}$ or the weights for each pair of locations are assigned by some preset rules which defines the spatial relations among locations. By convention, $w_{ij} = 0$ for the diagonal elements. Weight matrices have been classified into three broad categories, which are often used in practice.Here is the following diagram:*

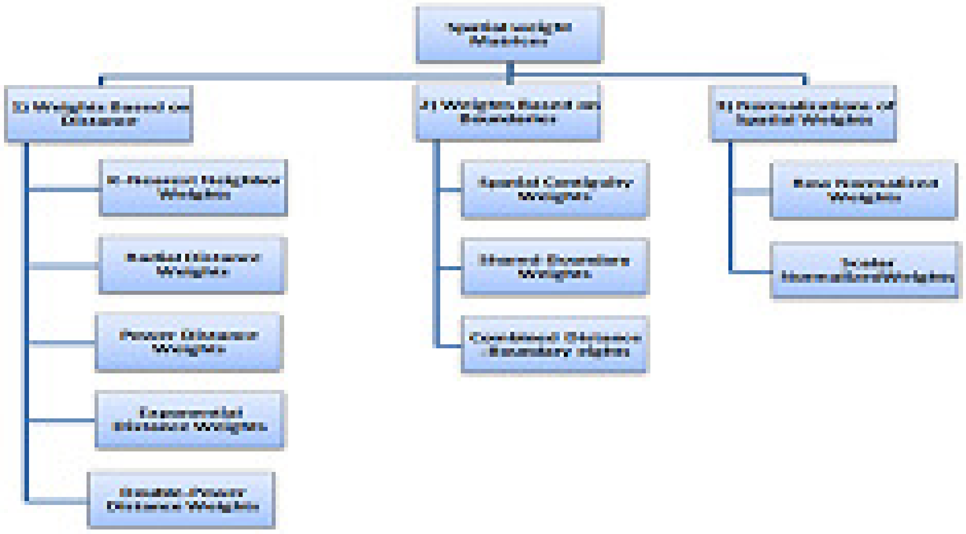

FIGURE 1. Diagram of different Spatial Weight Matrix

Choose a weight matrix for each category to compare the causality test with the weight matrix deferens.

2.4.1. *Weights Based on Distance.* Weights may be also defined as a function of the distance between region $i$ and $j$, $d_{ij}$. Let $x_i$ an $x_j$ be the longitud and $y_i$ and $y_j$ the latitude coordinates for region $i$ and $j$, respectively.

**Definition 2.2.** *(Great Circle Distance). Let two point i and j, with respective coordinates* $(x_i, y_i)$ *and* $(x_j, y_j)$

$$d_{ij} = r \times \arccos^{-1}[\cos|x_i - x_j| \cos y_i \cos\ y_j + \sin y_i \sin y_j],$$

*where $r$ is the Earth's radius. The arc distance is obtained in miles with $r = 3959$ and in kilometers with $r = 6371$.*

2.4.2. *Weights Based on Boundaries.*

**Definition 2.3.** *(Spatial Contiguity Weights). The simplest of these weights simply indicate whether spatial units share a boundary or not. If the set of boundary points of unit i is denoted by $bnd(i)$ then the so-called queen contiguity weights are defined by :*

$$w_{ij} = \begin{cases} 1 & bnd(i) \cap bnd(j) \neq \emptyset \\ 0 & bnd(i) \cap bnd(j) = \emptyset \end{cases} .$$

**Binary Contiguity**.

a) Rook criterion (Common Border)

| | | |
|---|---|---|
| 1 | red2 | 3 |
| red4 | 5 | red6 |
| 7 | red8 | 9 |

Common border: 2, 4, 6,8

b) Bishop criterion (Common Vertex)

| | | |
|---|---|---|
| red1 | 2 | red3 |
| 4 | 5 | 6 |
| red7 | 8 | red9 |

Common vertex: 1, 3, 7, 9

c) Queen criterion (Either common border or vertex)

| | | |
|---|---|---|
| red1 | red2 | red3 |
| red4 | 5 | red6 |
| red7 | red8 | red9 |

Common vertex and border: 1, 2, 3, 4, 6, 7, 8, 9.

## 3. Causality

The test in the sense of Granger (1969), used in econometric studies of causality, is constructed in a simple way on the following idea: If a phenomenon is the cause of another phenomenon, called "effect", then the latter cannot precede the cause. In other words, Granger considers that there is a causal relationship between two variables if the presence of the past of a variable $z$ provides information in the explanation of the present.

Looking behind Granger causality(ref)

**Definition 3.1.** *Let $z_{i,t}$ and $z_{j,t}$ be two time series in $z_t$. Let $z_{i.}$ collect all lagged variables of $z_{i,t}$, i.e. $z_{i.} = (z_{i,t-1}, z_{i,t-2}, \ldots)$ and similarly $x_{j.}$ collect all lagged variables of $x_{j,t}$. We say $x_{j,t}$ is not a bivariate Granger cause for $x_{i,t}$ if and only if conditional on $x_{i.}.x_{i,t}$ is independent of $x_{j.}$. If conditional on $x_{i.}$, $x_{i,t}$ is dependent of $x_{j.}$, we say $z_{j,t}$ is a bivariate Granger cause of $z_{i,t}$ .*

The multivariate Granger causality can be defined similarly.

**Definition 3.2.** *Let $z_{i,t}$ and $z_{j,t}$ be two component in $z_{t}$. Let $z_{j.}$ collect all lagged variables of $z_{j,t}$, i.e. $z_{j.} = (z_{j,t-1}, z_{j,t-2}, \ldots)$, and let $z_{j.}$ collect all lagged variables of $z_t$ except $z_{j.}$, i.e. $z_j = (z_1, z_2, \ldots, z_{j-1}, z_j, z_{j+1}, \ldots, z_N)$ . We say $z_{j,t}$ is not a multivariate Granger cause for $z_{i,t}$ if and only if conditional on $Z_{\jmath}$, $Z_{i,t}$is independent of $z_{j.}$. If conditional on $z_{\jmath}$, $z_{i,t}$ is dependent of $z_{j.}$, we say $z_{j,t}$ is a multivariate Granger cause of $z_{i,t}$*

3.1. **Instant non-causality.** Instantaneous non-causation is a different concept from Granger non-causation. It looks to see if for a given instant of time, that is to say at t, two or more variables evolve in a independent. That is, if a shock on one variable has no instantaneous repercussion on the other variables. We guess without it being necessary to do any mathematical developments that this will be the case if the innovations of the process are independent. More precisely, there will be no instantaneous causality between $z_{1,t}$ and $z_{2,t}$ if $\varepsilon_t$ and $\varepsilon_{2t}$ are uncorrelated, i.e. if $E\left(\varepsilon_t \varepsilon_{2t}\right) = 0$.

Granger (1969) proposed the concepts of causality and exogeneity: the variable $z_{2t}$ is the cause of $z_{1t}$, if the predictability of $z_{1t}$ is improved when $z_{2t}$ information is incorporated into the analysis. Either the model $STAR(p)$ for which the variables $z_{1t}$ and $z_{2t}$ are stationary:

$$z_t = \begin{pmatrix} z_{1,t} \\ z_{2,t} \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} + \begin{pmatrix} a_1^1 & b_1^1 \\ a_1^2 & b_1^2 \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \end{pmatrix} + \cdots + \begin{pmatrix} a_p^1 & b_p^1 \\ a_p^2 & b_p^2 \end{pmatrix} \begin{pmatrix} z_{1,t-p} \\ z_{2,t-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$$

The variable block $(z_{2t-1}, z_{2t-2}, \ldots, z_{2t-p})$ is considered exogenous by relation to variable block $(z_{1t-1}, z_{1t-2,\ldots,}z_{1t-p})$ if adding the block $z_{2t}$does not significantly improve the determination of the variables $z_{1t}$.This consists in carrying out a restriction test on the coefficients of the variables $z_{2t}$ of the $STAR$ representation

$z_{2t}$ does not cause $z_{1t}$ if the following hypothesis is accepted

$H_0 : b_1^1 = b_2^1 = \cdots h = b_p^1$

$z_{1t}$ does not cause $z_{2t}$ if the following hypothesis is accepted

$H_0 : a_1^2 = a_2^2 = \cdots = a_p^2$

3.2. **Determination of the causal structure with a GSTAR model.** In this subsection, we give some examples.

**Example 1.** *Application to the case of a $VAR(1)$ with $n = 2$*

$$z_t = a + \phi_{11} z_{t-1} + \varepsilon_t$$

$$z_t = \begin{pmatrix} z_{1,t} \\ z_{2,t} \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$$

*The $z_{2,t}$ variable does not cause the $z_{1,t}$ variable if and only if: $\phi_{12} = 0$.*

*The $z_{1,t}$ variable does not cause the $z_{2,t}$ variable if and only if: $\phi_{21} = 0$.If we are led to accept the two hypotheses that $z_{1,t}$ causes $z_{2,t}$ and $z_{2,t}$ causes $z_{1,t}$, we speak of a feedback loop "feedback effect".*

**Example 2.** *For a $STAR(1_1)$ with $n = 2$, the condition of Granger causality is immediate to obtain:*

$$z_t = \phi_{10} z_{t-1} + \phi_{11} w^{(1)} z_{t-1} + \varepsilon_t$$

$$\begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} = \left[ \begin{pmatrix} \phi_{10}^{(1)} & 0 \\ 0 & \phi_{10}^{(2)} \end{pmatrix} + \begin{pmatrix} \phi_{11}^{(1)} & 0 \\ 0 & \phi_{11}^{(2)} \end{pmatrix} \begin{pmatrix} 0 & w_{12} \\ w_{21} & 0 \end{pmatrix} \right] \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

$$\begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} = \begin{pmatrix} \phi_{10}^{(1)} & \phi_{11}^{(1)} w_{12} \\ \phi_{11}^{(2)} w_{21} & \phi_{10}^{(2)} \end{pmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{x,t} \\ \varepsilon_{y,t} \end{bmatrix}$$

$$z_{1,t} = \phi_{10}^{(1)} z_{1,t-1} + \phi_{11}^{(1)} w_{12} z_{2,t-1} + \varepsilon_{1,t}$$

$$z_{2,t} = \phi_{11}^{(2)} w_{21} z_{1,t-1} + \phi_{10}^{(2)} z_{2,t-1} + \varepsilon_{2,t}.$$

*The $z_{2,t}$ variable does not cause the $z_{1,t}$ variable if and only if $\phi_{11}^{(1)} w_{12} = 0 \Longrightarrow \phi_{11}^{(1)} = 0$ or $w_{12} = 0$.*

*The $z_{1,t}$ variable does not cause the $z_{2,t}$ variable if and only if $\phi_{11}^{(2)} w_{21} = 0 \Longrightarrow \phi_{11}^{(2)} = 0$ or $w_{21} = 0$.*

***In the case of a weight matrix is sympatric and $w_{ij} = 0$ therefore there is Bidirectional not causality $z_{i,t} \nrightarrow z_{j,t}$ and $z_{j,t} \nrightarrow z_{i,t}$ $(z_{i,t} \nrightarrow z_{j,t})$*

**Example 3.** *For a $STAR(2_2)$ with $n = 2$, the condition of Granger causality is immediate to obtain:*

$$z_t = \phi_{10} z_{t-1} + \phi_{11} w^{(1)} z_{t-1} + \phi_{12} w^{(2)} z_{t-2} + \varepsilon_t$$

$$z_t = \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} = \left[ \begin{pmatrix} \phi_{10} & 0 \\ 0 & \phi_{10} \end{pmatrix} + \begin{pmatrix} \phi_{11} & 0 \\ 0 & \phi_{11} \end{pmatrix} \begin{pmatrix} 0 & w_{12}^{(1)} \\ w_{21}^{(2)} & 0 \end{pmatrix} \right] \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \end{bmatrix}$$

$$+ \left[ \begin{pmatrix} \phi_{12} & 0 \\ 0 & \phi_{12} \end{pmatrix} \begin{pmatrix} 0 & w_{12}^{(2)} \\ w_{21}^{(2)} & 0 \end{pmatrix} \right] \begin{bmatrix} z_{1,t-2} \\ z_{2,t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

$$z_t = \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} = \begin{pmatrix} \phi_{10} & \phi_{11}w_{12} \\ \phi_{11}w_{21} & \phi_{10} \end{pmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \end{bmatrix}$$
$$+ \left[ \begin{pmatrix} 0 & \phi_{12}w_{12}^{(2)} \\ \phi_{12}w_{21}^{(2)} & 0 \end{pmatrix} \right] \begin{bmatrix} z_{1,t-2} \\ z_{2,t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$
$$z_{1,t} = \phi_{10}z_{1,t-1} + \phi_{11}w_{12}z_{2,t-1} + \phi_{12}w_{12}^{(2)}z_{2,t-2} + \varepsilon_{1,t}$$
$$z_{2,t} = \phi_{11}w_{21}^{(1)}z_{1,t-1} + \phi_{10}z_{2,t-1} + \phi_{12}w_{21}^{(2)}z_{1,t-2} + \varepsilon_{2,t}$$

*The $z_{2,t}$ variable does not cause the $z_{1,t}$ variable if and only if* $\phi_{11}w_{12}^{(1)} = 0 \Longrightarrow \phi_{11} = 0$ *or* $w_{12}^{(1)} = 0$ *and* $\phi_{12}w_{12}^{(2)} = 0 \Longrightarrow \phi_{12} = 0$ *or* $w_{12}^{(2)} = 0$.

*The $z_{1,t}$ variable does not cause the $z_{2,t}$ variable if and only if* $\phi_{11}^{(2)}w_{21} = 0 \Longrightarrow \phi_{11}^{(2)} = 0$ *or* $w_{21} = 0$ *and* $\phi_{12}w_{21}^{(2)} = 0 \Longrightarrow \phi_{12} = 0$ *or* $w_{21}^{(2)} = 0$.

## 4. Results and Discussions

Data was collected from World Health Organization. Data is from 2020-01-23 to 2020-08-29 from 4 countries (Canada, United States, Mexico, Greenland). confirmed cases of COVID-19 per day were used in our studie

The series following were used in our study

$Z_1$ : number of coronavirus cases in Canada

$Z_2$ : number of coronavirus cases in United States

$Z_3$ : number of coronavirus cases in Mexico

$Z_4$ : number of coronavirus cases in Greenland

TABLE 1. Summary descriptive statistics for the number of confirmed cases of covid-19

| | Canada | United States | Mexico | Greenland |
|---|---|---|---|---|
| Min | 0 | 1 | 0 | 0 |
| 1st Qu | 478 | 6141 | 93 | 0 |
| Median | 71264 | 1352962 | 36327 | 13 |
| Mean | 60732 | 1794162 | 142583 | 9 |
| 3rd Qu | 107394 | 2891124 | 256848 | 13 |
| Max | 129639 | 5961094 | 591712 | 14 |

TABLE 2. Longitude and Latitude of locations under consideration

| Countries | Latitude | Longitude |
|---|---|---|
| Canada | 56.13036 | −106.34677 |
| United States | 37.09024 | −95.712891 |
| Mexico | 23, 634501 | −102.552788 |
| Greenland | 71.70694 | −42.604301 |

TABLE 3. Standardized weighted matrix

| | Canada | United States | Mexico | Greenland |
|---|---|---|---|---|
| Canada | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| United States | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ |
| Mexico | $\frac{1}{3}$ | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ |
| Greenland | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 0 |

TABLE 4. Row normalized Inverse distance spatial weight matrix

| | Canada | United States | Mexico | Greenland |
|---|---|---|---|---|
| Canada | 0 | 21.8084 | 32.7166 | 65.6180 |
| United States | 21.8084 | 0 | 15.0944 | 63.3943 |
| Mexico | 32.7166 | 15.0944 | 0 | 76.8425 |
| Greenland | 65.6180 | 63.3943 | 76.8425 | 0 |

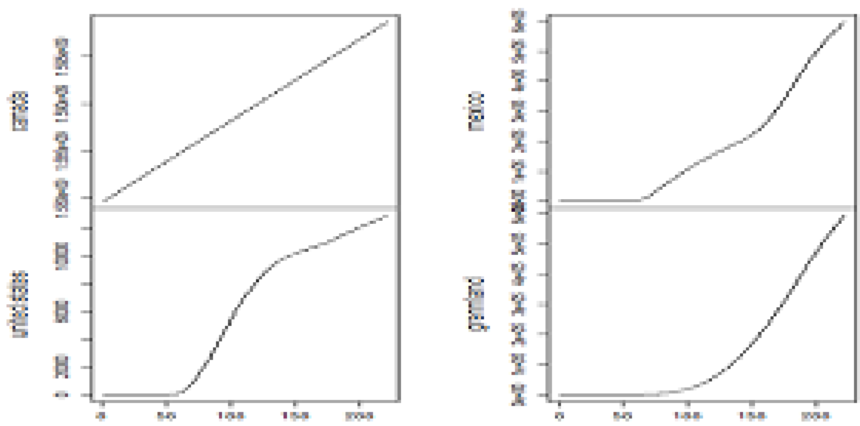

FIGURE 2. The increase in the number of covid-19 cases in four countries

TABLE 5. Fisher Test of Causality de Granger

| | **Fisher Test** | **P-value** | **Conclusion** |
|---|---|---|---|
| Canada$\nrightarrow$United.States | 25.409 | 6.822e-07 | Bidirectional |
| United.States $\nrightarrow$Canada | 24.833 | 9.047e-07 | Causality |
| Canada $\nrightarrow$ Mexico | 310.43 | < 2.2e-16 | Bidirectional |
| Mexico$\nrightarrow$ Canada | 59.775 | 7.483e-14 | Causality |
| Canada$\nrightarrow$ Greenland | 5.9955 | 0.01474 | Unidirectional |
| Greenland $\nrightarrow$ Canada | 730.18 | < 2.2e-16 | Causality |
| United.States $\nrightarrow$Mexico | 89.468 | < 2.2e-16 | Bidirectional |
| Mexico$\nrightarrow$United.States | 16.731 | 5.134e-05 | Causality |
| United.States $\nrightarrow$ Greenland | 1.9097 | 0.1677 | unidirectional |
| Greenland $\nrightarrow$ United.States | 91.855 | < 2.2e-16 | Causality |
| Mexico $\nrightarrow$Greenland | 1.1768 | 0.2786 | No Causality |
| Greenland $\nrightarrow$Mexico | 2.3482 | 0.1254 | Causality |

Notes rejection of the nul hypothesis at the $1\%$,if the probability $> 0.01$, the null hypothesis is accepted.

TABLE 6. Longrank test of Causality de Granger

| $H_0$: No instantaneous causality between | **Longrank test** | **P-value** | **Conclusion** |
|---|---|---|---|
| Canada and United States | $26.991$ | $2.044e^{-7}$ | Causality |
| Canada and Mexico | $27.117$ | $1.915e^{-7}$ | Causality |
| Canada and Greenland | $0.23177$ | $0.6302$ | No Causality |
| United States and Mexico | $37.627$ | $8.565e^{-10}$ | Causality |
| United States and Greenland | $2.1049$ | $0.1468$ | No Causality |
| Greenland and Mexico | $2.3482$ | $0.1254$ | No Causality |

Granger causality test reveals bidirectional Causality from number confirmed cases of corona-virus in Canada to number confirmed cases of corona virus in United. States, number confirmed cases of corona-virus in Canada to number confirmed cases of corona-virus in Mexico and number confirmed cases of corona-virus in Mexico to number confirmed cases of corona-virus in United States, Canada-United States, Canada-Mexico, Mexico- United States.

Granger causality test reveals unidirectional causality from number confirmed cases of corona-virus in Canada to number confirmed cases of corona-virus in Greenland and number confirmed cases of corona-virus in United. States to number confirmed cases of corona-virus Greenland. Canada→Greenland, United.States→Greenland.

Granger causality test reveals no causality from number confirmed cases of corona-virus in Mexico to number confirmed cases of corona-virus in Greenland. Mexico $\nleftrightarrow$Greenland.

*Estimated coefficients for equation Canada*

$z_{1,t} = 0.976032945 z_{1,t-1} + 0.001343703 z_{2,t-1} - 0.010709207 z_{3,t-1} + 124.428041412 z_{4,t-1} + 25.875325726$

Canada = Canada.l1 + United States.l1 + Mexico.l1 + Greenland.l1 + const

*Estimated coefficients for equation United States*

$z_{2,t} = 9.368883e^{-2} z_{1,t-1} + -9.653702e^{-1} z_{2,t-1} + 3.383258e^{-1} z_{3,t-1} + 3.84508e^{+3} z_{4,t-1} + 4.06841e^{+2}$

United States = Canada.l1 + United States.l1 + Mexico.l1 + Greenland.l1 + const

*Estimated coefficients for equation Mexico*

$z_{3,t} = 0.076676110 z_{1,t-1} - 0.003553168 z_{2,t-1} + 1.029450958 z_{3,t-1} + 24.50168267 z_{4,t-1} + 2.778536914$

Mexico = Canada.l1 + United States.l1 + Mexico.l1 + Greenland.l1 + const

*Estimated coefficients for equation Greenland*

$z_{4,t} = -2.411315e^{-6} z_{1,t-1} - 2.444835e^{-7} z_{2,t-1} + 2.257638e^{-6} z_{3,t-1} + 1.025547e + 00 z_{4,t-1} + 9.379688e^{-2}$

Greenland = Canada.l1 + United.States.l1 + Mexico.l1 + Greenland.l1 + const

TABLE 7. Test Residuals VAR (1)

| | $\chi^2$ *test* | *p-value* |
|---|---|---|
| *Portmanteau Test* | 1792.8 | < 2.2e-16 |
| *JB-Test* | 15288 | < 2.2e-16 |
| *Skewness* | 1267 | < 2.2e-16 |
| *Kurtosis* | 14021 | < 2.2e-16 |

From Table 3 the following weight matrix is obtained:

$$w = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix}.$$

By using this weight Matrix, the resulted parameter estimates and significant test of GSTAR $(1_1)^1$model is shown in Table 8.

TABLE 8. Least Squares Estimator of GSTAR $(1_1)^1$model

| Parameter | Estimate | Std. Err. | t Value | Pr > \| |
|---|---|---|---|---|
| $\phi_{10}$(Canada) | 1.020e+00 | 2.866e-01 | 3.559 | 0.000395 *** |
| $\phi_{10}$(United States) | 1.032e+00 | 2.328e-03 | 443.116 | < 2e-16 *** |
| $\phi_{10}$(Mexico) | 1.000e+00 | 6.829e-02 | 14.647 | < 2e-16 *** |
| $\phi_{10}$(Greenland) | 1.011e+00 | 4.335e+06 | 0.000 | 1.000000 |
| $\phi_{11}$(Canada) | -1.336e-03 | 2.850e-03 | -0.469 | 0.639258 |
| $\phi_{11}$(United States) | -4.101e-01 | 1.520e+00 | -0.270 | 0.787347 |
| $\phi_{11}$(Mexico) | 4.966e-03 | 4.087e-03 | 1.215 | 0.224708 |
| $\phi_{11}$(Greenland) | -1.063e-07 | 8.603e-04 | 0.000 | 0.999901 |

AIC =15847.22

Signif. codes: '***' 0.001 '**' 0.01 '*' 0.05 . From Table 4 the following weight matrix is obtained:

$$w = \begin{bmatrix} 0 & 21.8084 & 32.7166 & 65.6180 \\ 21.8084 & 0 & 15.0944 & 63.3943 \\ 32.7166 & 15.0944 & 0 & 76.8425 \\ 65.6180 & 63.3943 & 76.8425 & 0 \end{bmatrix}.$$

By using this weight Matrix, the resulted parameter estimates and significant test of GSTAR $(1_1)^2$model is shown in Table 9.

Table 9: Least Squares Estimator of GSTAR $(1_1)^2$model

| Parameter | Estimate | Std. Err. | t Value | Pr > t \| |
|---|---|---|---|---|
| $\phi_{10}$ (Canada) | 1.020e+00 | 2.888e-01 | 3.532 | 0.000436 *** |
| $\phi_{10}$(United States) | 1.031e+00 | 2.120e-03 | 486.306 | < 2e-16 *** |

| | | | | |
|---|---|---|---|---|
| $\phi_{10}$(Mexico) | 1.000e+00 | 6.901e-02 | 14.494 | < 2e-16 *** |
| $\phi_{10}$(Greenland) | 1.011e+00 | 4.357e+06 | 0.000 | 1.000000 |
| $\phi_{11}$(Canada) | -1.143e-03 | 2.103e-03 | -0.543 | 0.586956 |
| $\phi_{11}$(United States) | -3.000e-01 | 8.285e-01 | -0.362 | 0.717354 |
| $\phi_{11}$(Mexico) | 3.925e-03 | 2.568e-03 | 1.528 | 0.126809 |
| $\phi_{11}$(Greenland) | -1.915e-07 | 2.781e-03 | 0.000 | 0.999945 |

AIC =15847.22

Signif. codes: '***' 0.001 '**' 0.01 '*' 0.05.

From Table 8 and Table 9, the GSTAR $(1_1)^1$and GSTAR $(1_1)^2$ models are as follows:

$$z_t^{(1)} = \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \\ z_{4,t} \end{bmatrix} = \begin{pmatrix} 1.0200 & -0.0006 & 0.0000 & 0.0000 \\ -0.4101 & 1.0320 & -0.4101 & 0.0000 \\ 0.0000 & 0.0024 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.011 \end{pmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \\ z_{3,t-1} \\ z_{4,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

$$z_t^{(2)} = \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \\ z_{4,t} \end{bmatrix} = \begin{pmatrix} 1.0200 & -8.467e-04 & -4.86e04 & -0.0003 \\ -1.604e-01 & 1.0320 & -1.7419e-01 & -0.0754 \\ 1.8313e-03 & 0.0024 & 1.0000 & 0.0009 \\ -3.6207e-08 & -3.6782e-08 & -3.33e-08 & 1.0110 \end{pmatrix}$$

$$\cdot \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \\ z_{3,t-1} \\ z_{4,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

In tables 8 and 9, we keep the same modeled GSTAR($1_1$) but played on the weight matrix. This will add confirmation of the effectiveness of our spatio causality test.

## 5. Conclusion

The simplest framework for the study of causality in the sense of Granger is that of a system consisting only of two variables x and y. It allows normally familiarizing oneself easily with the tools and representations which will be mobilized in more general frameworks.

We will first see what Granger's lack of causation implies. of the four variables on the coefficients of VAR and GSTAR. Secondly, we will show the possibility of treatment of causality according to Granger within the framework of the multivariate Spatio-temporal chronological series model (GSTAR model).

In a third step, we will present the implementation of the tests of causality of the VAR and GSTAR model with a small change in the weight matrix, we have chosen for the $\mathrm{GSTAR}(1_1)^1$ model Standardized weighted matrix and for the $\mathrm{GSTAR}(1_1)^2$ model the Row normalized Inverse distance spatial weight matrix.

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LaPS laboratory
Badji-Mokhtar University
BP12, Annaba 23000
Algeria

LaPS laboratory
Badji-Mokhtar University
BP12, Annaba 23000
Algeria
*Email address*: `halimzeghdoudi77@gmail.com`