AN OPTIMUM PRODUCTION INVENTORY MODEL FOR DETERIORATING PRODUCT WITH SHORTAGES IS PARTIALLY BACKLOGGED

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ABSTRACT. The author analyses a manufacture inventory prototypical for fading products with persistent demand and partially backlogged shortages. And also the model gives the length of the cycle time for the next order level. The impartial of method is to get the optimizing the shortage time, total repeated time and the average total inventory cost. The proposed strategy is projected with the help of arithmetical examples.

1. INTRODUCTION


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The time changing interest considered in a large portion of the papers referenced above expects request rate to be either expanding or diminishing all through with time, while practically speaking, it settles at the developed phase of the item life cycle once the item has been acknowledged on the lookout. This sort of adjustment has been named as "incline type" and has been considered in the writing since Ritchie [13]. Yao, M et al. [2] concentrated on survey of multi-provider inventory models in store network the board. Birim, S [5] introduced assessing seller oversaw inventory frameworks: how impetuses can help store network accomplices. Esmaeili et al. [16] read up an inventory model for only merchant vender production network under various situations and exchange credit. Muniyappen, P et al. [12] concentrated on an Ideal EOQ framework for purchaser merchant with value disruptions and static croft charge. Sana [14] considered a creation inventory model of flawed value items in a layer of three production network. Mishra, U. [17] fostered an enhancing a paces of-creation inventory model on market place cost and promoting charge with decaying things. Shah, N. H., and Vaghela [18] concentrated on an Imperfect creation inventory model for time and exertion subordinate interest under expansion and most extreme unwavering quality. Beraand Jana [19] analyzed the Multi-thing blemished creation fluffy conditions. Rastogi [20] examined a creation for disintegrating items with selling value subordinate utilization proportion and deficiencies under climate. Barron, Y [21] concentrated on the exhibition investigation of a reflected liquid
creation/inventory model. Adventure, R. S et al [22] examined an investigating fossil fuel byproduct in a creation inventory model under flawed creation, assessment blunders and administration level limitation. Singh, D [23] examined the creation of weakening things with stock, selling cost, and holding cost with excess.

2. INVENTORY MODEL FORMULATION

Notation.

(1) D describes the demand rate that is constant
(2) K=D defines the Production rate
(3) ts denotes shortage time
(4) θ is the deterioration rate
(5) T is the total time
(6) k denotes charge of production per unit time
(7) h denotes holding cost per unit time
(8) s denotes cost shortage per unit time
(9) φ denotes the opportunity cost per unit time
(10) I1(t) is the inventory stage at time 0 ≤ t ≤ ts
(11) I2(t) is the inventory level at time ts ≤ t ≤ T
(12) Backlogging parameter is \( \frac{1}{1+\delta(T-t)} \)

Mathematical model as follows.

\[
\frac{dI_1(t)}{dt} + \theta I_1(t) = K - D; \theta \leq t \leq ts
\]
\[
\frac{dI_2(t)}{dt} = \frac{-D}{1+\delta(T-t)}; ts \leq t \leq T.
\]

With limit Circumstances are \( I_1(t_s) = 0 \) and \( I_2(T) = 0 \).

The inventory levels in the different stages are

\[
I_1(t) = \frac{(K-D)}{\theta(1-e^{\theta(t_s-t)})}; 0 \leq t \leq ts
\]
\[
I_2(t) = \frac{D}{\delta}\{\log[1+\delta(T-t)] - \log[1+\delta(T-t_s)]\}; ts \leq t \leq T.
\]

Production quantity is \( Q_0 = \frac{(K-D)}{\theta}(1-e^{\theta t_s}), I(0) = I_1(0) + I_2(0). \)
Average total inventory cost, which is the sum of holding cost, format cost, production cost, deficiency cost and opportunity cost

\[
TC(t_s, T) = \frac{A}{T} + \frac{h}{T} \left\{ \frac{K - D}{\theta^2} [1 - \theta t_s] + \frac{K - D}{\theta} t_s + \frac{D}{\delta^2} \delta (T - t_s) - \log [1 + \theta (T - t_s)] \right\} + \frac{k}{T} \left\{ \frac{K - D}{\theta} (1 - e^{\theta t_s}) \right\} + \frac{Ds}{T \delta^2} \left\{ \theta (T - t_s) - \log [1 + \delta (T - t_s)] \right\} + \frac{D \phi}{T \delta} \left\{ \delta (T - t_s) - \log [1 + \delta (T - t_s)] \right\}
\]

Average setup cost is \( \frac{A}{T} \).
Average holding cost is

\[
\frac{h}{T} \left\{ \frac{K - D}{\theta^2} [1 - \theta t_s] + \frac{K - D}{\theta} t_s + \frac{D}{\delta^2} \delta (T - t_s) - \log [1 + \theta (T - t_s)] \right\} + \frac{k}{T} \left\{ \frac{K - D}{\theta} (1 - e^{\theta t_s}) \right\} + \frac{Ds}{T \delta^2} \left\{ \theta (T - t_s) - \log [1 + \delta (T - t_s)] \right\} + \frac{D \phi}{T \delta} \left\{ \delta (T - t_s) - \log [1 + \delta (T - t_s)] \right\}
\]

Average shortage cost is \( \frac{Ds}{T \delta^2} \left\{ \theta (T - t_s) - \log [1 + \delta (T - t_s)] \right\} + \frac{D \phi}{T \delta} \left\{ \delta (T - t_s) - \log [1 + \delta (T - t_s)] \right\} \).

Opportunity cost is

\[
\frac{k}{T} Q_0 = \frac{k}{T} \left\{ \frac{K - D}{\theta} (1 - e^{\theta t_s}) \right\}
\]

For optimality,

\[
\frac{\partial TC(t_s, T)}{\partial t_s} = 0, \quad \frac{\partial TC(t_s, T)}{\partial T} = 0
\]

and

\[
\frac{\partial^2 TC(t_s, T)}{\partial t_s^2} > 0, \quad \frac{\partial^2 TC(t_s, T)}{\partial T^2} > 0,
\]

\[
\frac{\partial TC(t_s, T)}{\partial t_s} = \frac{1}{T} \left\{ \frac{K - D}{\theta} \left[ h \left( 1 - \frac{e^{\theta t_s}}{\theta^2} \right) - \frac{k}{\theta} e^{\theta t_s} \right] - \frac{D (T - t_s) (h + s + \delta \pi)}{1 + \delta (T - t_s)} \right\}
\]

\[
\frac{\partial^2 TC(t_s, T)}{\partial t_s^2} = \frac{1}{T} \left\{ - \frac{K - D}{\theta^3} (h + k \theta) e^{\theta t_s} + \frac{D (h + s + \delta \pi)}{(1 + \delta (T - t_s))^2} \right\}
\]

\[
\frac{\partial TC(t_s, T)}{\partial t_s} = 0.
\]

This gives

\[
t_s = \frac{\frac{(K - D)}{\theta} \left[ \frac{k}{h} - h \left( 1 - \frac{1}{\theta^2} \right) \right] + DT (h + s + \delta \pi)}{\frac{(K - D)}{\theta} \left[ \frac{(h + k \theta)}{\theta} + D (h + s + \delta \pi) \right]}
\]
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and
\[
\frac{\partial TC(t_s, T)}{\partial T} = -\frac{1}{T^2} \left\{ X + Y \left[ \delta(T - t_s) - \log(1 + \delta(T - t_s)) \right] \right\} + \frac{1}{T} \left\{ \frac{Y \delta^2(T - t_s)}{1 + \delta(T - t_s)} \right\}.
\]

Let \( X = A + \frac{K - D}{\theta} \left[ h \left( \frac{1 - e^{\theta t_s}}{\theta} + t_s \right) + k(1 - e^{\theta t_s}) \right] \) and \( Y = \frac{D}{\delta^2} (h + s + \delta \pi) \). Then,
\[
\frac{\partial^2 TC(t_s, T)}{\partial T^2} = \frac{2}{T^3} X + Y \left[ \delta(T - t_s) - \log(1 + \delta(T - t_s)) \right] - \frac{1}{T^2} \left\{ \frac{Y \delta^2(T - t_s)}{1 + \delta(T - t_s)} \right\}
\]
\[
+ \frac{T Y \delta^2(1 + \delta(T - t_s)) - Y \delta^2(T - t_s)(1 + \delta(2T - t_s))}{T^6(1 + \delta(T - t_s))^2}
\]
\[
\frac{\partial TC(t_s, T)}{\partial T} = 0.
\]

This gives.
\[
T = \frac{2Y \delta t_s - \delta t_s - X - 2Y \delta^2 t_s^2}{t_s + 2Y \delta + \delta - 4Y \delta^2 t_s}.
\]

Algorithm.

1. Fixing a trial value of \( T \).
2. From the Equation (7), find \( t_s \).
3. From the Equation (9), find \( T \).
4. Repeat the steps 2–3 until \( TC_n \leq TC_{n-1} \). Set \( t_s^* = t_s \) and \( T^* = T \)
5. Compute the corresponding \( TC(t_s^*, T^*) \)

3. MATHEMATICAL EXAMPLE

Example 1. Let \( A = 100, P = 2000, D = 1000, h = 0.6, \theta = 0.05, k = 3, s = 0.5, \delta = 2, \pi = 0.2, T = 0.05 \). Hence optimum decision variables are
\( t_s^* = 0.2994, T^* = 1.9380 \) and \( TC(t_s^*, T^*) = 169.5797 \).

Example 2. Let \( A = 200, P = 200, D = 100, h = 1.2, \theta = 0.08, k = 20, s = 30, \delta = 2, \pi = 15, T = 0.05 \). Hence optimum decision variables are
\( t_s^* = 0.0834, T^* = 0.0963 \) and \( TC(t_s^*, T^*) = 342.0704 \).
4. CONCLUSION

The author developed production inventory model for weakening goods with constant claim and partially backlogged deficiencies. This method considered as shortages are partially backlogged. Additionally, mathematical examples are given to reveal the proposed model and a process is proposed to find optimum decision parameters. The model can further extended in different situation such as stock dependent demand, credit periods, price break, quantity discount etc.

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