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# ON INEQUALITIES RELATED TO GENERALIZED SIGMOID FUNCTIONS

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ABSTRACT. The sigmoid function is widely utilized across various fields, such as statistics and biochemistry. It has been the subject of extensive research aimed at discovering its properties and extending its definitions. This paper examines the generalized sigmoid function, where the exponent $-x$ is replaced to a smooth function $-g(x)$. Building upon Nantomah's work in 2019, we derive and analyze several inequalities that characterize the convexity of this new definition.

## 1. INTRODUCTION

The sigmoid function [6], characterized by its S-shaped curve, is a fundamental mathematical construct defined by the formula

$$\sigma(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{1+e^x}$$

for any $x \in \mathbb{R}$. This function is distinguished by its smooth transition between asymptotic limits, providing a bounded, continuous mapping from the real line to the interval $(0, 1)$.

The sigmoid function appears to be beneficial in various fields. In statistics and machine learning, it is known as the activation function of the logistic regression [10]. This regression is a data analysis technique used to model the relationship

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between a dependent binary variable and one or more independent variables. Adopting standard notations, the corresponding key equation reads as follows:

$$y = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}.$$

In biochemistry and pharmacology, the Hill-Langmuir equation [7], modelled by the sigmoid function, denotes two interrelated equations that describe how ligands bind to macromolecules depending on the concentration of the ligand. Adopting standard notations, the corresponding key equation reads as follows:

$$\theta = \frac{[L]^n}{K_d + [L]^n}.$$

In ecology, the sigmoid function is used to describe a logistic population growth [12]. This type of growth happens when the growth rate declines as the population reaches the carrying capacity. Adopting standard notations, the corresponding key equation reads as follows:

$$N = \frac{N_0 K}{N_0 + (K - N_0)e^{-\lambda K t}}.$$

Due to its usefulness, many mathematicians have attempted to derive properties related to this function. For example, U. A. Ezeafulukwe et al. [4] considered the sigmoid function defined on a complex plane and deduced several properties. One result is they proved that, for any $z \in \{w \in \mathbb{C} : |w| < 1\}$,

$$\text{Re}\left(1 + \frac{z\sigma''(z)}{\sigma'(z)}\right) > 0$$

if and only if $\text{Re}(\sigma(z)) > 0$. Now, one interesting direction is to study its convexity. It is well-known that the sigmoid function is convex on $(-\infty, 0)$ and concave on $(0, \infty)$. Nantomah [6] discovered various results related to the sigmoid function and the convexity of the means functions. For example, he showed that $\sigma(x)$ is $AH$-convex. In other words, for any $x, y > 0$,

$$\sigma\left(\frac{x+y}{2}\right) \geq \frac{2\sigma(x)\sigma(y)}{\sigma(x)+\sigma(y)}.$$

In the end of the paper by Valdés [11], he asked a new research topic by constructing a general version of a sigmoid function as shown below.

**Definition 1.1.** *Let $g : \mathbb{R} \to \mathbb{R}$ be a smooth function. We define the generalized sigmoid function $\sigma : \mathbb{R} \to \mathbb{R}$ by the formula*

$$\sigma(x) = \frac{1}{1+e^{-g(x)}} = \frac{e^{g(x)}}{1+e^{g(x)}}.$$

Its first and second derivatives are given as follows:

$$\sigma'(x) = \frac{e^{g(x)}g'(x)}{(1+e^{g(x)})^2},$$
$$\sigma''(x) = \frac{e^{g(x)}(g''(x)(1+e^{g(x)}) + (g'(x))^2(1-e^{g(x)}))}{(1+e^{g(x)})^3}.$$

In this paper, we explore the various inequalities associated with the new definition of generalized sigmoid functions. In particular, we examine properties such as sub-additivity and the convexity of mean functions, as well as Grumbaum-type inequalities and other miscellaneous results. We establish conditions for $g(x)$ that ensure the inequalities hold. For example, we have shown that if $g(x)$ is concave, then $\sigma(x)$ is logarithmically concave.

The rest of the paper is organized as follows: Section 2 provides the foundational definitions and lemmas. Section 3 presents the main results of our study, while Section 4 offers a discussion and conclusion.

## 2. Preliminaries

**Definition 2.1.** *[8] Let $I \subseteq \mathbb{R}$ and $f : I \to \mathbb{R}$. We say that $f$ is sub-additive (or super-additive) if $f(x) + f(y) >$ (or $<$) $f(x+y)$ for any $x, y, x+y \in I$.*

**Lemma 2.1.** *[8] Let $I \subseteq \mathbb{R}$ and $f : I \to \mathbb{R}$. If $\dfrac{f(x)}{x}$ is decreasing (or increasing), then $f$ is sub-additive (or super-additive).*

From now on, unless specified, $I$ that we are considering in this section will be an open sub-interval of $(0, \infty)$, which makes it a convex set.

**Definition 2.2.** *[3] Let $f : I \to \mathbb{R}$. We say that $f$ is convex (or concave) if*

$$f(\lambda x + (1-\lambda)y) \leq (\text{or } \geq)\ \lambda f(x) + (1-\lambda) f(y)$$

*for any $x, y \in I$ and $\lambda \in (0,1)$.*

**Lemma 2.2.** *[9] Let $f : I \to \mathbb{R}$ be a twice differentiable function. If $f''(x) \geq$ (or $\leq$) $0$ for any $x \in I$, then $f$ is convex (or concave).*

**Definition 2.3.** *[3] Let $f : I \to \mathbb{R}$. We say that $f(x)$ is logarithmically concave on $I$ if for any $x, y \in I$,*

$$f\left(\frac{x}{a} + \frac{y}{b}\right) \geq (f(x))^{\frac{1}{a}}(f(y))^{\frac{1}{b}},$$

*where $a, b > 1$ and $\dfrac{1}{a} + \dfrac{1}{b} = 1$.*

**Lemma 2.3.** *[3] Let $f : I \to \mathbb{R}$. If $\ln(f(x))$ is concave, then $f(x)$ is logarithmically concave.*

**Definition 2.4.** *[1] We say that a function $M : (0, \infty) \times (0, \infty) \to (0, \infty)$ is a mean function if it satisfies each of the following.*

$(i)$ $M(x, y) = M(y, x)$.
$(ii)$ $M(x, x) = x$.
$(iii)$ $x < M(x, y) < y$ *for* $x < y$.
$(iv)$ $M(\lambda x, \lambda y) = \lambda M(x, y)$ *for* $\lambda > 0$.

**Example 1.** *[5] Below are examples of well-known mean functions.*

$(i)$ *Arithmetic mean:* $A(x, y) = \dfrac{x + y}{2}$.
$(ii)$ *Geometric mean:* $G(x, y) = \sqrt{xy}$.
$(iii)$ *Harmonic Mean:* $H(x, y) = \dfrac{2xy}{x + y}$.
$(iv)$ *Quadratic Mean:* $Q(x, y) = \sqrt{\dfrac{x^2 + y^2}{2}}$.

**Definition 2.5.** *[1] Let $f : I \to (0, \infty)$ be a continuous function. Let $M$ and $N$ be any two mean functions. Then we say that $f$ is $MN$-convex (or $MN$-concave) if*

$$f(M(x, y)) \leq (\text{or } \geq)\ N(f(x), f(y))$$

*for any $x, y \in I$.*

**Example 2.** *Below are examples of how we define $MN$-convex functions when $M$ and $N$ are arithmetic, geometric, and harmonic means.*

$(i)$ *$f$ is GG-convex on $I$ if for any $x, y \in I$,*

$$f(\sqrt{xy}) \leq \sqrt{f(x)f(y)}.$$

$(ii)$ $f$ *is* $AH$*-convex on* $I$ *if for any* $x, y \in I$,

$$f\left(\frac{x+y}{2}\right) \leq \frac{2f(x)f(y)}{f(x)+f(y)}.$$

$(iii)$ $f$ *is* $HH$*-convex on* $I$ *if for any* $x, y \in I$,

$$f\left(\frac{2xy}{x+y}\right) \leq \frac{2f(x)f(y)}{f(x)+f(y)}.$$

**Lemma 2.4.** *[1] Let* $f : I \to (0, \infty)$ *be a differentiable function. Then we have each of the following. In parts (iv)-(ix),* $I$ *is of the form* $(0, \beta)$*, where* $\beta \in (0, \infty)$.

$(i)$ $f(x)$ *is* $AA$*-convex (or* $AA$*-concave) on* $I$ *if and only if* $f'(x)$ *is increasing (or decreasing).*

$(ii)$ $f(x)$ *is* $AG$*-convex (or* $AG$*-concave) on* $I$ *if and only if* $\dfrac{f'(x)}{f(x)}$ *is increasing (or decreasing).*

$(iii)$ $f(x)$ *is* $AH$*-convex (or* $AH$*-concave) on* $I$ *if and only if* $\dfrac{f'(x)}{(f(x))^2}$ *is increasing (or decreasing).*

$(iv)$ $f(x)$ *is* $GG$*-convex (or* $GG$*-concave) on* $I$ *if and only if* $\dfrac{xf'(x)}{f(x)}$ *is increasing (or decreasing).*

$(v)$ $f(x)$ *is* $GA$*-convex (or* $GA$*-concave) on* $I$ *if and only if* $xf'(x)$ *is increasing (or decreasing).*

$(vi)$ $f(x)$ *is* $GH$*-convex (or* $GH$*-concave) on* $I$ *if and only if* $\dfrac{xf'(x)}{(f(x))^2}$ *is increasing (or decreasing).*

$(vii)$ $f(x)$ *is* $HA$*-convex (or* $HA$*-concave) on* $I$ *if and only if* $x^2 f'(x)$ *is increasing (or decreasing).*

$(viii)$ $f(x)$ *is* $HG$*-convex (or* $HG$*-concave) on* $I$ *if and only if* $\dfrac{x^2 f'(x)}{f(x)}$ *is increasing (or decreasing).*

$(ix)$ $f(x)$ *is* $HH$*-convex (or* $HH$*-concave) on* $I$ *if and only if* $\dfrac{x^2 f'(x)}{(f(x))^2}$ *is increasing (or decreasing).*

**Lemma 2.5.** *[2] Let* $\alpha \geq 0$ *and* $f : (\alpha, \infty) \to \mathbb{R}$. *If the function* $h_1(x) = \dfrac{f(x)-1}{x}$ *is increasing on* $(\alpha, \infty)$*, then the function* $h_2(x) = f(x^2)$ *satisfies the Grumbaum-type inequality*

$$1 + h_2(z) \geq h_2(x) + h_2(y),$$

*where $x, y \geq \alpha$ and $z^2 = x^2 + y^2$. If $h_1$ is decreasing, then the inequality is reversed.*

## 3. Main Results

**Theorem 3.1.** *If $e^{g(x)} + 1 - xg'(x) \geq$ (or $\leq$) $0$, then $\sigma(x)$ is sub-additive (or super-additive) on $\mathbb{R}$.*

*Proof.* We will only prove the first case. Note that

$$\left(\frac{\sigma(x)}{x}\right)' = \left(\frac{e^{g(x)}}{x(1+e^{g(x)})}\right)' = -\frac{e^{g(x)}(e^{g(x)} + 1 - xg'(x))}{x^2(1+e^{g(x)})^2}.$$

Also $e^{g(x)}$ and $x^2(1+e^{g(x)})^2$ are positive for any $x \in \mathbb{R}$. Thus for $\sigma(x)$ to be sub-additive, we require $e^{g(x)} + 1 - xg'(x) \geq 0$ from Lemma 2.1. □

**Corollary 3.1.** *We have each of the following.*

($i$) *If $g(x)$ is decreasing on $(0, \infty)$, then $\sigma(x)$ is sub-additive on $(0, \infty)$.*
($ii$) *If $g(x)$ is increasing on $(-\infty, 0)$, then $\sigma(x)$ is sub-additive on $(-\infty, 0)$.*

*Proof.* For (i), as $g'(x) \geq 0$ and $x \geq 0$, we obviously have $e^{g(x)} + 1 - xg'(x) \geq 0$. Thus $\sigma(x)$ is sub-additive on $(0, \infty)$ from Theorem 3.1. Part (ii) can be proven analogously. □

**Example 3.** *Let $g(x) = x^n$, where $n \in \mathbb{N}$. Then we have each of the following.*

($i$) *$\sigma(x)$ is sub-additive on $\mathbb{R}$ if and only if $1 \leq n \leq 3$.*
($ii$) *If $n$ is odd, then $\sigma(x)$ is sub-additive on $(-\infty, 0)$.*

From here, $I$, $I_1$, $I_2$ that we are considering will be open-intervals.

**Theorem 3.2.** *Define $I_1 = \{x \in \mathbb{R} : g(x) < 0\}$ and $I_2 = \{x \in \mathbb{R} : g(x) > 0\}$. If $I_1, I_2 \neq \emptyset$, then we have each of the following.*

($i$) *If $g(x)$ is convex on $I_1$, then $\sigma(x)$ is convex and $AA$-convex on $I_1$.*
($ii$) *If $g(x)$ is concave on $I_2$, then $\sigma(x)$ is concave and $AA$-concave on $I_2$.*

*Proof.* We will only prove (i). As $g(x)$ is convex, $g''(x) \geq 0$. Also, for $x \in I_1$, we have $e^{g(x)} \leq 1$. So

$$\sigma''(x) = \frac{e^{g(x)}(g''(x)(1+e^{g(x)}) + (g'(x))^2(1-e^{g(x)}))}{(1+e^{g(x)})^3} \geq 0$$

for any $x \in I_1$. Hence $\sigma(x)$ is convex on $I_1$. $\sigma(x)$ is then $AA$-convex consequently from Lemma 2.4. □

As a remark, the condition $I \neq \emptyset$ guarantees that we can construct $I$ which is an interval. If there is $x_0 \in \mathbb{R}$ such that $g(x_0) < 0$, then due to the continuity of $g$, there exists an open ball $B$ centered at $x_0$ such that $g(x) < 0$ for any $x \in B$.

**Example 4.** *The function* $\sigma(x) = \dfrac{1}{1+e^{2-(x+1)^2}}$ *is convex on* $(-1,0)$ *as* $g(x) = (x+1)^2 - 2$ *is convex and negative on* $(-1,0)$.

**Theorem 3.3.** *If* $g(x)$ *is concave for* $x \in \mathbb{R}$*, then* $\sigma(x)$ *is logarithmically concave.*

*Proof.* As $g(x)$ is concave, $g''(x) \leq 0$. Observe that

$$(\ln \sigma(x))'' = \left(\frac{\sigma'(x)}{\sigma(x)}\right)' = \frac{(1+e^{g(x)})g''(x) - (g'(x))^2(e^{g(x)})}{(1+e^{g(x)})^2} \leq 0$$

for any $x \in \mathbb{R}$. So $\ln(\sigma(x))$ is concave. Hence $\sigma(x)$ is logarithmically concave by Lemma 3.3. □

**Theorem 3.4.** *If* $g(x)$ *is concave for* $(0,\infty)$*, then* $\sigma(x)$ *is* $AG$*- and* $AH$*-concave.*

*Proof.* Being $AG$-concave is a direct consequence from Lemma 2.4 and Theorem 3.3. Now, as $g(x)$ is concave, we have $g''(x) \leq 0$. Note that

$$\left(\frac{\sigma'(x)}{(\sigma(x))^2}\right)' = \left(\frac{g'(x)}{e^{g(x)}}\right)' = \frac{g''(x) - (g'(x))^2}{e^{g(x)}} \leq 0$$

for any $x \in (0,\infty)$. So $\dfrac{\sigma'(x)}{(\sigma(x))^2}$ is decreasing. Hence $\sigma(x)$ is $AH$-concave from Lemma 2.4. □

From Theorems 3.5 to 3.7, we restrict $I$, $I_1$, and $I_2$ to be of the form $(0,\beta)$, where $\beta \in (0,\infty)$, as suggested in Lemma 2.4.

**Theorem 3.5.** *Define* $I = \{x \in (0,\infty) : xg'(x) < 1\}$. *If* $I \neq \emptyset$*, then we have each of the following.*

$(i)$ *If* $g(x)$ *is convex and increasing on* $I$*, then* $\sigma(x)$ *is* $GG$*- and* $GH$*-convex on* $I$.

$(ii)$ *If* $g(x)$ *is concave and decreasing on* $I$*, then* $\sigma(x)$ *is* $GG$*- and* $GH$*-concave on* $I$.

*Proof.* We will only prove (i). As $g(x)$ is convex and increasing, $g''(x) \geq 0$ and $g'(x) \geq 0$. So

$$\left(\frac{x\sigma'(x)}{\sigma(x)}\right)' = \left(\frac{xg'(x)}{1+e^{g(x)}}\right)'$$
$$= \frac{xg''(x) + g'(x) + xe^{g(x)}g''(x) + e^{g(x)}g'(x)(1 - xg'(x))}{(1+e^{g(x)})^2} \geq 0$$

and

$$\left(\frac{x\sigma'(x)}{(\sigma(x))^2}\right)' = \left(\frac{xg'(x)}{e^{g(x)}}\right)' = \frac{xg''(x) + g'(x)(1 - xg'(x))}{e^{g(x)}} \geq 0$$

for any $x \in I$. So $\frac{x\sigma'(x)}{\sigma(x)}$ and $\frac{x\sigma'(x)}{(\sigma(x))^2}$ are increasing. Hence $\sigma(x)$ is $GG$- and $GH$-convex on $I$ from Lemma 2.4. □

**Theorem 3.6.** *Define* $I_1 = \{x \in (0,\infty) : g(x) < 0\}$ *and* $I_2 = \{x \in (0,\infty) : g(x) > 0\}$. *If* $I_1, I_2 \neq \emptyset$, *then we have each of the following.*

- $(i)$ *If* $g(x)$ *is convex and increasing on* $I_1$, *then* $\sigma(x)$ *is* $GA$- *and* $HA$-*convex on* $I_1$.
- $(ii)$ *If* $g(x)$ *is concave and decreasing on* $I_2$, *then* $\sigma(x)$ *is* $GA$- *and* $HA$-*concave on* $I_2$.

*Proof.* We will only prove (i). As $g(x)$ is convex and increasing, $g''(x) \geq 0$ and $g'(x) \geq 0$. Also, for $x \in I_1$, we have $e^{g(x)} \leq 1$. So

$$(x\sigma'(x))' = \left(\frac{xe^{g(x)}g'(x)}{(1+e^{g(x)})^2}\right)'$$
$$= \frac{e^{g(x)}(x(g''(x)(1+e^{g(x)}) + (g'(x))^2(1-e^{g(x)})) + g'(x)(1+e^{g(x)}))}{(1+e^{g(x)})^3} \geq 0$$

and

$$(x^2\sigma'(x))' = \left(\frac{x^2e^{g(x)}g'(x)}{(1+e^{g(x)})^2}\right)'$$
$$= \frac{e^{g(x)}x(x(g''(x)(1+e^{g(x)}) + x(g'(x))^2(1-e^{g(x)})) + 2g'(x)(1+e^{g(x)}))}{(1+e^{g(x)})^3} \geq 0$$

for any $x \in I_1$. So $x\sigma'(x)$ and $x^2\sigma'(x)$ are increasing. Hence $\sigma(x)$ is $GA$- and $HA$-convex by on $I_1$ from Lemma 2.4. □

**Theorem 3.7.** *Define* $I = \{x \in (0,\infty) : xg'(x) < 2\}$. *If* $I \neq \emptyset$, *then we have each of the following.*

($i$) *If* $g(x)$ *is convex and increasing on* $I$, *then* $\sigma(x)$ *is* $HG$- *and* $HH$-*convex on* $I$.

($ii$) *If* $g(x)$ *is concave and decreasing on* $I$, *then* $\sigma(x)$ *is* $HG$- *and* $HH$-*concave on* $I$.

*Proof.* We will only prove (i). As $g(x)$ is convex and increasing, $g''(x) \geq 0$ and $g'(x) \geq 0$. So

$$\begin{aligned}\left(\frac{x^2\sigma'(x)}{\sigma(x)}\right)' &= \left(\frac{x^2 g'(x)}{1+e^{g(x)}}\right)' \\ &= \frac{x^2 g''(x) + 2xg'(x) + x^2 e^{g(x)} g''(x) + xe^{g(x)} g'(x)(2 - xg'(x))}{(1+e^{g(x)})^2} \geq 0\end{aligned}$$

and

$$\left(\frac{x^2\sigma'(x)}{(\sigma(x))^2}\right)' = \left(\frac{x^2 g'(x)}{e^{g(x)}}\right)' = \frac{x(g''(x) + g'(x)(2 - xg'(x)))}{e^{g(x)}} \geq 0$$

for any $x \in I$. So $\dfrac{x^2\sigma'(x)}{\sigma(x)}$ and $\dfrac{x^2\sigma'(x)}{(\sigma(x))^2}$ are increasing. Hence $\sigma(x)$ is $HG$-convex on $I$ from Lemma 2.4. □

**Theorem 3.8.** *If* $g(x)$ *is increasing, then* $\sigma(x)$ *satisfies the Grumbaum-type inequality*

$$1 + \sigma(z^2) \geq \sigma(x^2) + \sigma(y^2),$$

*where* $x, y \in (0,\infty)$ *and* $z^2 = x^2 + y^2$.

*Proof.* Note that $g'(x) \geq 0$ as $g(x)$ is increasing. Define $\vartheta : (0,\infty) \to \mathbb{R}$ by

$$\vartheta(x) = \frac{\sigma(x) - 1}{x} = -\frac{1}{x(1+e^{g(x)})}.$$

Then we have

$$\vartheta'(x) = \frac{1}{x^2(1+e^{g(x)})} + \frac{e^{g(x)} g'(x)}{x(1+e^{g(x)})^2} \geq 0.$$

This means $\vartheta(x)$ is increasing. Hence by Lemma 2.5, we obtain the desired result. □

**Theorem 3.9.** *Define* $\xi(x) = \dfrac{\sigma(x+1)}{\sigma(x)}$ *for* $x \in \mathbb{R}$. *If* $g(x)$ *is concave, then* $\xi(x)$ *is decreasing.*

*Proof.* As $g(x)$ is concave, we have $g''(x) \leq 0$. Note that

$$\left(\frac{\sigma'(x)}{\sigma(x)}\right)' = \left(\frac{g'(x)}{1+e^{g(x)}}\right)' = \frac{(1+e^{g(x)})g''(x) - (g'(x))^2(e^{g(x)})}{(1+e^{g(x)})^2} \leq 0$$

for any $x \in \mathbb{R}$. Thus the function $\dfrac{\sigma'(x)}{\sigma(x)}$ is decreasing on $\mathbb{R}$. Now, observe that

$$\xi'(x) = \frac{\sigma(x+1)}{\sigma(x)}\left(\frac{\sigma'(x+1)}{\sigma(x+1)} - \frac{\sigma'(x)}{\sigma(x)}\right) \leq 0$$

as $\dfrac{\sigma'(x)}{\sigma(x)}$ is decreasing. Since $\sigma(x)$ is always positive, hence $\xi(x)$ is also decreasing. $\square$

**Theorem 3.10.** *Define* $\varrho(x) = \ln(1+e^{g(x)}) - \sigma(x)$ *for* $x \in \mathbb{R}$. *If* $g(x)$ *is increasing (or decreasing), then* $\varrho(x) - \sigma(x)$ *is increasing (or decreasing).*

*Proof.* We will only prove the first case. As $g(x)$ is increasing, we have $g'(x) \geq 0$. Note that

$$(\varrho(x) - \sigma(x))' = \frac{e^{g(x)}g'(x)}{(1+e^{g(x)})} - \frac{e^{g(x)}g'(x)}{(1+e^{g(x)})^2} = \frac{(e^{g(x)})^2 g'(x)}{(1+e^{g(x)})^2} \geq 0$$

for any $x \in \mathbb{R}$. Hence $\varrho(x) - \sigma(x)$ is increasing. $\square$

## 4. Discussion and Conclusion

In this paper, we have presented many inequalities involving a generalized sigmoid function. We can see that the convexity and concavity of $\sigma(x)$ mostly depend on $g(x)$. These results could be used for modelling in various fields. For future research, we could consider applying the sigmoid function to a complex domain.

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