J COMP SCI
APPL MATH
Journal of Computer Science and Applied Mathematics
Vol. 6, no.1, (2024), 13–31
ISSN : 1857-9582 https://doi.org/10.37418/jcsam.6.1.2

NEW APPROACH TO PROXIMITY ANALYSIS BETWEEN TWO HORIZONTAL MULTI-TABLES: DSCIA2 METHOD

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RÉSUMÉ. In this paper, we propose a new method called Dual Simultaneous Co-Inertia Analysis Type 2 (DSCIA2), which is both a new approach to DSCIA in the context of two horizontal multi-tables and Dual of SCIA2 method. The aim is to study the variability of the proximity between pairs of tables with the same variables, by constructing the common axes of representation of the variables and the individuals simultaneously, i.e. constructing in one go an orthogonal matrix containing the solutions. An example of this method consisting being studied of the compared economic evolution of the countries of the Economic Monetary Community of Central Africa (EMCCA) and the West African Economic Monetary Union(WAEMU) is given.

1. INTRODUCTION

The DSTATICO method (Combination of Dual Co-Inertia Analysis (DCIA, Kissita et al 2017) and the Partial Triadic Analysis (PTA, Thioulouse and Chessel (1987)) by Makany et al (2017) has made it possible to establish the proximities between the series of pairs of tables having the variables in common (two horizontal multi-tables). In other words, it is a question of studying, on the one hand, the evolution in time or in space of the proximities between two groups

Key words and phrases. Horizontal multitables, Co-inertia analysis, Dual Co-inertia analysis, EMCCA, WAEMU.
Submitted : 18.03.2024 ; *Accepted :* 03.04.2024 ; *Published :* 06.05.2024.

of individuals relating to the same variables and, on the other hand, to analyse the proximities between individuals within each group of variables and to study how these evolve from one group of variables to another. Recently, in the same context as DSTATICO, Kossa-Kongbowali (2019) proposed some methods (DsCIA 1.3 and DsOCIA 1.3), which search for solutions to problems in a successive (step-by-step) manner, which could lead to a gradual loss of some information. To overcome this problem, Kossa-Kongbowali et al (2020) proposed a simultaneous approach (DSCIA). In this paper, we will propose a new approach of DSCIA named DSCIA2 which is Dual of SCIA2 method (Kossa-Kongbowali, 2023) allows to find the solutions simultaneously, i.e. to build in one shot an orthogonal matrix containing the solutions. An application of this method is presented, with a view to understanding the proximities between EMCCA and WAEMU countries. These data have already been used by Makany et al (2017) and by Kossa-Kongbowali (2019, 2020).

2. Recall of Dual Simultaneous Co-Inertia Analysis : DSCIA

2.1. **Data and notations.** In this subsection, we present the method of simultaneous factorial analysis of pairs of tables with the same variables (two horizontal multi-tables). But first, we will define the context of the methods. A' designates the transpose of the matrix A.

The context of the dual methods is that where we have two horizontal multi-tables $X = [X_1|X_2|\dots|X_M]$ and $Y = [Y_1|Y_2|\dots|Y_M]$ whose sub-tables X_i and Y_i are respectively of format (n^X, p_i) and (n^Y, p_i), where the p_i variables are measured on the n^X individuals of the table X_i and n^Y individuals of the table Y_i.

In this context, the tables X_i and Y_i $(i = 1, \cdots, M)$ are respectively defined on the products $I^X \times J_i$ and $I^Y \times J_i$, or else, the variables for each pair of the tables (X_i, Y_i) are the same but the individuals are different. We denote by n^X the cardinal of I^X and by n^Y that of I^Y. We denote by p_i the cardinal of J_i.

Consider the first M statistical triplets (X_i, Q_i, D^X) where X_i is an array of dimension (n_X, p_i) and the second M statistical triplets (Y_i, Q_i, D^Y) where :

- Y_i is an array of dimension (n^Y, p_i).
- Q_i is a metric defined in the space of individuals $\mathbb{R}^{p_i}$.

- D^X is the metric of the weights of the individuals defined in $\mathbb{R}^{n^X}$.
- D^Y is the metric of the weights of the individuals defined in $\mathbb{R}^{n^Y}$.
- $Q_{bd} = diag(Q_i/i = 1, \cdots, M)$ the block-diagonal metric of the individuals defined in $\mathbb{R}^p$ of the metrics Q_i with $p = \sum_{i=1}^{M} p_i$.

The horizontal multi-table X is the concatenation of the arrays X_i which are centred and possibly reduced. The multi-table Y is also the concatenation of the arrays Y_i. which are also centred and possibly reduced. We can also weight the arrays X_i and Y_i by the weights π_i such that $\pi_i > 0$.

$\Delta_\Pi = diag(\pi_i Id_{p_i}/i = 1, \cdots, M)$ is the block-diagonal matrix of the products of weights π_i of the arrays X_i and Y_i with the identity matrices Id_{p_i}.

We note here by $X_c = X\Delta_\Pi^{\frac{1}{2}} = [\sqrt{\pi_1}X_1|\ldots|\sqrt{\pi_i}X_i|\ldots|\sqrt{\pi_M}X_M]$ and $Y_c = Y\Delta_\Pi^{\frac{1}{2}} = [\sqrt{\pi_1}Y_1|\ldots.|\sqrt{\pi_i}Y_i|\ldots|\sqrt{\pi_M}Y_M]$ the horizontal multi-tables associated respectively with the horizontal multi-tables X and Y.

Let $T_i = W_{Y_iX_i} = Y_iQ_iX_i'$ (see Kissita, 2017) be the matrix of scalar inter-products between individuals of Y_i and those of X_i of dimension $n^Y \times n^X$ for all $i = 1, \cdots, M$.

$W_{X_i} = X_iQ_iX_i'$ (respectively $W_{Y_i} = Y_iQ_iY_i'$) the matrix of scalar products of the array X_i (respectively the array Y_i).

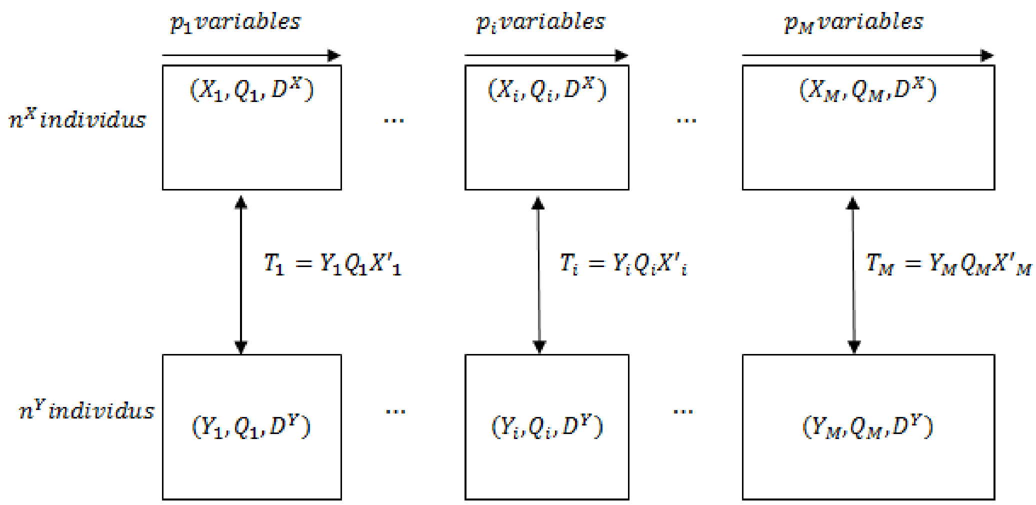


FIGURE 1. Context of two horizontal multi-tables

2.2. **Dual Simultaneous Co-Inertia Analysis : DSCIA.** The Dual Simultaneous Co-Innertia Analysis (DSCIA) was proposed by Kossa-Kongbowali et al (2020). The aim is to study the stability of the proximity between pairs of tables $\begin{pmatrix} X_i \\ Y_i \end{pmatrix}$, $i = 1, \dots, M$

We note $A_{X_i'} = X_i' D^X U_X$ and $A_{Y_i'} = Y_i' D^Y U_Y$, the matrices containing in columns the linear combinations of the individuals of the table X_i and Y_i respectively of format (p_i, r) where U_X and U_Y are matrices associated with the two groups of individuals and containing respectively in columns the axes $u_X^{(s)}$ and $u_Y^{(s)}$ $(s = 1, \dots, r)$ with $r = \min(n^X, n^Y)$.

DSCIA is the search for the matrices U_X and U_Y maximizing the function

$$f(U_X, U_Y) = \sum_{i=1}^{M} \pi_i ||diag(U_X' D^X T_i D^Y U_Y)||^2$$

under the constraints

$$U_X' D^X U_X = U_Y' D^Y U_Y = I_r$$

.

The function can still be written as

$$\text{Maximise} \quad f(U_X, U_Y) = \sum_{i=1}^{M} \pi_i tr[(U_X' D^X T_i D^Y U_Y) diag(U_X' D^X T_i D^Y U_Y)]$$

under the constraints

$$U_X' D^X U_X = U_Y' D^Y U_Y = I_r,$$

where U_X and U_Y are of format (n^X, r) and (n^Y, r) respectively.

Note that we can still write

$$\begin{aligned} f(U_X, U_Y) &= \sum_{i=1}^{M} \sum_{s=1}^{r} \pi_i (u_X^{(s)'} D^X X_i Q_i Y_i' D^Y u_Y^{(s)})^2 \\ &= \sum_{i=1}^{M} \sum_{s=1}^{r} \pi_i < X_i D^X u_X^{(s)} / Y_i' D^Y u_Y^{(s)} >_{Q_i}^2 . \end{aligned}$$

Maximising the function f under the constraints requires the construction of updates U_X^* and U_Y^* of U_X and U_Y respectively. To do this, an iterative algorithm associated with the function f and the constraints is given. We first make a change of variables to get back to the conditions of Cliff's theorem (1966).

Let $B_X = (D^X)^{\frac{1}{2}}U_X$ and $B_Y = (D^Y)^{\frac{1}{2}}U_Y$ or $U_X = (D^X)^{-\frac{1}{2}}B_X$ et $U_Y = (D^Y)^{-\frac{1}{2}}B_Y$. Thus the function f to be maximized becomes :

$$\text{Maximize} \quad f(B_X, B_Y) = \sum_{i=1}^{M} \pi_i tr[(B'_X(D^X)^{\frac{1}{2}}T_i(D^Y)^{\frac{1}{2}}B_Y)diag(B'_X(D^X)^{\frac{1}{2}}T_i(D^Y)^{\frac{1}{2}}B_Y)]$$

under the constraints

$$B'_X B_X = B'_Y B_Y = I_r.$$

After centring and reducing the arrays X_i and Y_i, we apply the following algorithm :

2.2.1. *The algorithm.*

1) Choose arbitrarily B_X and B_Y such that $B'_X B_X = B'_Y B_Y = I_r$ and ε (example $\varepsilon = 0.00001$).
2) Determine the update of B_X.
   a. Calculate $T_{B_X B_Y} = \sum_{i=1}^{M} \pi_i((D^X)^{\frac{1}{2}}T'_i(D^Y)^{\frac{1}{2}}B_Y)diag(B'_X(D^X)^{\frac{1}{2}}T'_i(D^Y)^{\frac{1}{2}}B_Y)$.
   b. Decompose into singular values of $T_{B_X B_Y} = P\Lambda S'$ where $P'P = S'S = SS' = I_r$.
   c. Apply the update for B_X, the matrix $B_X^* = PS'$.
3) Determine the update of B_Y.
   a. Calculate $H_{B_X B_Y} = \sum_{i=1}^{M} \pi_i((D^Y)^{\frac{1}{2}}T_i(D^X)^{\frac{1}{2}}B_X)diag(B'_X(D^X)^{\frac{1}{2}}T'_i(D^Y)^{\frac{1}{2}}B_Y)$ where $B_X = B_X^*$ is the update of B_X in the previous step.
   b. Decompose into singular values of $H_{B_X B_Y} = L\Delta R'$ where $L'L = R'R = RR' = I_r$.
   c. Ask for the update of B_Y, the matrix $B_Y^* = LR'$.
4) If $f(B_X^*, B_Y^*) - f(B_X, B_Y) \leq \varepsilon$, then the algorithm converges, otherwise we set $B_X = B_X^*$ and $B_Y = B_Y^*$ and go to 2).
5) Calculate $U_X^* = (D^X)^{-\frac{1}{2}}B_X$ and $U_Y^* = (D^Y)^{-\frac{1}{2}}B_Y$.
6) Calculate $A_{X'_i} = X'_i D^X U_X^*$ and $A_{Y'_i} = Y'_i D^Y U_Y^*$.

3. Dual Simultaneous Co-Inertia Analysis Type 2 : DSCIA2

The context here is the same as that of the Dual Simultaneous Co-Innertia Analysis (DSCIA). In this section, we propose another version of DSCIA, which we call Simultaneous Co-Innertia Analysis Type 2 (DSCIA2). The aim is to study the stability of the proximity between the pairs of tables $\begin{pmatrix} X_i \\ Y_i \end{pmatrix}$, $i = 1, \ldots, M$.

We note $A_{X_i'} = X_i' D^X U_X$ and $A_{Y_i'} = Y_i' D^Y U_Y$, the matrices containing in columns the linear combinations of the individuals of the table X_i and Y_i respectively of format (p_i, r) where U_X and U_Y are matrices associated with the two groups of individuals and containing respectively in columns the axes $u_X^{(s)}$ and $u_Y^{(s)}$ $(s = 1, \ldots, r)$ with $r = \min(n^X, n^Y)$.

3.1. **Definition.** DSCIA2 is the search for the matrices U_X and U_Y maximising the function

$$f(U_X, U_Y) = \sum_{i=1}^{M} tr^2(U_X' D^X T_i D^Y U_Y),$$

under the constraints

$$U_X' D^X U_X = U_Y' D^Y U_Y = I_r.$$

3.2. **Solution.** Maximising the function f under the constraints requires the construction of updates U_X^* and U_Y^* of U_X and U_Y respectively. To do this, an iterative algorithm associated with the function f and the constraints is given. We first make a change of variables to get back to the conditions of Cliff's theorem (1966).

Let $B_X = (D^X)^{\frac{1}{2}} U_X$ and $B_Y = (D^Y)^{\frac{1}{2}} U_Y$ or $U_X = (D^X)^{-\frac{1}{2}} B_X$ and $U_Y = (D^Y)^{-\frac{1}{2}} B_Y$. Thus the function f to be maximized becomes :

$$\text{Maximize} \quad f(B_X, B_Y) =$$
$$\sum_{i=1}^{M} \pi_i tr[(B_X'(D^X)^{\frac{1}{2}} T_i (D^Y)^{\frac{1}{2}} B_Y) diag(B_X'(D^X)^{\frac{1}{2}} T_i (D^Y)^{\frac{1}{2}} B_Y)]$$

under the constraints

$$B_X' B_X = B_Y' B_Y = I_r.$$

After centring and reducing the arrays X_i and Y_i, we apply the following algorithm :

3.2.1. *The algorithm.*

1) Choose arbitrarily B_X and B_Y such that $B_X' B_X = B_Y' B_Y = I_r$ and ε (example $\varepsilon = 0,00001$).

2) Determine the update of B_X.
   a. Calculate $T_{B_X B_Y} = E^{B_X B_Y} B_Y$.
   b. Do the singular value decomposition of $T_{B_X B_Y} = P'P = S'S = SS' = I_r$.
   c. Posit the update of B_X, the matrix $B_X^* = PS'$.
3) Determine the update of B_Y.
   a. Calculate $F_{B_X B_Y} = (E^{B_X B_Y})' B_X$ where $B_X = B_X^*$ is the update of B_X in the previous step.
   b. Do the singular value decomposition of $F_{B_X B_Y} = L\Delta R'$ where $L'L = R'R = RR' = I_r$.
   c. Posit the update of B_Y, the matrix $B_Y^* = LR'$.
4) If $f(B_X^*, B_Y^*) - f(B_X, B_Y) \leq \varepsilon$, then the algorithm converges, otherwise we pose $B_X = B_X^*$ and $B_Y = B_Y^*$ and go to 2.
5) Compute $U_X^* = (D^X)^{-\frac{1}{2}} B_X$ and $U_Y^* = (D^Y)^{-\frac{1}{2}} B_Y$.
6) Calculate $A_{X_i'} = X_i' D^X U_X^*$ and $A_{Y_i'} = Y_i' D^Y U_Y^*$.

3.2.2. *Monotony.* We show the monotonicity of the algorithm only with respect to U_X, knowing that the monotonicity of U_Y is demonstrated in the same way by only exchanging the roles of U_X and U_Y. It is thus a question of showing that

$$f(U_X, U_Y) \leq f(U_X^*, U_Y) \leq f(U_X^*, U_Y^*).$$

To do this, it is sufficient to show that

$$f(U_X, U_Y) \leq f(U_X^*, U_Y).$$

Indeed, according to Cliff (1966), we have, from the update U_X^* of U_X the relation

$$tr[U_X' \sum_{i=1}^{M} tr(U_X' K_i U_Y) K_i U_Y] \leq tr[U_X^{*'} \sum_{i=1}^{M} tr(U_X' K_i U_Y) K_i U_Y], \tag{3.1}$$

or

$$tr[U_X' tr(U_X' K_i U_Y) K_i U_Y] \leq tr[U_X^{*'} tr(U_X' K_i U_Y) K_i U_Y], \tag{3.2}$$

or again

$$tr(U_X' K_i U_Y) tr(U_X' K_i U_Y) \leq tr(U_X^{*'} K_i U_Y) tr(U_X' K_i U_Y). \tag{3.3}$$

If $tr(U'_X K_i U_Y) > 0$ for all $i = 1, \dots, M$, then

$$0 < tr(U'_X K_i U_Y) \leq tr(U^{*'}_X K_i U_Y). \tag{3.4}$$

Multiplying (3.4) by $tr(U^{*'}_X K_i U_Y)$, we have :

$$tr(U^{*'}_X K_i U_Y) tr(U'_X K_i U_Y) \leq tr(U^{*'}_X K_i U_Y) tr(U^{*'}_X K_i U_Y).$$

According to (3.3), we have :

$$tr(U'_X K_i U_Y) tr(U'_X K_i U_Y) \leq tr(U^{*'}_X K_i U_Y) tr(U^{*'}_X K_i U_Y),$$

or

$$tr(U'_X tr(U'_X K_i U_Y) K_i U_Y) \leq tr(U^{*'}_X tr(U^{*'}_X K_i U_Y) K_i U_Y).$$

Summing with respect to i, we have :

$$tr[U'_X \sum_{i=1}^{M} tr(U'_X K_i U_Y) K_i U_Y] \leq tr[U^{*'}_X \sum_{i=1}^{M} tr(U^{*'}_X K_i U_Y) K_i U_Y].$$

Therefore

$$f(U_X, U_Y) \leq f(U^*_X, U_Y).$$

If $tr(U'_X K_i U_Y) < 0$ for all $i = 1, \dots, M$ then we have :

$$tr(U^{*'}_X K_i U_Y) \leq tr(U'_X K_i U_Y) < 0. \tag{3.5}$$

Multiplying (3.5) by $tr(U^{*'}_X K_i U_Y)$, we have :

$$tr(U^{*'}_X K_i U_Y) tr(U'_X K_i U_Y) \leq tr(U^{*'}_X K_i U_Y) tr(U^{*'}_X K_i U_Y).$$

According to (3.3), we have :

$$tr(U'_X K_i U_Y) tr(U'_X K_i U_Y) \leq tr(U^{*'}_X K_i U_Y) tr(U^{*'}_X K_i U_Y),$$

or

$$tr(U'_X tr(U'_X K_i U_Y) K_i U_Y) \leq tr(U^{*'}_X tr(U^{*'}_X K_i U_Y) K_i U_Y).$$

Summing with respect to i, we have :

$$tr[U'_X \sum_{i=1}^{M} tr(U'_X K_i U_Y) K_i U_Y] \leq tr[U^{*'}_X \sum_{i=1}^{M} tr(U^{*'}_X K_i U_Y) K_i U_Y].$$

This allows us to write $f(U_X, U_Y) \leq f(U^*_X, U_Y)$. Hence the monotony of the algorithm with respect to U_X.

3.2.3. *Comments.*
The matrices U_X^* and U_Y^* found, verify the constraints $U_X^{*\prime} D^X U_X^* = U_Y^{*\prime} D^Y U_Y^* = I_r$.Thus, the systems of axes $\{u_X^{(s)}\}_s$ and $\{u_Y^{(s)}\}_s$ are D^X-orthonormal and D^Y-orthonormal in $(\mathbb{R}^{n^X})$ and $(\mathbb{R}^{n^Y})$. The matrices $A_{X_i'} = X_i' D^X U_X^*$ contain the coordinates of the variables in the X_i arrays and the matrices $A_{Y_i'} = Y_i' D^Y U_Y^*$ contain the coordinates of the variables in the Y_i array. Thus, the variables of two tables can be represented in the reference frame defined by these co-inertia axes. Moreover, the partial component systems $\{a_{X_i'}^{(s)}\}_{s=1\dots r}$ and $\{a_{Y_i'}^{(s)}\}_{s=1\dots r}$ are not Q_i-orthogonal. To represent the individuals X_i, we use the technique of additional elements, i.e. we project the rows of the tables T_i on the first two columns of the matrix U_Y^*. For the individuals in the Y_i tables, the columns of the T_i tables, i.e. the rows of the T_i' tables are projected onto the first two columns of the U_X^* matrix. The specific weights are determined by the quantities $\rho_{X_i} = u_X' D^X W_{X_i} D^X u_X$ and $\rho_{Y_i} = u_Y' D^Y W_{Y_i} D^Y u_Y$, which are projected inertias of the clouds of variables associated with the X_i and Y_i arrays onto the $u_{X,s}$ and $u_{Y,s}$ axes respectively.

On the other hand, the quantities defined by

$$\rho_{X_i Y_i} = \frac{(u_X' D^X W_{X_i Y_i} D^Y u_Y)^2}{(u_X' D^X W_{X_i} D^X u_X)(u_Y' D^Y W_{Y_i} D^Y u_Y)},$$

which are the coefficients of proximities, allow to estimate the stability of the proximity between the tables X_i and Y_i in time and space.

4. Application

4.1. **The data.** The data that are analysed in this article are published by the Bank of Central African States (BEAC). They were used in the application of the DSTATICO method (see Makany et al, 2017) and the DsOCIA 1,3 and DSCIA methods (see Kossa-Kongbowali et al, 2019;2020). They concern the economic evolution of eight countries of the West African Economic and Monetary Union (WAEMU) (Benin, Burkina-Faso, Ivory Coast, Guinea-Bissau, Mali, Niger, Senegal, Togo) and six countries of the Economic and Monetary Community of central Africa (EMCCA) (Cameroon, Central African Republic, Congo, Gabon, Equatorial Guinea, Chad) in the 10-year period from 2003 to 2012. Seventeen macroeconomic variables were measured for each country (Table 1). We dispose thus of two horizontal multi-tables, each consisting of ten tables with the same variables. The rows of the first multi-table are the eight WAEMU countries, while those of the second multi-table are the six EMCCA countries.

TABLE 1. List of the variables and abbreviations

Variables	Abbreviations
Nominal Gross Domestic Product (Mds FCFA)	GDP
Real Growth Rate (in %)	RGR
Changes in the Consumer Price Index	CPI
Exports of Goods (Mds FCFA)	EG
Imports of Goods (Mds FCFA)	IG
Trade Balance (Mds FCFA)	TB
Exterior Current Account Balance (Mds FCFA)	ECAB
Budgetary Balance (Mds FCFA)	BB
Budgetary Balance (in % of GDP)	BBG
Investment Rate (in % of GDP)	IR
Total External Debt (Mds USD)	TED
Total External Debt (in % of GDP)	TEDG
Net Foreign Assets (Mds FCFA)	NFA
Credits for the Economy (Mds FCFA)	CE
Net Claim on the States (Mds FCFA)	NCS
Money Supply (Mds FCFA)	MS
Money Supply (in % of GDP)	MSG

TABLE 2. Projected inertias (specific weights) for each year on the first two axes.

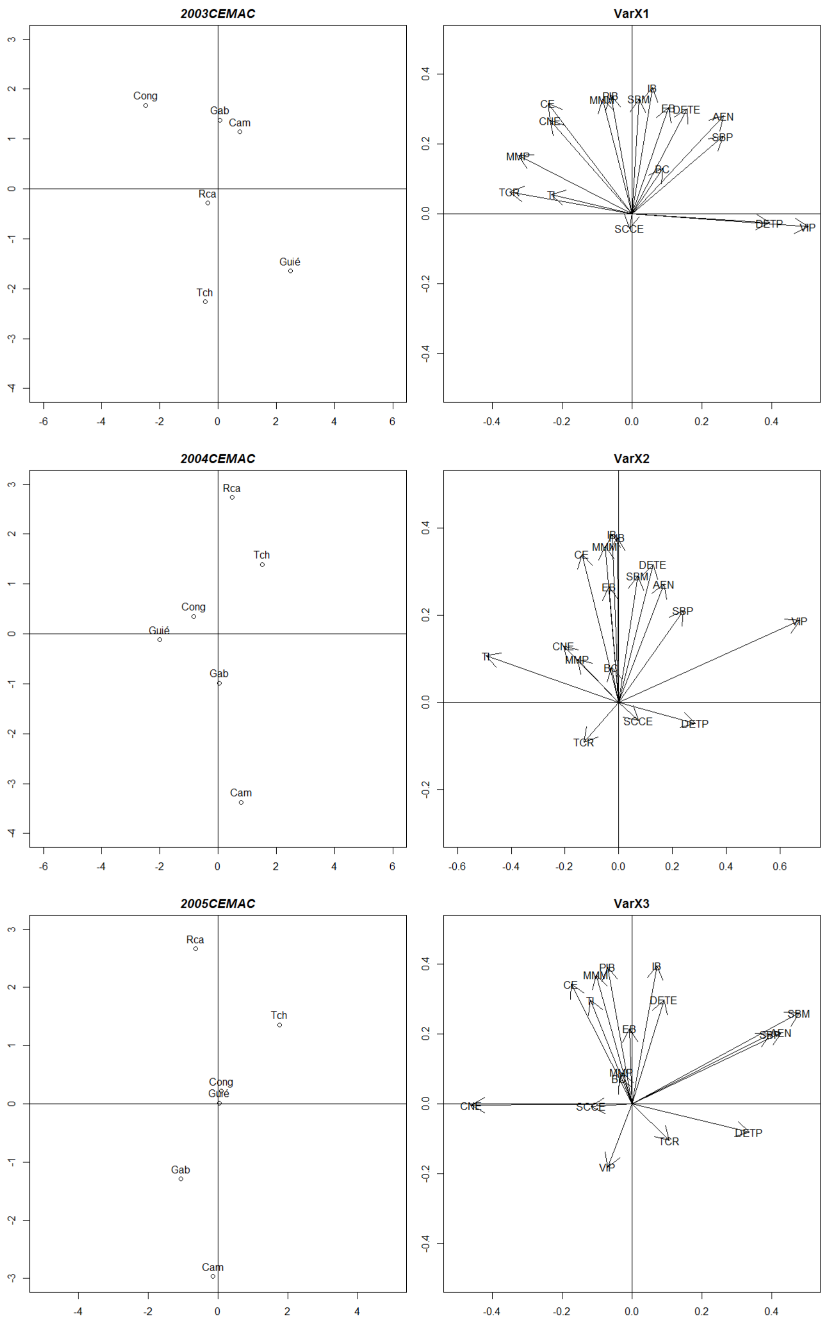
Years / Method	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
DSCIA2 (X)	0.46	0.37	0.30	0.57	0.76	0.77	0.55	0.85	0.95	0.87
	2.53	2.82	2.92	3.38	2.61	3.56	2.95	2.92	3.82	3.15
DSCIA2 (Y)	0.32	0.23	0.19	0.34	0.23	0.32	0.51	0.27	0.22	0.34
	6.14	5.33	4.65	4.92	5.54	5.04	4.62	4.05	5.06	3.32

TABLE 3. The proximity coefficients between linear combinations of individuals in the tables associating the EMCCA and WAEMU countries for DSCIA2.

Method \ Years	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
DSCIA2	0.65	0.28	0.39	0.01	0.02	0.00	0.00	0.00	0.03	0.01
	0.37	0.42	0.19	0.20	0.13	0.33	0.20	0.28	0.05	0.14

4.2. **Data Analysis.** Given the differences in the dispersion of the variables, we opted to work with centred and scale data within each table. In addition, we have adopted the same weighting for each individual for each group of countries. A uniform weighting is chosen for each table. Table 2 contains the projected inertia of each year on the first two axes. According to the DSCIA2 method and for EMCCA, there are 3 groups of years on the first axis : First, from 2003 to 2005, then from 2007 to 2008 and finally, from 2010 to 2012. With respect to the second axis, the inertia of all years is roughly in the same order. For WAEMU, there are also 3 groups of years with respect to the first axis : First, from 2003 to 2004, then from 2006 to 2008 and finally from 2010 to 2012. With respect to the second axis, the inertia of all years is roughly in the same order, except for the years 2003 and 2012. In Table 3, we present the coefficients of proximity between linear combinations of individuals in the tables associating the EMCCA and WAEMU countries for the DSCIA2. The proximity between a pair of tables will be strong if the coefficient of proximity between linear combinations is close to 1 and weak if the coefficient between linear combinations is close to 0. These coefficients of proximities make it possible to describe the evolution of the EMCCA-WAEMU proximity. A constancy of these coefficients of proximities makes it possible to conclude to the stability of the proximity. Table 3 reveals that there is no stability of proximity between the two groups of countries during almost all the years in step 1 and 2 for DSCIA2. In the same systems of axes from DSCIA2, we have represented the positions of the EMCCA countries (Figures 2 to 4) and the WAEMU (Figures 5 to 7). We note that axis 2 of DSCIA2 plays the same role as axis 1 of the DsOCIA1,3 methods. There is stability in dimension 2 for the EMCCA group, unfortunately this is not the case for the

WAEMU group. DSCIA2 also "destabilises" the WAEMU group like DSCIA. For the EMCCA, Axis 2 opposes the CAR and Cameroon for most of the period of our study.



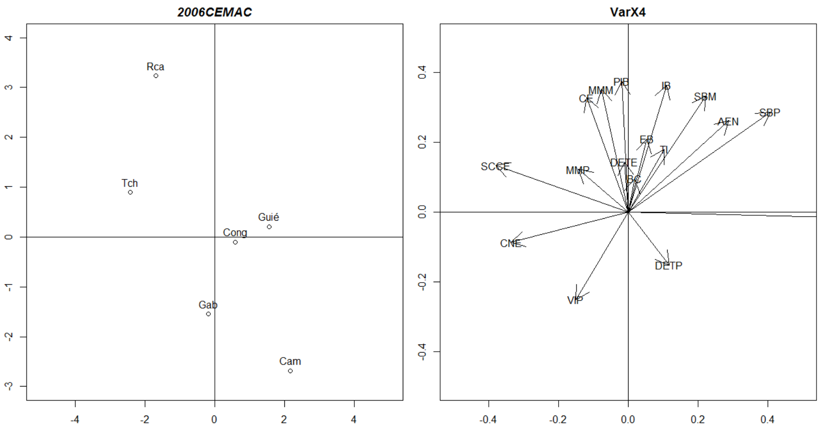
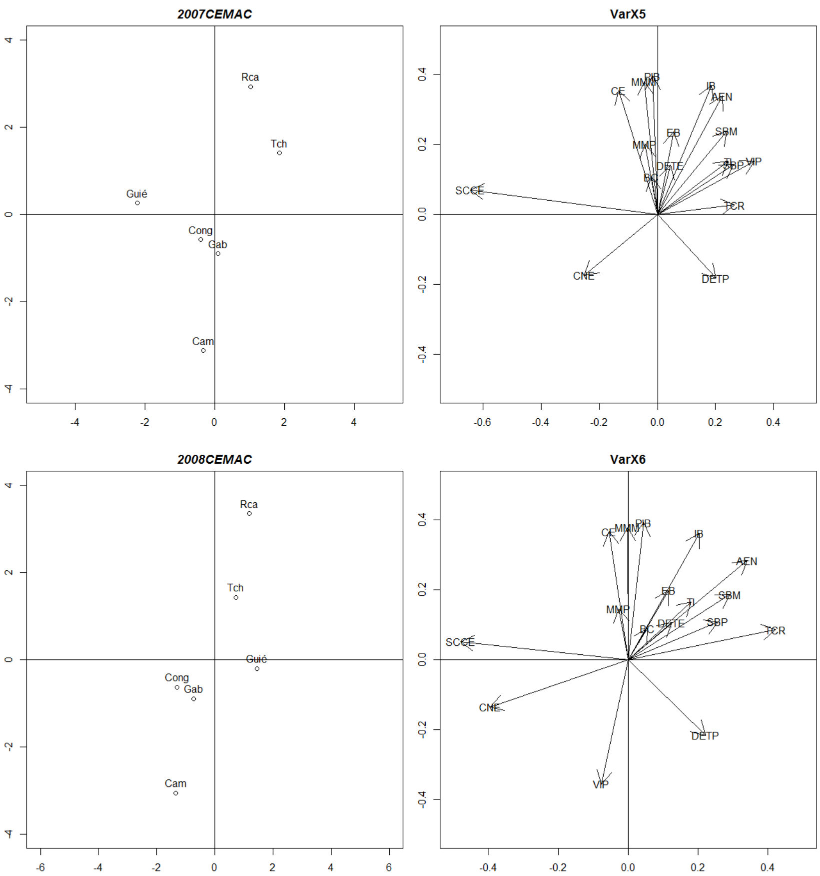


FIGURE 2. Countries of the EMCCA and Macroeconomics variables



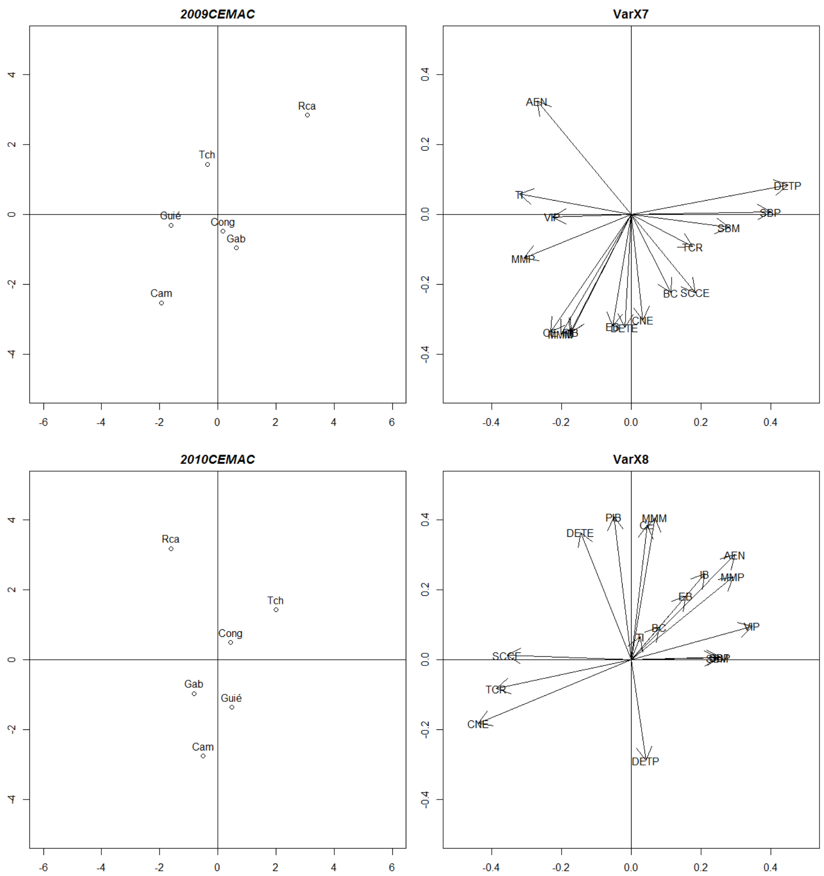
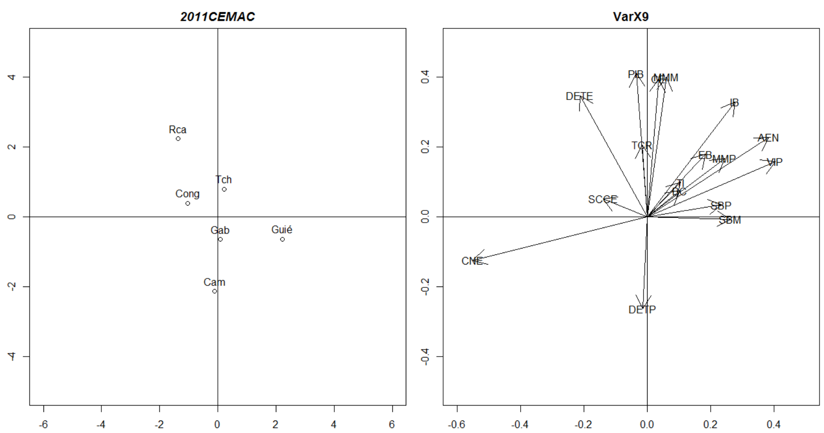


FIGURE 3. Countries of the EMCCA and Macroeconomics variables (Following)



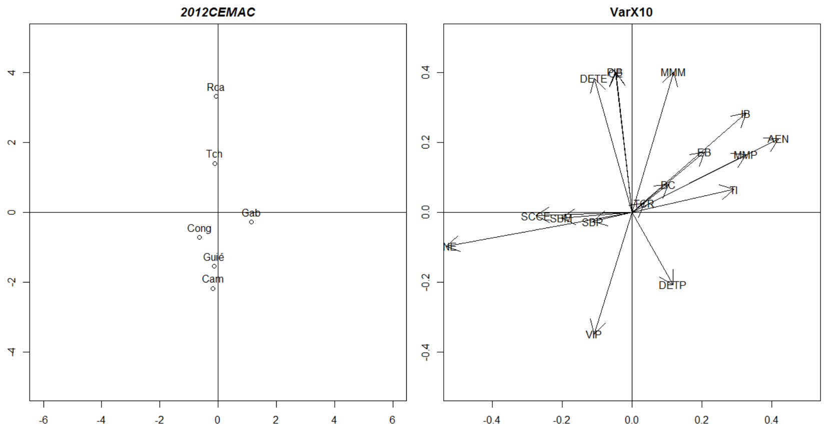
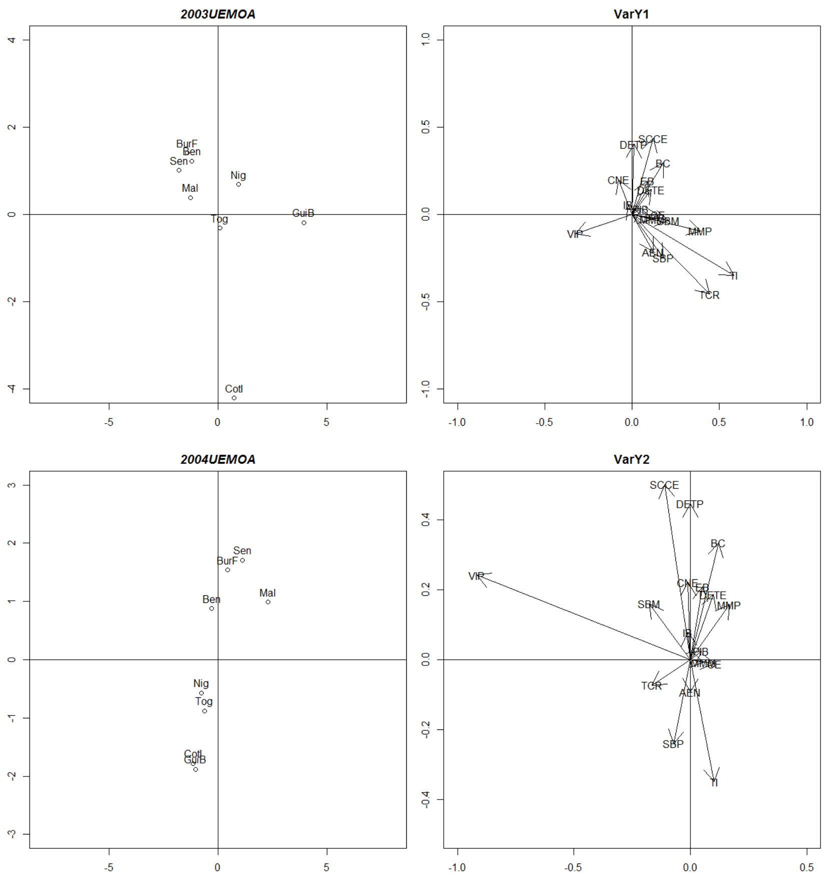


FIGURE 4. Countries of the EMCCA and Macroeconomics variables (End)



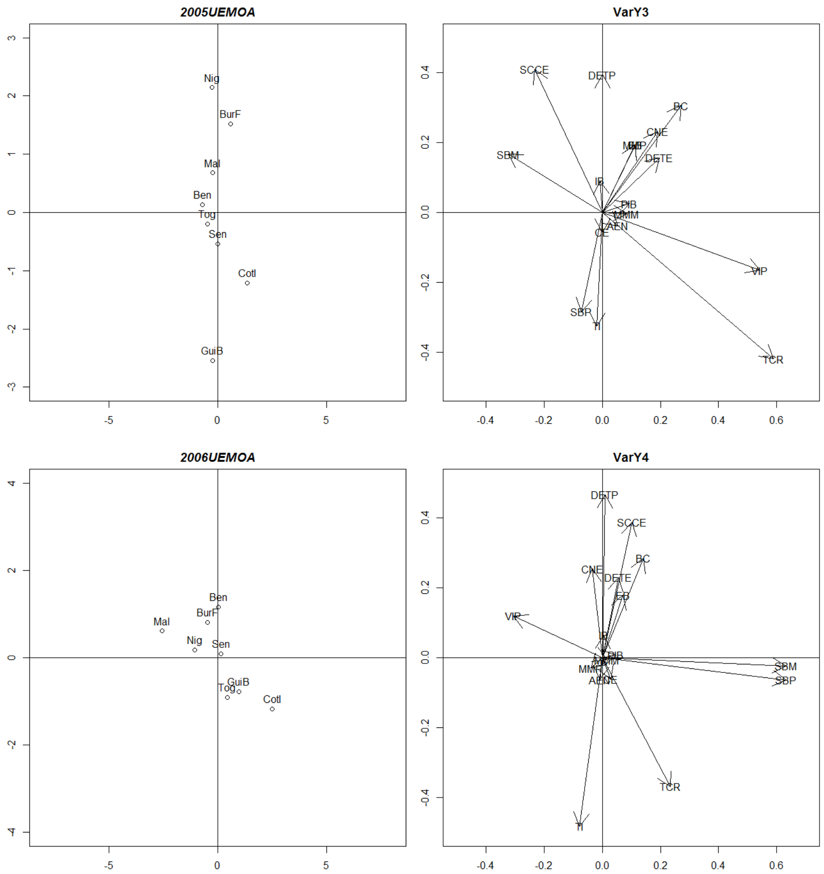
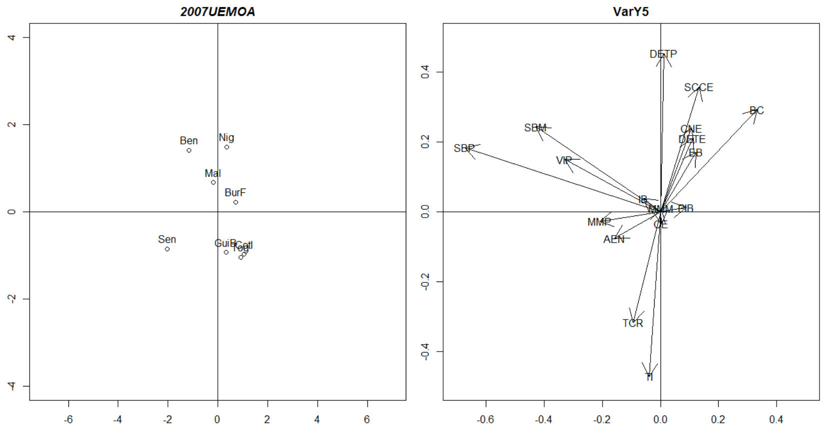


FIGURE 5. Countries of the WAEMU and Macroeconomics variables



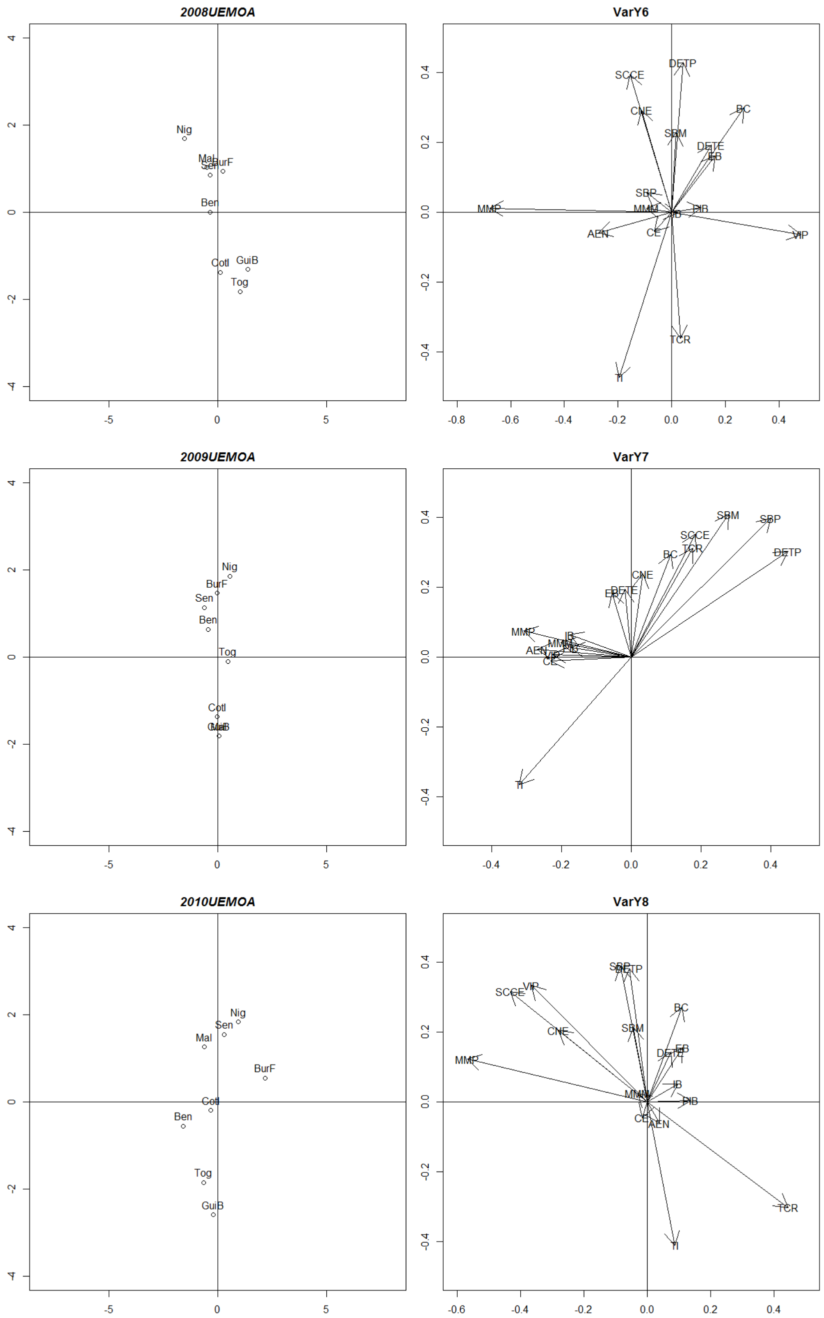


FIGURE 6. Countries of the WAEMU and Macroeconomics variables (Followind)

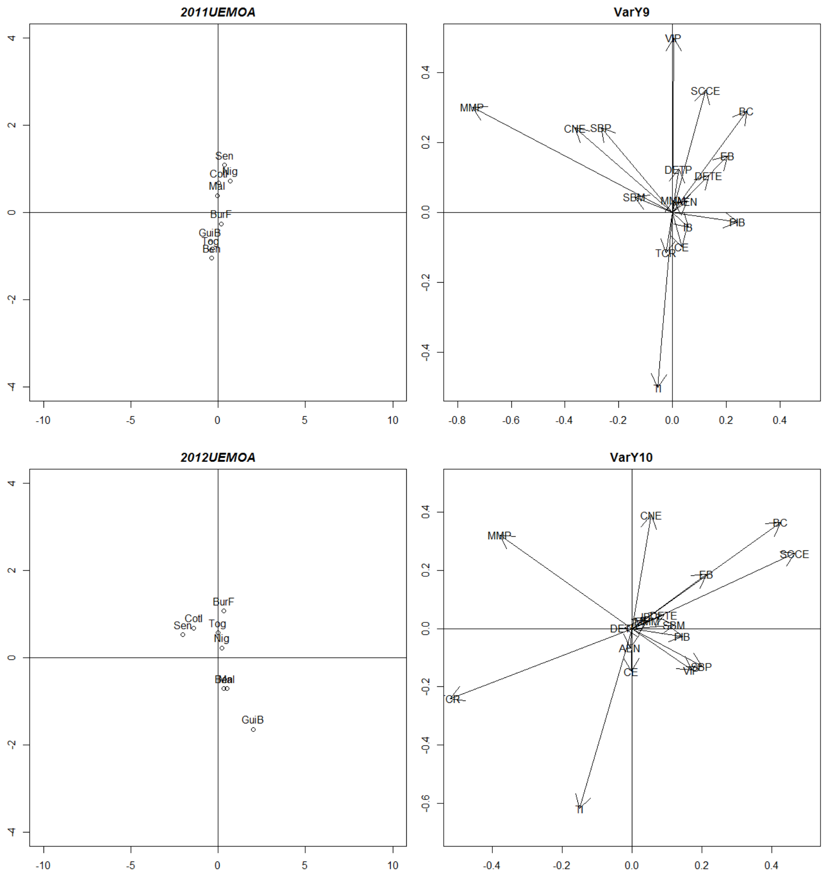


FIGURE 7. Countries of the WAEMU and Macroeconomics variables (End)

5. CONCLUSION

Motivated by the highlighting of proximities between two sets of individuals on which the same variables are observed (two horizontal multi-tables), we have just proposed in this paper the Dual Simultaneous Co-Inertia Analysis method of type 2 (DSCIA2), which is a new approach to ACISD in the context of two horizontal multi-tables. It is an alternative to DsOCIA 1,3 methods in that it allows solutions to be found simultaneously. It is perfectly suited to the analysis of evolutionary data between two different groups of individuals, partitioned into columns. We noticed that the DSCIA and DSCIA2 methods did not give the same results as the DSTATICO (see Makany et al., 2017) and DsOCIA 1,3 methods (see Kossa-Kongbowali, 2019). According to computer scientists, this is a problem related to the initialization of the algorithm.

6. ACKNOWLEDGEMENTS

The author would like to thank Professors Roger Armand Makany and Gabriel Kissita, for their suggestions and guidance in the construction and implementation of these methods.

RÉFÉRENCES

[1] G. KISSITA, I.P.L. MOULOGHO, R.A. MAKANY, D. MIZÃĹRE : *Proposal of analyses : DCA, DCIA, DGCA and DGPCA*, Applied mathematical Sciences, **11**(18) (2017), 859-886.

[2] J.S. KOSSA-KONGBOWALI : *Proposal of analysis of proximity between two horizontal multi-tables : DsCIA 1, DsOCIA 1, DsCIA 3, DsOCIA 3*, Hikari Ltd Applied mathematical Sciences, **13**(20) (2019), 957-981.

[3] J.S. KOSSA-KONGBOWALI, B. MFOURGA, R.A. MAKANY, G. KISSITA : *Proposal of Dual Simultaneous Co-Inertia Analysis : DSCIA*, Advances and applications in statistics, **61**(1) (2020), 95-118.

[4] J.S. KOSSA-KONGBOWALI : *New approach of analysis of the relationship between two vertical multi-tables : SCIA2 Method*, Hikari Ltd Applied mathematical Sciences, **17**(14) (2023), 655-669.

[5] J. THIOULOUSE, AND D. CHESSEL : *Les analyses multi-taleaux en Ãľcologie factorielle. De la typologie d'Ãľtat Ãă la typologie de fonctionnement par l'analyse triadique*, Acta Ecologica, Ecologia Generalis, **8**(4) (1987), 463-480.

[6] R.A. MAKANY, J.S. KOSSA-KONGBOWALI, G. KISSITA, D. MIZÃĹRE : *DSTATICO METHOD : Application to the compared economic evolution of the countries of the EMCCA and the WAEMU*, Far East Journal of Theoritical Statistics, **53**(2) (2017), 89-109.

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