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TWO INEQUALITIES DERIVED FROM THE NESBITT INEQUALITY

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ABSTRACT. This paper is devoted to establishing two new inequalities derived from the Nesbitt inequality. Complete proofs of these inequalities are provided.

1. INTRODUCTION

Inequalities play a fundamental role in mathematics, particularly those involving several parameters. They often lead to deeper insights and broad generalizations of classical results. In this paper, we establish two new multiparameter inequalities derived from the well-known Nesbitt inequality. These results extend the scope of the classical inequality and provide new perspectives on its applications. The precise statement of the Nesbitt inequality is recalled in the theorem below.

Theorem 1.1 (The Nesbitt inequality). *For any $a, b, c > 0$, we have*

$$\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} \geq \frac{3}{2}.$$

For the sake of completeness, two classical proofs of this inequality are included in Appendix. For further developments and related results concerning this inequality, we refer the reader to [1–4].

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2. Inequalities

Our first inequality involves powers of functions and the exponential function. It is stated in the theorem below.

Theorem 2.1. *For any $b, c > 0$ and $x \geq 0$, we have*
$$\left(1+\frac{x}{c}\right)^{b}\left(1+\frac{x}{b}\right)^{c} \geq \exp\left(\frac{3}{2}x-\frac{x^2}{2(b+c)}\right).$$

Proof. For any $a, b, c > 0$, the Nesbitt inequality gives
$$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b} \geq \frac{3}{2}.$$
For any $x \geq 0$, integrating both sides with respect to a over $[0, x]$, we have
$$\int_0^x \left(\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}\right) da \geq \frac{3}{2}\int_0^x da.$$
Using standard primitives, we get
$$\left[\frac{a^2}{2(b+c)}+b\log(a+c)+c\log(a+b)\right]_{a=0}^{a=x} \geq \frac{3}{2}x,$$
so that
$$\frac{x^2}{2(b+c)}+b\log(x+c)+c\log(x+b)-0-b\log(c)-c\log(b) \geq \frac{3}{2}x.$$
Using logarithmic manipulations, we obtain
$$b\log\left(1+\frac{x}{c}\right)+c\log\left(1+\frac{x}{b}\right) \geq \frac{3}{2}x-\frac{x^2}{2(b+c)},$$
so that
$$\log\left(\left(1+\frac{x}{c}\right)^{b}\left(1+\frac{x}{b}\right)^{c}\right) \geq \frac{3}{2}x-\frac{x^2}{2(b+c)}$$
and
$$\left(1+\frac{x}{c}\right)^{b}\left(1+\frac{x}{b}\right)^{c} \geq \exp\left(\frac{3}{2}x-\frac{x^2}{2(b+c)}\right).$$
This concludes the proof. □

Our second inequality involves arctangent functions. It is stated in the theorem below.

Theorem 2.2. *For any $b, c > 0$ and $x \geq 0$, we have*

$$\frac{b}{\sqrt{c}} \arctan\left(\frac{x}{\sqrt{c}}\right) + \frac{c}{\sqrt{b}} \arctan\left(\frac{x}{\sqrt{b}}\right) \geq \frac{3}{2}x - \frac{x^3}{3(b+c)}.$$

Proof. For any $a, b, c > 0$, the Nesbitt inequality yields

$$\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} \geq \frac{3}{2}.$$

Replacing a by a^2, we also have

$$\frac{a^2}{b+c} + \frac{b}{a^2+c} + \frac{c}{a^2+b} \geq \frac{3}{2}.$$

For any $x \geq 0$, integrating both sides with respect to a over $[0, x]$, we have

$$\int_0^x \left(\frac{a^2}{b+c} + \frac{b}{a^2+c} + \frac{c}{a^2+b}\right) da \geq \frac{3}{2}\int_0^x da.$$

Using standard primitives, we get

$$\left[\frac{a^3}{3(b+c)} + \frac{b}{\sqrt{c}} \arctan\left(\frac{a}{\sqrt{c}}\right) + \frac{c}{\sqrt{b}} \arctan\left(\frac{a}{\sqrt{b}}\right)\right]_{a=0}^{a=x} \geq \frac{3}{2}x,$$

so that

$$\frac{x^3}{3(b+c)} + \frac{b}{\sqrt{c}} \arctan\left(\frac{x}{\sqrt{c}}\right) + \frac{c}{\sqrt{b}} \arctan\left(\frac{x}{\sqrt{b}}\right) - 0 \geq \frac{3}{2}x$$

and

$$\frac{b}{\sqrt{c}} \arctan\left(\frac{x}{\sqrt{c}}\right) + \frac{c}{\sqrt{b}} \arctan\left(\frac{x}{\sqrt{b}}\right) \geq \frac{3}{2}x - \frac{x^3}{3(b+c)}.$$

This concludes the proof. □

To the best of our knowledge, these two inequalities are new in the literature. We emphasize that the variable x is not necessarily the primary quantity of interest; it may equally be b, c, or a combination of several parameters. This flexibility makes the obtained inequalities of particular interest.

APPENDIX: PROOFS OF THE NESBITT INEQUALITY

For completeness, two well-known proofs of the Nesbitt inequality are presented below.

First proof. Let $S = a + b + c$ and

$$T = \frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b}.$$

We can write

$$\begin{aligned} T &= \left(\frac{a}{b+c} + 1\right) + \left(\frac{b}{a+c} + 1\right) + \left(\frac{c}{a+b} + 1\right) - 3 \\ &= \frac{a+(b+c)}{b+c} + \frac{b+(a+c)}{a+c} + \frac{c+(a+b)}{a+b} - 3 \\ &= S\left(\frac{1}{b+c} + \frac{1}{a+c} + \frac{1}{a+b}\right) - 3. \end{aligned}$$

Since $2S = 2(a+b+c) = (b+c) + (a+c) + (a+b)$, we have

$$T = \frac{1}{2}\Big((b+c) + (a+c) + (a+b)\Big)\left(\frac{1}{b+c} + \frac{1}{a+c} + \frac{1}{a+b}\right) - 3.$$

By the Arithmetic Mean-Harmonic Mean (AM-HM) inequality, for any $x, y, z > 0$, the following relation holds:

$$\frac{x+y+z}{3} \geq \frac{3}{1/x + 1/y + 1/z}.$$

Multiplying both sides by $3\,(1/x + 1/y + 1/z)$ gives

$$(x+y+z)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) \geq 9.$$

Letting $x = b + c$, $y = a + c$, and $z = a + b$, we get

$$\Big((b+c) + (a+c) + (a+b)\Big)\left(\frac{1}{b+c} + \frac{1}{a+c} + \frac{1}{a+b}\right) \geq 9.$$

Therefore, we derive

$$T \geq \frac{1}{2} \cdot 9 - 3 = \frac{3}{2}.$$

This concludes the proof. □

A more direct proof is given below.

Second proof. A non-trivial decomposition gives

$$\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} = \frac{3}{2} + \frac{1}{2}\left(\frac{(a-b)^2}{(a+c)(b+c)}\right.$$

$$+\frac{(a-c)^2}{(a+b)(b+c)}+\frac{(b-c)^2}{(a+b)(a+c)}\Bigg)\geq\frac{3}{2}.$$

$\square$

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