

NEW TRENDS IN THE MATHEMATICAL EDUCATION

Biljana Jolevska-Tuneska

ABSTRACT. The explosion of technology over the past two decades has not left the education sector behind. Computers and the internet have changed how students can not only access information but even classes themselves. The growth of technological capabilities means that a variety of media and learning-support tools now exist to help students receive a high-quality education through the Internet. This trend presents a number of benefits and drawbacks for teachers and institutions who want to continue to offer their students the rigorous education.

In this paper we will consider the new trends in the mathematical education, with emphasis on the teaching supported by the use of digital technology.

1. INTRODUCTION

The increased use of technology in today's education gives the following questions:

- How can technology-supported learning help to move beyond content delivery and truly enhance education so that students develop a broad mix of skills?
- Could innovative and technology supported teaching and learning approaches spark thinking and creativity, enhance student engagement, strengthen communication, and build collaboration?

¹*corresponding author*

Key words and phrases. teaching mathematics, project based learning, e-learning.

Submitted: 10.05.2022; *Accepted:* 24.05.2022; *Published:* 02.06.2022.

Trough this paper we will try to give the answers of these questions. We believe that technology-supported education can improve students' learning outcomes, and expand the range of learning opportunities made available to students. Also, students' higher-order thinking – above and beyond content learning – can be fostered by specific technology-supported pedagogic models, and in addition, students' creativity, imagination and problem-solving skills will be enhanced.

Lately, many studies on the effects of the technology supported learning are done, see [1,5,7]. Within these studies both potential advantages and risks from the use of technology are considered and analyzed. In 2016 OECD made a report on innovation in education concerning the power of digital technologies, see [6], where they clearly stated that new technologies can facilitate innovative pedagogical models, simulation and virtual online laboratories, international collaboration, real time formative assessment and e-learning.

New technology-enhanced educational models present not so much a technological challenge or cost challenge but a pedagogic challenge. To adopt these new models requires teachers to revisit their pedagogy and this may amount to the greatest cost and challenge. The efficacy of the technology-supported models does not come from technology alone, but from the pedagogy that it supports. Without good pedagogic resources and a good understanding of how to use technology to foster deeper learning, these models may not yield the expected outcomes.

2. MATHEMATICS TECHNOLOGY

Advances in the capabilities and user-friendliness of mathematical software mean that a whole range of problems which previously would have needed graduate level skills to solve can now be accessed by first year undergraduates. For example, to solve an integral in *Mathematica* these days is enough to know the Integrate command, and the syntax in this software. There is no need for complex rules of the integration process. The question that arise is: What we consider to be a technology for mathematical education, or "mathematics technology"? In this set we can put the following items:

- Pocket calculators with different symbolic and/or numerical and/or graphical capabilities.

- Mathematical computer programs: symbolic and numerical ones, e.g. Computer Algebra Systems (CAS) like Maple, Mathematica, or MathCad;
- Numerical programs like Matlab.
- Dynamic geometry programs like Cabri and Geogebra.
- Spreadsheet programs.
- Engineering programs based on mathematical models.

The effect of computer technology on education seems to be greater in mathematics than in any other subject. There are two distinct ways in which developments in technology affect learning and teaching in mathematics.

- New technology provides opportunities for new approaches to teaching and learning.
- Advances in technology impact not only on how mathematics is taught but also on what mathematics is taught.

Modern technology also creates some new challenges and tasks in the mathematical education that must be addressed for technology to really improve teaching and learning:

- One large new challenge is to strike the right balance between work with and without technology in a blended learning approach. Some participants advocated an approach where concepts come first and their application using technology comes later.
- Learners should discuss and experience an adequate use of technology. They should for example see that there is no need for technology when it comes to computing $\cos(0)$. Students should also experience the limitations and pitfalls of mathematical technology (for example: are all possible cases covered in a computational procedure?) such that they develop a critical attitude towards technology and can make goal-oriented use of technology.

Technology will be there and cannot be ignored.

We recognize the following new opportunities and potential advantages provided by technology:

- Better visualization of mathematical concepts by using mathematical programmes.

In some area of mathematical education the visualization and demonstration is very important for the effectiveness of the teaching process, for example, in calculus or multivariable calculus.

- Explorative approach to learning.
Using technology, the technical dimension of mathematical activities are been facilitated. It allow the user to take action on mathematical objects or representations of those objects. This feature can be utilised to enable students to explore objects and structures and to discover properties and connections e.g. by performing parameter variations.
- Experimental approach to problem solving.
Mathematics programs provide new ways of problem solving. In classical paper and pencil work students had to know a certain procedure in order to solve a problem and they could not advance once they got stuck in the process. Mathematics programs allow students to select different ways of investigating a problem (for example finding an approximate solution by looking at the graph of a function instead of getting an exact solution; investigating several related examples to derive a hypothesis or to discover a counter-examples and thus increase the students chances in making a progress with a problem.
- Realistic modeling.
Mathematics and engineering programs allow the earlier introduction of more interesting modelling tasks since some parts of the computation can be delegated to the program.
- Change of roles.
Using mathematics programs can help to bring about changes in the way classes are conducted, as their usage requires student active participation and bigger activity. Such activity can also be designed to require interaction among students. The result is that the process of acquiring and developing mathematical knowledge becomes more student-centred. This also changes the role of teachers, who become tutors and instructors rather than lecturers.
- Large data manipulation in more realistic projects.
Experiential learning is most likely to provide expected improvements in

conceptual understanding and scientific inquiry skills if teachers encourage students to repeat their experiments and provide students with a robust scaffolding to understand them.

- Easier assessment.

Real-time formative assessment (with a help of the technology) allows teachers to see in real time what students think and know, but they still have to use this information in their teaching to encourage students to reflect more deeply and to challenge their misconceptions.

- Use of chat facilities.

The use of the chat facility allowed students to participate more actively in lectures by posing questions or giving answers to questions posed by the lecturer. Why students are more comfortable with the chat option rather than asking the teacher directly is yet to be investigated. Maybe is because whereas in class only one student gives an answer, the chat allows for many students to give answers simultaneously. Moreover, the chat seemed to constitute a smaller hurdle for asking questions than the classroom situation. The question is how this can be transferred to normal class teaching. Even if the technology is available, it is hard for a lecturer to give the lecture and at the same time to observe the chat.

- Use of smartphones for activating students by voting.

Polling or voting systems are also a means to involve students more. The polling can be offered via smart phones.

- Usage of videos for flipped classroom scenarios (lecture becomes tutorial).

Many short video clips or longer recorded lectures are now available. These digital offerings increase the flexibility of learning and were hence appreciated by students.

- Motivational aspects.

Most students are accustomed to using technology such as smart phones in their daily life; consequently simply having technology involved can make a huge difference in students attitudes and feelings towards mathematics. Therefore increased use of mathematics technology may help to improve student motivation for learning.

For sure, the use of technology in the mathematical education has its cons. The following risks have been identified:

- Loss of basic capabilities.

When adapting the mathematical educational process to make use of new technological tools one must be aware of the risk that this computer-based learning environment may cause an unexpected reduction of students knowledge in basic "mathematical culture". This is not just a loss of fluency in carrying out procedural mathematical tasks brought about by a reduced amount of practice (due to using computer programs to carry out these tasks), but can also be a more limited understanding of core mathematical concepts as the reduced practice may bring with it reduced need to think about the basic concepts thereby impacting on overall mathematical reasoning skills.

- Loss of connection between procedures and understanding.

Extensive and exclusive usage of mathematical technology can potentially prevent students from making proper connections between the techniques used for calculations and conceptual understanding, for example the Gauss algorithm for solving a linear system of equations also provides insight into the possible solution types; simple use of "solve" command does not give this insight.

- Pure trial and error working style without thinking.

There is a danger that students may use mathematics and application programs in a largely thoughtless trial and error mode, making variations without any particular strategy in the hope that somehow they will achieve what is required without having any idea why what they are doing. Problems must be found where such a strategy is not productive so that students are forced to think about the effects of possible variations.

- Tool dependence.

When students are no longer able to compute even simple examples by hand, they depend totally on what the tool they are using provides. They also have no idea of what to do when a program fails to give them an answer to a problem because they do not know what the program is able to do. Although the students do not need to know in detail what a program

does, they should know which model a program is appropriate to use or not appropriate.

3. MATHEMATICS EDUCATION

What should Mathematics education look like now and in the future? This section presents five innovative models of technology-supported mathematical teaching and learning. These are educational gaming, online laboratories, collaboration through technology, real-time formative assessment and skills-based curriculum alignment. The emphasis is on practices that would be difficult to implement without technology and that can improve not just traditional learning outcomes, but also motivation, social, behavioural, thinking and creativity skills and their assessment.

3.1. Educational gaming. Educational gaming offers a promising model to enhance student learning in mathematical education, not just improving content knowledge, but also motivation and thinking and creativity skills, see [2,3]. Educators and policy makers should consider using it to enhance mathematical learning outcomes and problem-solving skills and motivation. Designing games appears to lead to even deeper learning than just using them for educational purposes.

In educational gaming students interact with video games, simulations or virtual worlds based on imaginary or real worlds, also seen as highly interactive virtual environments. Educational gaming also includes collaborative project-based learning experiences where students themselves become game designers and content producers.

As a promising model for various disciplines and education levels, educational gaming may promote:

- Learning by doing.

The interactive, reactive and often collaborative nature of educational gaming enable learning by doing of complex topics by allowing students to (repeatedly) make mistakes and learn from them. Real-life based gaming allows experimentation that would otherwise be too costly or dangerous.

- Student learning.

Educational gaming which covers specific topics or subject areas and take place within a set of rules can increase students' achievements and subject-specific knowledge. Constructing educational games seems to increase deep learning more than just using existing games.

- Student engagement and motivation.

Based on play and increasing challenges, educational gaming can foster student engagement and motivation in various subjects and education levels. Low-achieving students may find the educational gaming experience more engaging than high achieving students. Students' motivation can increase more when they construct games themselves as opposed to just playing an existing game.

- Students' thinking skills.

Games have the potential to help students find new ways around challenges, use knowledge in new ways and "think like a professional". Educational gaming may also improve students' skills such as problem solving.

3.2. Mathematics Laboratories. What is mathematics laboratory? Can we speak these days about Mathematics Laboratories, or maybe the Mathematics Laboratories existed some years ago, see [4].

By mathematics laboratories, we mean learning scenarios where students work in a PC laboratory on tasks requiring the use of mathematical software such as numerical programs Matlab, Maple, Mathematica or spreadsheets Excel. In such laboratory sessions, students practise the usage of the programs and see how they can be used for standard tasks. They might also be used for experimenting with more open tasks of an investigative nature.

Mathematics laboratories, whether remote or virtual, are another promising innovation to enhance the teaching and learning of mathematics at all levels of education. Virtual mathematics laboratories allow students to simulate experiments while remote ones allow students to use real laboratory equipment from a distance through the Internet.

Educators and policy makers should consider mathematics laboratories as a promising way to increase access to a wide range of experimental learning. The

use of mathematics laboratories only requires access to the Internet and allows teachers and students to get access to more experimental equipment than a single school can generally provide, see [8].

3.3. Collaboration through technology. Collaboration through technology can enhance students' interaction, engagement, learning and thinking skills, in addition to increasing flexibility and diversity in educational experience, see [9]. Technology-supported collaboration can enhance students' awareness of global challenges and develop their understanding of other cultures.

In technology-enabled collaboration, students work together (in groups) and/or interact with each other to enhance their learning with the help of various technologies and often with facilitation from the teacher. When combined with other learning approaches, technology-enabled collaboration can form a part of project- or problem based learning or supplement face-to-face learning. Technology-enabled collaboration models may include in-built assessment features taking into account also team performance and/or collaborative activity.

As a promising model for Mathematics education and other disciplines at various education levels, collaboration through technology may improve:

- Flexibility.
Technology enables students to collaborate and practice at "their own pace", beyond the formal classroom hours and without limitations of physical location.
- Cultural diversity.
Technology can significantly increase possibilities for intercultural interactions by broadening the scope of collaborations to distant locations, even across borders.
- Student learning.
Technology-enabled collaboration may support student learning, in both individual and group outcomes, although not necessarily more than face-to-face interaction. There can also be cross-cultural differences. In general, positive results of co-operative learning on student achievement have been shown to depend on group learning goals and individual accountability.

- Student interaction and engagement.

Technology-enabled collaboration can encourage student group work skills, interaction and engagement. Yet, "active learning strategies" are not automatically adopted and activity may differ across cultures. In general, co-operative learning has shown clearly beneficial results on affective student outcomes.

- Students' thinking skills.

Online collaboration may enhance higher order thinking even more than face-to-face collaboration through "more complex, and more cognitively challenging discussions". This can also be the case for "questioning behaviours" and "project performance".

REFERENCES

- [1] V. AHMED, A. OPOKU: *Technology supported learning and pedagogy in times of crisis: the case of COVID-19 pandemic*, Education and Information Technologies, **27** (2022), 365–405.
- [2] L. BERTRAM: *Digital Learning Games for Mathematics and Computer Science Education: The Need for Preregistered RCTs, Standardized Methodology, and Advanced Technology*, Frontiers in Psychology, **11** 15 October (2020), doi=10.3389/fpsyg.2020.02127
- [3] F. CHIZARY, A. FARHANGI: *Efficiency of Educational Games on Mathematics Learning of Students at Second Grade of Primary School*, Journal of History Culture and Art Research, **6**(1) (2017), doi:http://dx.doi.org/10.7596/taksad.v6i1.738
- [4] M. KLUTTZ: *The mathematics laboratory—a meaningful approach to mathematics instruction*, The Mathematics Teacher, **56**(3) (1963), 141–145.
- [5] F. A. MULDER, T. WULF: *Technology-supported management education: a systematic review of antecedents of learning effectiveness*, International Journal of Educational Technology in Higher Education, **17** 47 (2020).
- [6] OECD: *Innovating Education and Educating for Innovation: The Power of Digital Technologies and Skills*, OECD publishing, Paris, 2016.
- [7] Q. WANG: *Developing a Technology-supported Learning Model for Elementary Education Level*, Mimbar Sekolah Dasar, **6**(1) (2019), 141–146.
- [8] B. NWOKE, C. OGOKE, A. ANYANWU: *Availability and Utilization of Mathematics Laboratory Facilities For Pedagogical Purposes in Secondary Schools*, Journal of CUDIMAC, **8**(1) (2020), 163–174.
- [9] D. SONNENWALD, M. IIVONEN, J. ALPI, H. KOKKINEN: *Collaborative Learning Using Collaboration Technology: Report from the Field*, Integrating Information and Communications Technology in Higher Education, Kluwer Publishers, (1999), 247–258.

DEPARTMENT OF MATHEMATICS AND PHYSICS
FACULTY OF ELECTRICAL ENGINEERING AND INFORMATIONAL TECHNOLOGIES
SS. CYRIL AND METHODIUS UNIVERSITY IN SKOPJE
KARPOŠ II B.B., 1000 SKOPJE
REPUBLIC OF NORTH MACEDONIA.
Email address: biljanaj@feit.ukim.edu.mk