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A MATHEMATICAL ANALYSIS OF HEAT AND MASS TRANSFER ON MHD BOUNDARY LAYER FLOW

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Abstract: The objective of this paper is to solve a system of highly non-linear differential equation governing MHD boundary layer flow over a moving vertical porous plate. An analytical expression for dimensionless velocity profile, temperature profile and concentration profile has been derived using Q-Homotopy Analysis Method. The impact of velocity, temperature and concentration on varying parameters that are influencing the flow are discussed graphically and compared with the numerical results.

Key Words: Porous plate, Heat transfer, Mass transfer, Non-linear differential equations, Q-Homotopy Analysis Method.

1.Introduction:

MHD free convection flows have noteworthy applications in the field of stellar and planetary magnetospheres, aeronautical plasma flows, chemical engineering and electronics. The summary of the applications are explained by many researchers.[9] made a mathematical analysis of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. The unsteady free convection flow past a vertical plate embedded in a porous medium was examined by [11]. The study of heat and mass transfer of the fluid was demonstrated by [2] to [8] under various circumstances.In many situations, such as in geothermal operations, petroleum industries, thermal insulation, design of solid-matrix heat exchangers, chemical catalytic reactors, the transportation of the fluid through porous media and their behaviours during the process plays vital roles. The importance of inertia effects for flows in porous media was discussed by [6]. [14]examined the MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. [7] made similarity solutions for boundary

layer near a vertical surface in a porous medium with constant temperature and concentration.There are many transport processes in nature and in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species.Designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over a agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution are some of the fields of interest in which combined heat and mass transfer plays an important role. In this context, [5] analysed the effects of mass transfer and free convection on the flow past an impulsively started vertical flat plate. A discussion was done by [2] on the effects of heat and Mass transfer in the laminar boundary layer flow of a moving flat surface with constant surface velocity and temperature focusing on the effects of suction/injection. With these prior works of [2] to [15], [1] depicted the chemical and thermal radiation effects of a MHD boundary layer flow over a moving vertical porous plate. [1] obtained a system of system of partial differential equations with boundary conditions, which on suitable simulations and appropriate usage of dimensionless quantities, converted to a system of non-linear differential equations with the corresponding reduced boundary conditions.
In this paper, Q-Homotopy method is adapted to solve analytically the given differential equations satisfying the boundary conditions that are proposed by [1] in his previous work.

2. Mathematical Formulation:

We have consider a steady laminar convective flow of an electrically conducting incompressible fluid along a semi-infinite vertical plate. Theassumption taken into consideration is that the velocity of the fluid far away from the plate surface are to be zero for a quiescent state fluid. There is a linear variation in the surface temperature and concentration. Except for the density variations in the buoyancy force term of the linear momentum equation, all other characteristics of the fluid are taken as constant. The induced magnetic field can be neglected as the Magnetic Reynolds number is assumed to be small. Under these assumptions and having Boussinesq approximations taken into account, the flow governing equations are given as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{k'}u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y} \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1(C-C_\infty) \quad . \qquad (4)$$

The boundary conditions are given as:

$$\begin{aligned} &u=Bx,\ v=V,\ T=T_w=T_\infty+ax,\ C=C_w=C_\infty+bx\ at\ y=0 \\ &u\to 0,\ T\to T_\infty,\ C\to C_\infty\ \ as\ y\to\infty \end{aligned} \qquad (5)$$

Here, x and y represent the coordinate axes along the continuous stretching surface in the direction of motion and perpendicular to it respectively, u and v are the velocity componentsalong the x and y axes respectively, ν is the kinematics viscosity, β,β^* are the thermal and concentration expansion coefficient respectively, σ is the electric conductivity, $B_0$is the uniform magnetic field, ρ is the density, κ' is the permeability of the porous medium, T is the temperature inside the boundary layer, C_∞ species concentration of the ambient fluid, α is the thermal diffusivity, k_1 is the rate constant of first order chemical reaction, C_P is the specific heat at constant pressure, q_r is the relative heat flux, $D$is the molecular diffusivity of the species concentration, B is a constant, a and b denotes the stratification rate of the gradient of ambient temperature and concentration profiles.

We introduce the non-dimensionless variables as follows:

$$\eta=\sqrt{\frac{B}{\nu}}y,\ u=\frac{\partial\psi}{\partial y}=xBf',v=-\frac{\partial\psi}{\partial x}=-\sqrt{B\nu}\,f,\theta=\frac{(T-T_\infty)}{(T_w-T_\infty)},\phi=\frac{(C-C_\infty)}{(C_w-C_\infty)},$$

$$M=\frac{\sigma B_0^2}{\rho B},K=\frac{\nu}{kB},\ Gr=\frac{g\beta(T_w-T_\infty)}{xB^2},Gc=\frac{g\beta^*(C_w-C_\infty)}{xB^2},\ \Pr=\frac{\nu}{\alpha},Sc=\frac{\nu}{D},\quad (6)$$

$$\gamma=\frac{k_1(C_w-C_\infty)}{bxB},R=\frac{\alpha k^*\rho C_P}{\sigma^* T_\infty^B}$$

The differential equations become:

$$f'''+ff''-f'^2+Gr\theta+Gc\phi-(M+K)f'=0 \quad (7)$$

$$(1+\frac{16}{3R})\theta''+\Pr(f\theta'-f'\theta)=0 \quad (8)$$

$$\phi''+Sc(f\phi'-f'\phi)-Sc\gamma\phi=0 \quad (9)$$

with the boundary conditions:

$$\begin{aligned}&f'=1,\ f=f_w,\theta=1,\phi=1\ \ at\ \ \eta=0\\&f'=0,\theta=0,\phi=0\ \ \ as\ \ \ \eta\to\infty\end{aligned} \quad (10)$$

where $f_w=\frac{v}{\sqrt{\nu B}}$ is the suction parameter.

3. Analytical Solution using Q-Homotopy Analysis Method:[16] to [28].

Consider the non-linear differential equation

$$N[u(t)]=0 \quad (11)$$

where, *N* is a nonlinear operator, *u(t)* is an unknown function.

Construct the so-called zero order deformation equation as:

$$(1-nq)L[\phi(x,t;q)-u_0(t)]=F(n)qN[\phi(x,t;q)] \quad (12)$$

where, *F(n)* is a nonzero auxiliary function, $n\geq 1$, $q \in [0,1]$ is the embedding parameter, *L* is an auxiliary linear operator. The function *F(n)* is to be chosen depending on the given problem.
when*q=0* and *q=1/n,*

$$\phi(x,t,0)=u_0(x,t)\ and\ \phi(x,t,\frac{1}{n})=u(x,t)$$

respectively. Thus as *q* increases from 0 to *1/n*, the solution $\phi(t;q)$ varies from the initial guess $u_0(t)$ to the solution $u(t)$. Having the freedom to choose $u_0(t)$, *L ,h, H(x,t,),* one can choose them appropriately, so that the solution $\phi(x,t;q)$ of (12) exists for $q \in [0,1/n]$. Expanding $\phi(x,t;q)$ in Taylor's series, we get:

$$\phi(x,t;q)=u_0(t)+\sum_1^\infty u_m(t)(q)^m \quad (13)$$

where$u_m(t)=\frac{1}{m!}\frac{\partial^m\phi(x,t;q)}{\partial q^m};q=0$. Assume that F(n), $u_0(t)$, L are properly chosen such that the series (12) converges at *q=1/n* and

$$u(t)=\phi(x,t;\frac{1}{n})=u_0(t)+\sum_1^\infty u_m(t)(\frac{1}{n})^m\ . \quad (14)$$

Defining the vector$u_r(t)=\{u_0(t),u_1(t),u_2(t),....u_r(t)\}$. Differentiating (12) m times with respect to q and then setting q=0 and finally dividing them by *m!* , we have the m^{th} order deformation equation:

$$L[u_m(t)-k_m u_{m-1}(t)]=F(n)R_m(u_{m-1}(t)) \quad (15)$$

where,

$$R_m(u_{m-1}(t)) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N\phi(x,t;q)}{\partial q^{m-1}}, q = 0 \qquad (16)$$

and km= 0 for $m \le 1, n$ otherwise .

It should be emphasized that $u_m(t)$ for $m \ge 1$ is governed by the linear equation (15) with the boundary conditions that come from the original problem. Due to the presence of the factor $(1/n)^m$, more chances for convergences may occur or even much faster convergence can be obtained better than the standard HAM. It should be noted that the cases of *n*=1 in (12), standard HAM can be reached. Solving (7) to (9) with boundary conditions (10) using Q-Homotopy Analysis Method, we obtain the results as follows:

$$f_0 = 1 - f_w - e^{-\eta} \qquad (17)$$

$$f_1 = C_1 + C_2 e^{-\eta} - f_w \eta e^{-\eta} - (Gr + Gc)\eta e^{-\eta} + (M + K)\eta e^{-\eta} \qquad (18)$$

$$f_2 = C_3 + C_4 e^{-\eta} + P_1 \eta e^{-\eta} + P_2 (\frac{\eta^2}{2} + 2\eta + 3)e^{-\eta} + P_3 \frac{e^{-2\eta}}{4} + P_4 (\eta + 2)\frac{e^{-2\eta}}{4} \qquad (19)$$

$$\theta_0 = e^{-\eta} \qquad (20)$$

$$\theta_1 = (1 + \frac{16}{3R})\eta e^{-\eta} - \Pr \eta e^{-\eta} + \Pr f_w \eta e^{-\eta} \qquad (21)$$

$$\theta_2 = C_5 e^{-\eta} - P_5 \eta e^{-\eta} - P_6 e^{-\eta}\left(\frac{\eta^2}{2} + \eta + 1\right) + P_7 \frac{e^{-2\eta}}{2} + P_8 \frac{e^{-2\eta}}{2}\left(\eta + \frac{3}{2}\right) \qquad (22)$$

$$\phi_0 = e^{-\eta} \qquad (23)$$

$$\phi_1 = \eta e^{-\eta} - Sc\eta e^{-\eta} + Scf_w \eta e^{-\eta} - Sc\gamma \eta e^{-\eta} \qquad (24)$$

$$\phi_2 = C_6 e^{-\eta} - P_9 \eta e^{-\eta} - P_{10} e^{-\eta}\left(\frac{\eta^2}{2} + \eta + 1\right) + P_{11}\frac{e^{-2\eta}}{2} \qquad (25)$$

where,

$$C_1 = f_w + (Gr + Gc) - (M + K) \qquad (26)$$

$$C_2 = - f_w - (Gr + Gc) + (M + K) \qquad (27)$$

$$C_3 = -P_1 - 2P_2 - \frac{1}{4}P_3 + \frac{1}{4}P_4 \qquad (28)$$

$$C_4 = P_1 - P_2 - \frac{1}{2}P_3 - \frac{3}{4}P_4 \qquad (29)$$

$$C_5 = P_6 - \frac{1}{2}P_7 - \frac{3}{4}P_8 \qquad (30)$$

$$C_6 = P_{10} - \frac{1}{2}P_{11} \qquad (31)$$

$$\begin{aligned} P_1 = &-nf_w - n(Gr + Gc) + n(M + K) + f_w + 2(Gr + Gc) - (M + K) + C_2 f_w \\ &+ 2 f_w^{\ 2} + 2 f_w (Gr + Gc) - 3 f_w (M + K) - C_2 (M + K) - (M + K)(Gr + Gc) \\ &+ (M + K)^2 \end{aligned} \qquad (32)$$

$$\begin{aligned} P_2 = &- f_w^{\ 2} - f_w (Gr + Gc) - Gr\left(1 + \frac{16}{3R}\right) + Gr\Pr - Gr\Pr f_w - Gc + GcSc - GcScf_w \\ &+ GcSc\gamma + 2(M + K) f_w + (M + K)(Gr + Gc) - (M + K)^2 \end{aligned} \qquad (33)$$

$$P_3 = -(M + K) + (Gr + Gc) \qquad (34)$$

$$P_4 = (M+K) - (Gr+Gc) \tag{35}$$

$$P_5 = -n\left(1+\frac{16}{3R}\right) + n\Pr - n\Pr f_w - \left(1+\frac{16}{3R}\right)^2 + 2\Pr f_w\left(1+\frac{16}{3R}\right) + \Pr^2 - 2\Pr^2 f_w + C_1\Pr + C_2\Pr + \Pr^2 f_w^{\,2} \tag{36}$$

$$P_6 = \left(1+\frac{16}{3R}\right) - 2 f_w \Pr\left(1+\frac{16}{3R}\right) - \Pr^2 + 2\Pr^2 f_w + \Pr^2 f_w^{\,2} \tag{37}$$

$$P_7 = \left(1+\frac{16}{3R}\right)\Pr - \Pr^2 + \Pr^2 f_w + C_2 + \Pr f_w + (Gr+Gc) - \Pr(M+K) \tag{38}$$

$$P_8 = 2\Pr^2 f_w - 2\Pr f_w - 2(Gr+Gc)\Pr + 2(M+K)\Pr \tag{39}$$

$$P_9 = nSc - nScf_w + nSc\gamma + n + 2 - 2Sc + 2Scf_w - 2Sc\gamma - Sc - f_w Sc^2 + Sc^2 f_w^{\,2} - Sc^2 f_w\gamma + C_1 Sc \tag{40}$$

$$P_{10} = -1 + 2Sc - 2Scf_w + 2Sc\gamma - Sc^2 + 2Sc^2 f_w - 2Sc^2\gamma - Scf_w - Sc^2 f_w^{\,2} + 2Sc^2 f_w\gamma - Sc^2\gamma^2 \tag{41}$$

$$P_{11} = -2n - Scf_w - (Gr+Gc)Sc + (M+K)Sc \tag{42}$$

Hence, the velocity profile, temperature profile and concentration profile are obtained as follows:

$$f = \sum_0^\infty f_m \left(\frac{1}{n}\right)^m \tag{43}$$

$$\theta = \sum_0^\infty \theta_m \left(\frac{1}{n}\right)^m \tag{44}$$

$$\phi = \sum_0^\infty \phi_m \left(\frac{1}{n}\right)^m \tag{45}$$

4.Results and Discussion:

The effects of velocity profile, temperature profile, concentration profile on varying the governing parameters are represented graphically. The graphs interpret the following observations:

Fig.1 to Fig.9 denotes the influence of the parameters on velocity profiles. It is evident that the velocity increases with an increase in *K*and *M*. These effects are observed from Fig. 1and Fig.2. Fig. 3 and Fig.4 Confirms that there is a gain in the velocity with a raise in *Gr* and*Gc*respectively. The fact that the velocity varies inversely with respect to *Sc* and *Pr* are validated from Fig.5 and Fig.6. It is proved from Fig.7 and Fig.8 that the velocity gets escalated with a hike in γ, whereas it slumps as R is incremented. It is demonstrated in Fig.9 that velocity gets amplified as the parameter $f_w$is increased. The effect of temperature profile while increasing the parameters are demonstrated from Fig.10 to Fig.18. There is a hike in temperature on enlarging the parameters K and M respectively. This is illustrated in Fig.10 and Fig.11. There is a fall in temperature on increasing the value of the parameter*Gr*but the increase in*Gc*increases the temperature. It is studied in Fig. 12 and Fig. 13. Fig. 14 to Fig.16 predict that the temperature gets incremented with a reinforce in *Sc, Pr*and λ. The upsurge in *R* lessen the temperature (Fig. 17). As the parameter f_w is amplified, the temperature boosted up (Fig. 18). From Fig.19 to Fig.27, the behaviour of concentration profile with parameter change can be studied. It is evident that the concentration of the fluid changes proportionally with the parameters *M, K, Gc,λ,* and *R.* That is, concentration increase with increase in these parameters(Fig.19, 20,22, 25, and 26). Whereas, the concentration decreases on increasing the value of the parameters *Gr, Sc, Pr*andf_w (Fig. 21, 23,24 and 27)

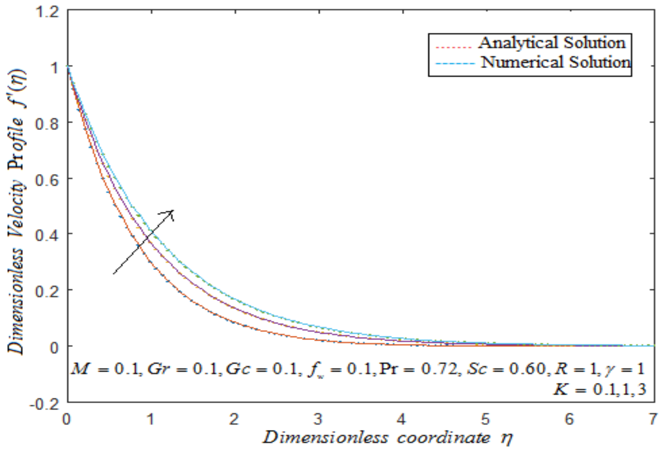

Fig:1. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, Gr, Gc*, f_w, γ ,*Pr* ,*R, Sc* and varying *K*=0.1, 1, 3.

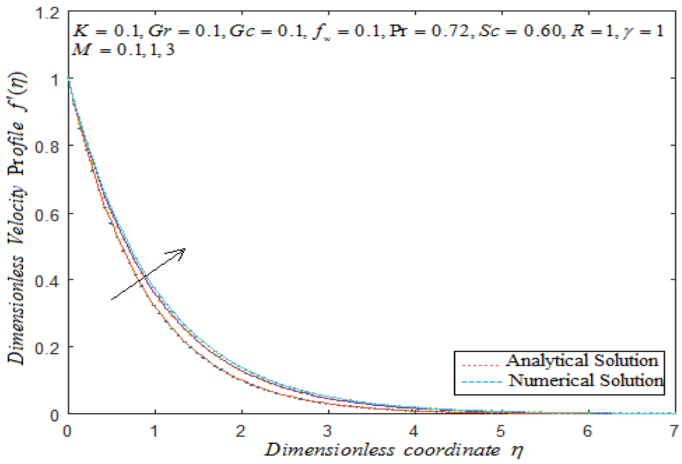

Fig:2. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *K, Gr, Gc*, f_w, γ , *Pr* ,*R, Sc* and varying *M*=0.1, 1, 3.

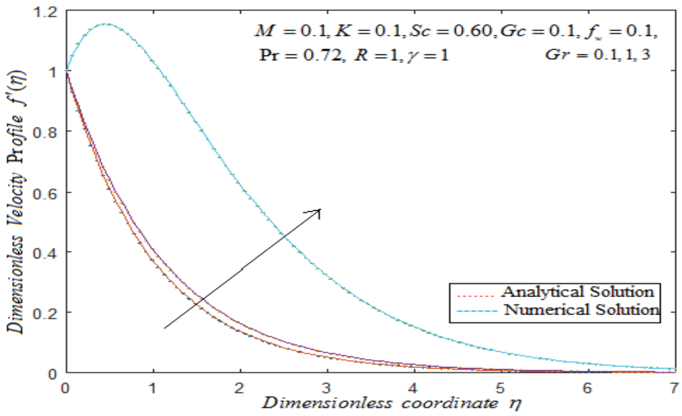

Fig:3. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, K, Gc*, f_w, γ ,*Pr* ,*R, Sc* and varying *Gr*=0.1, 1, 3.

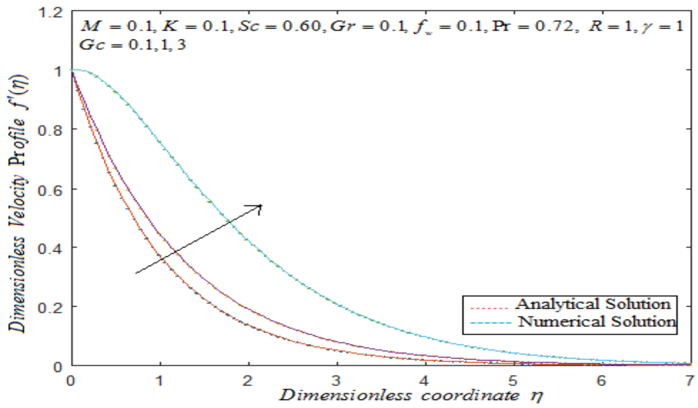

Fig:4. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, Gr, K,* f_w,, γ ,*Pr* ,*R, Sc* and varying *Gc*=0.1, 1, 3.

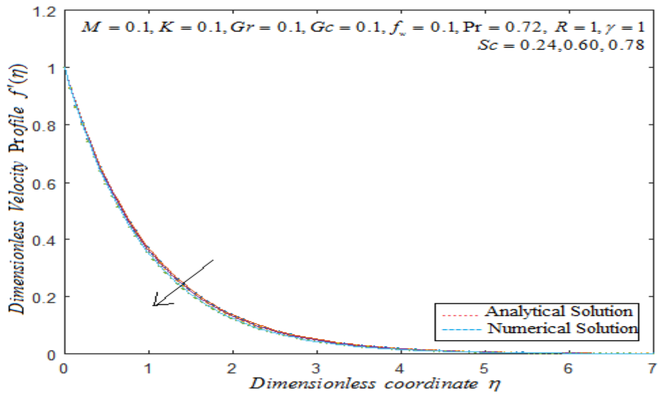

Fig:5. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, Gr, Gc,* f_w, γ ,*Pr* ,*R, K* and varying *Sc*=0.24, 0.60, 0.78

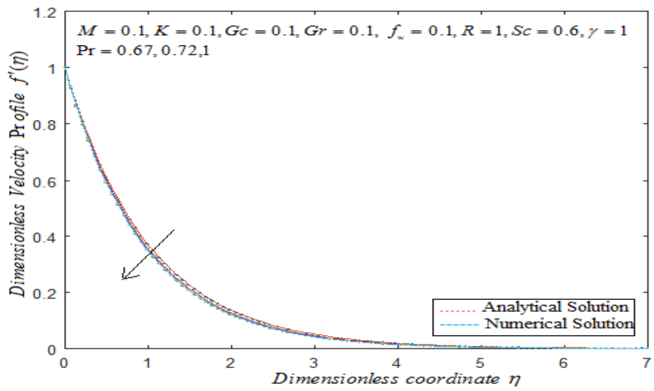

Fig:6. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, Gr, Gc,* f_w, γ ,*K* ,*R, Sc* and varying *Pr*=0.67, 0.72, 1

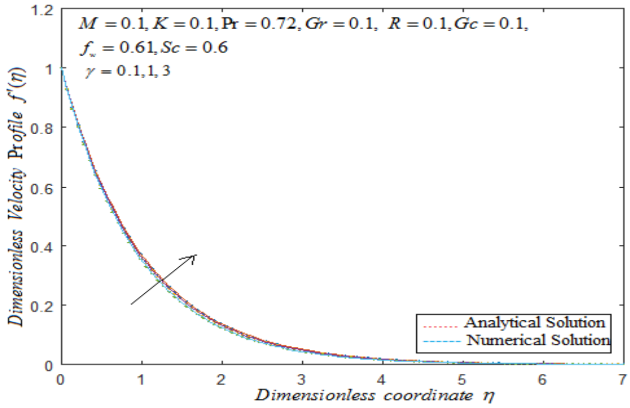

Fig:7. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, Gr, Gc,* f_w,*K, Pr ,R, Sc* and varying γ =0.1, 1, 3.

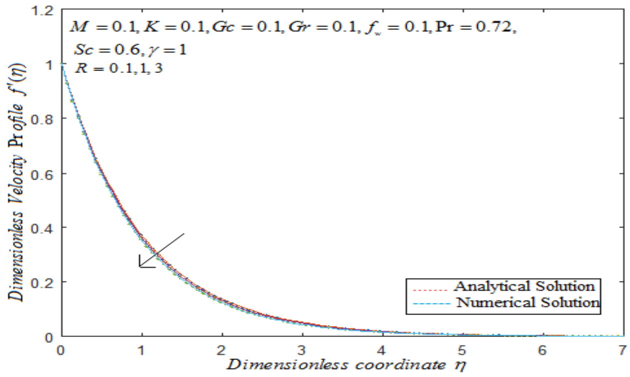

Fig:8. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, Gr, Gc,* f_w, γ ,*Pr,K, Sc* and varying *R*=0.1, 1, 3.

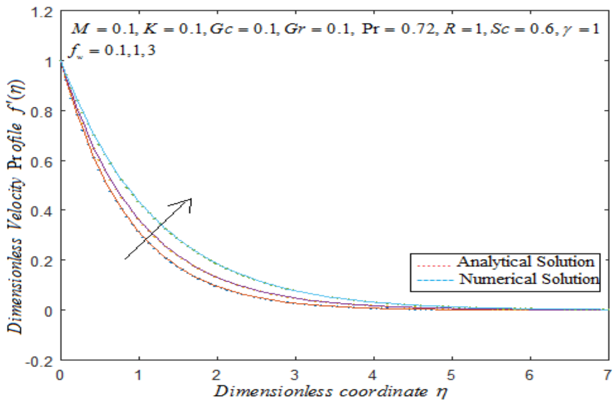

Fig:9. Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (43) for fixed *M, Gr, Gc, K* , γ , *Pr ,R, Sc* and varying f_w=0.1, 1, 3.

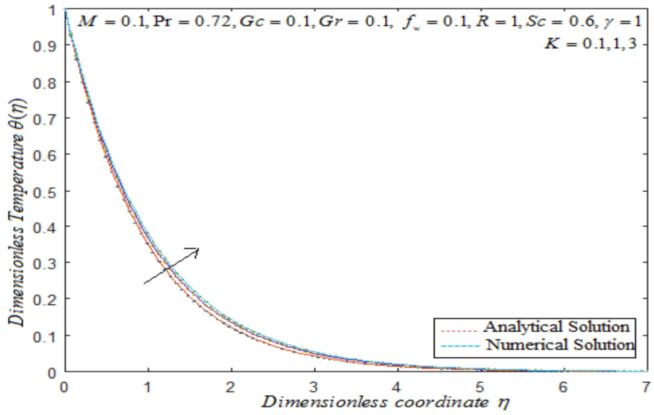

Fig:10. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R, Gr, Gc,* f_w, γ, *Pr ,Sc, M* and varying *K*=0.1, 1, 3

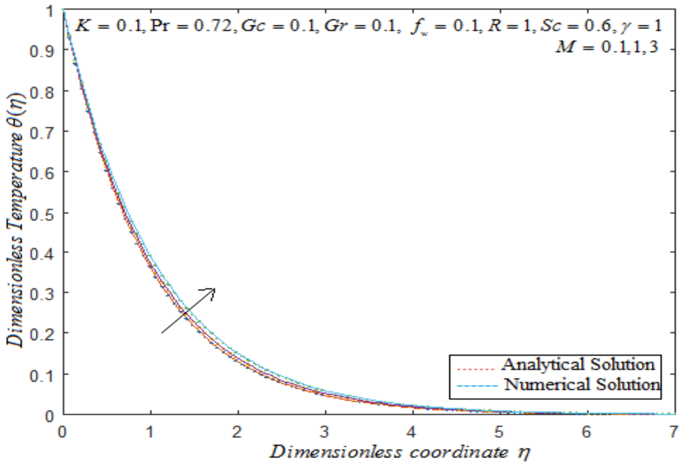

Fig:11. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R, Gr, Gc,* f_w, γ, *Pr ,Sc, K* and varying *M*=0.1, 1, 3

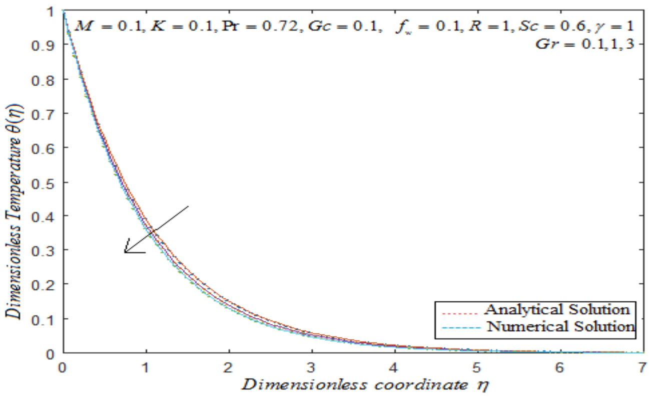

Fig:12. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R, K ,Gc,* f_w, γ, *Pr ,Sc, M* and varying *Gr*=0.1, 1, 3

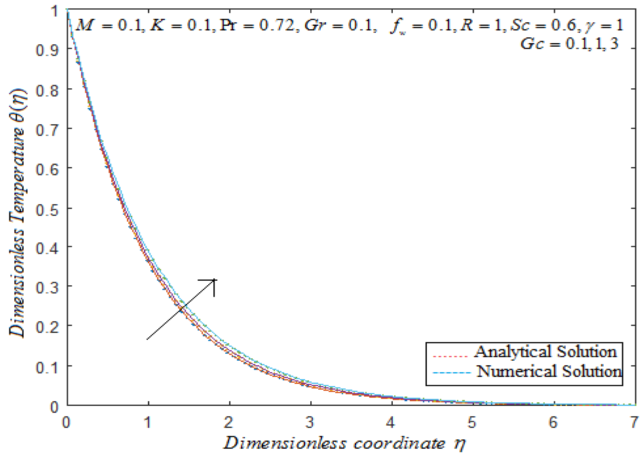

Fig:13. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R, Gr, K,* f_w, γ , *Pr* ,*Sc, M* and varying *Gc*=0.1, 1, 3

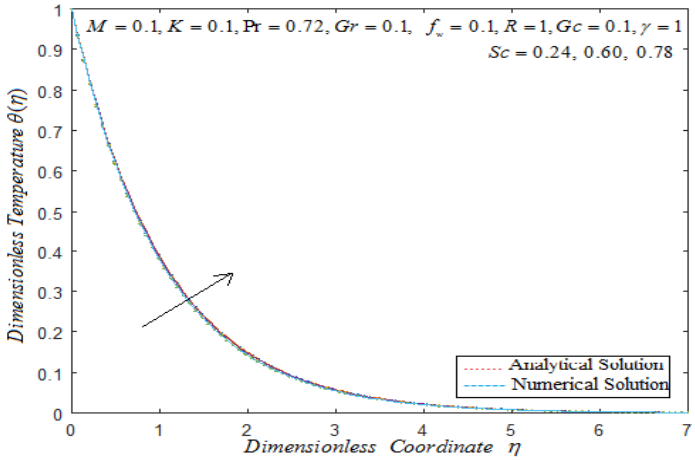

Fig:14. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R, Gr, Gc,* f_w, γ , *Pr* ,*K, M* and varying *Sc*=0.24, 0.60, 0.78

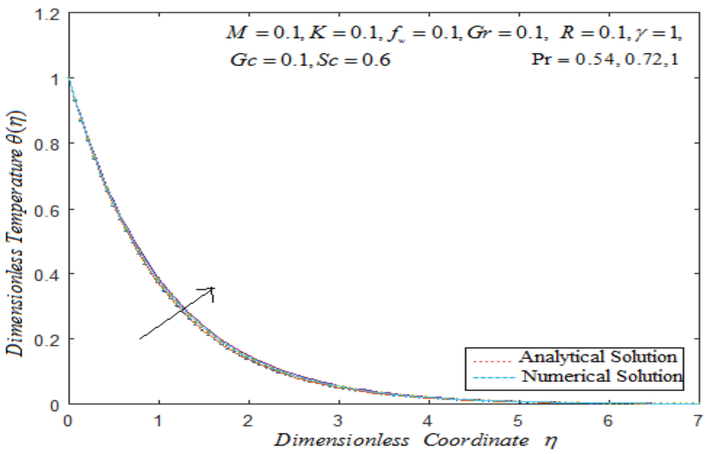

Fig:15. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R, Gr, Gc,* f_w, γ ,*K,Sc, M* and varying *Pr*=0.54, 0.72, 1

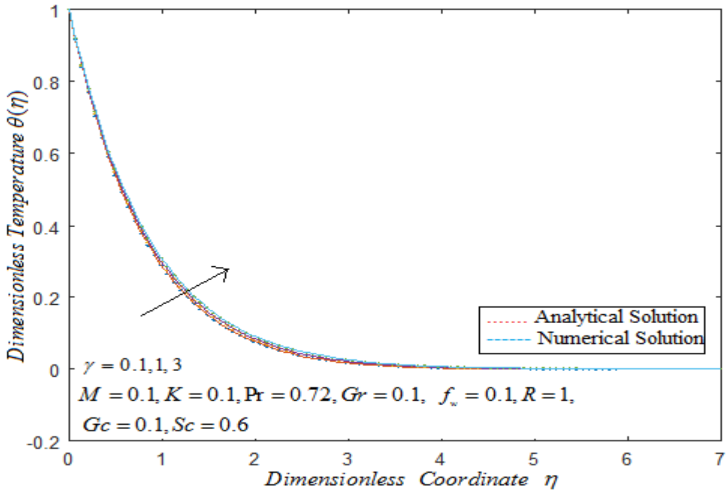

Fig:16. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R*, *Gr*, *Gc*, f_w,*K*, *Pr* ,*Sc*, *M* and varying γ =0.1, 1, 3

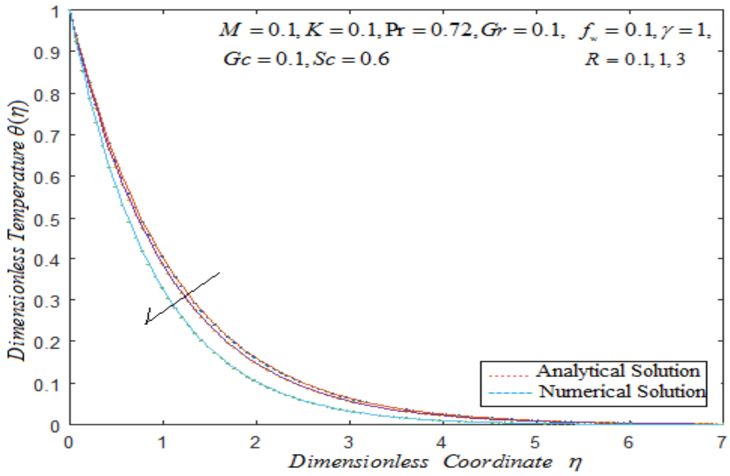

Fig:17. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed*K*, *Gr*, *Gc*, f_w, γ , *Pr* ,*Sc*, *M* and varying *R*=0.1, 1, 3

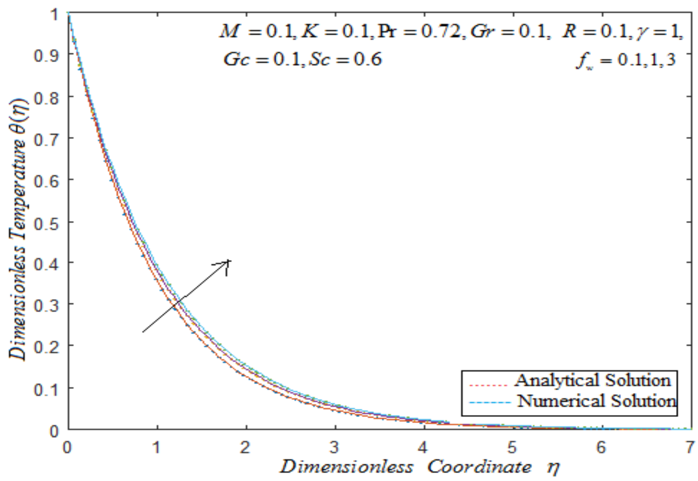

Fig:18. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (44) for fixed *R, Gr, Gc, K,* γ , *Pr ,Sc, M* and varying f_w=0.1, 1, 3

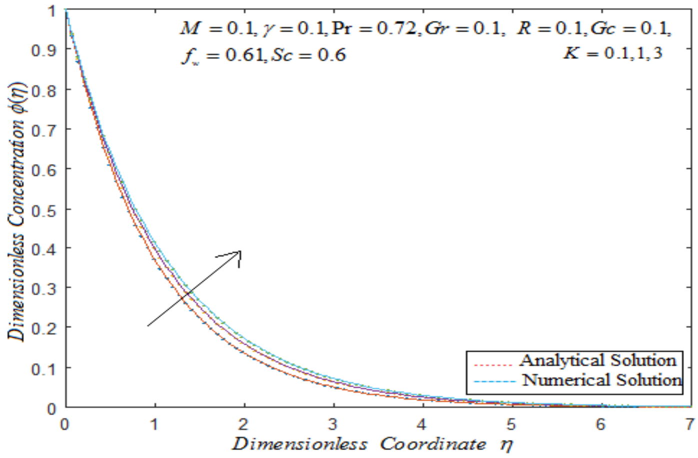

Fig:19. Dimensionless coordinate η versus concentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, Gr, Gc,* f_w, γ , *Pr ,Sc, M* and varying *K*=0.1, 1, 3.

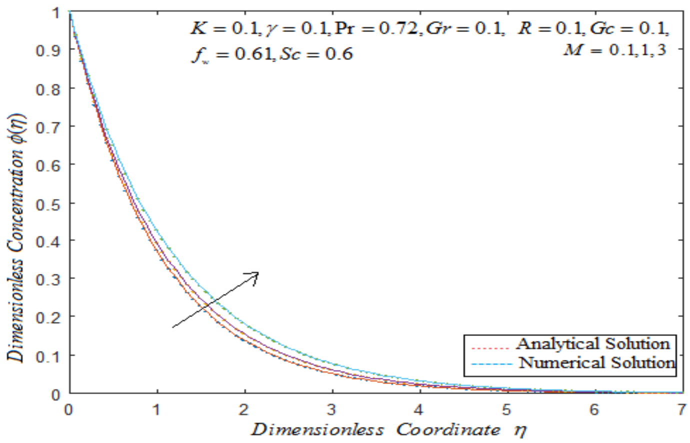

Fig:20. Dimensionless coordinate η versus concentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, Gr, Gc,* f_w, γ , *Pr ,Sc, K* and varying *M*=0.1, 1, 3.

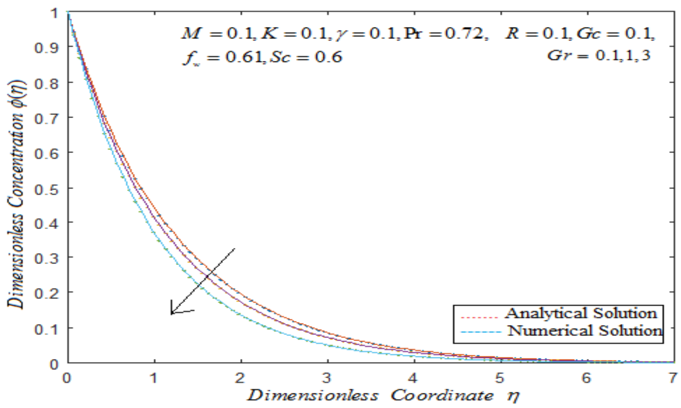

Fig:21. Dimensionless coordinate η versus concentration profile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, K, Gc,* f_w, γ , *Pr ,Sc, M* and varying *Gr*=0.1, 1, 3.

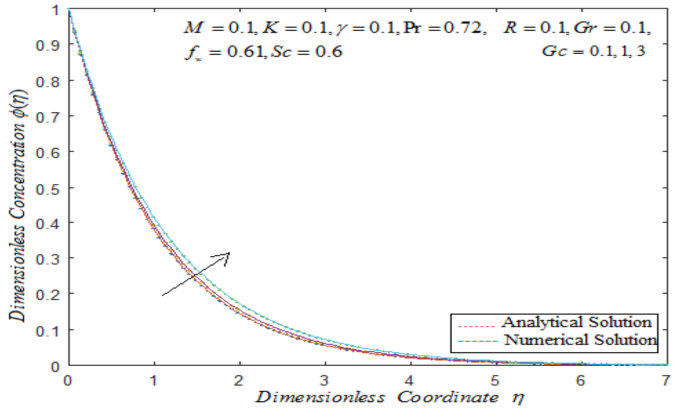

Fig:22. Dimensionless coordinate η versus concentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, Gr, K,* f_w, γ , *Pr ,Sc, M* and varying *Gc*=0.1, 1, 3.

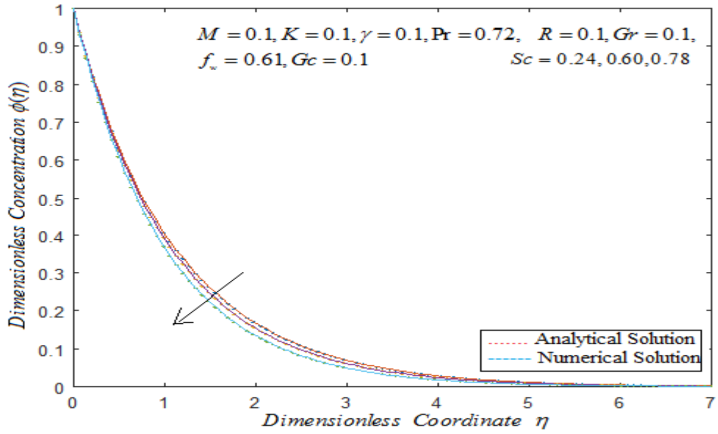

Fig:23. Dimensionless coordinate η versusconcentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, Gr, Gc,* f_w, γ , *Pr ,K, M* and varying *Sc*=0.24, 0.60, 0.78

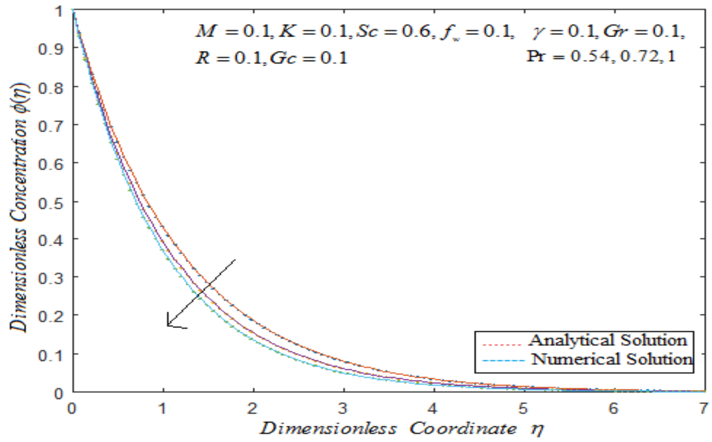

Fig:24. Dimensionless coordinate η versus concentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, Gr, Gc,* f_w, γ , *K,Sc, M* and varying *Pr*=0.54, 0.72, 1

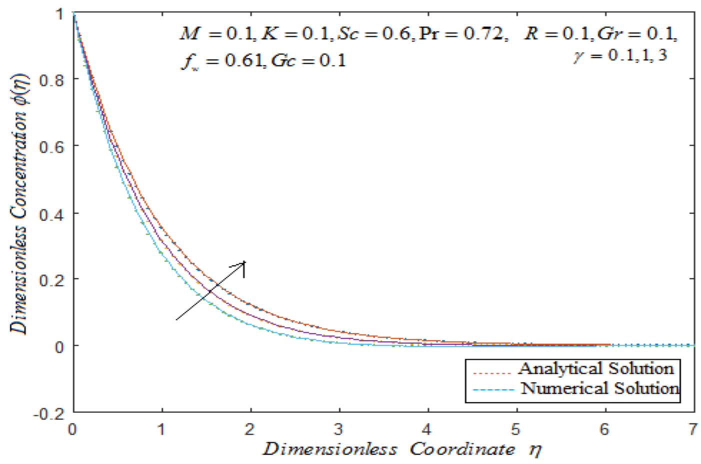

Fig:25. Dimensionless coordinate η versus concentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, Gr, Gc,* f_w,*K, Pr ,Sc, M* and varying γ =0.1, 1, 3.

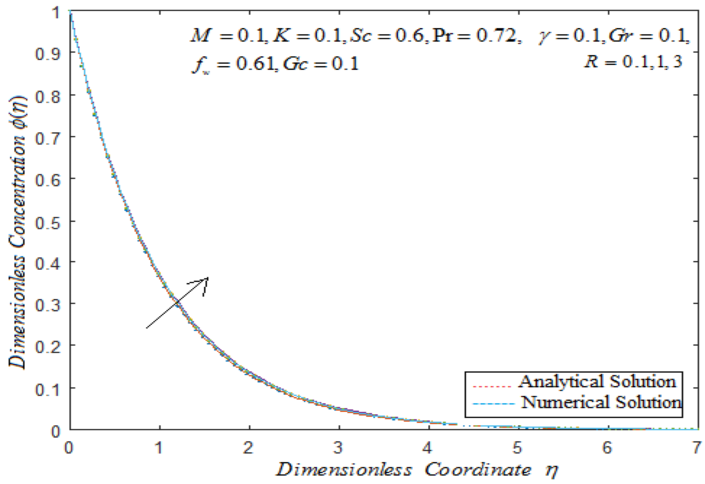

Fig:26. Dimensionless coordinate η versus concentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed*K, Gr, Gc,* f_w, γ , *Pr ,Sc, M* and varying *R*=0.1, 1, 3.

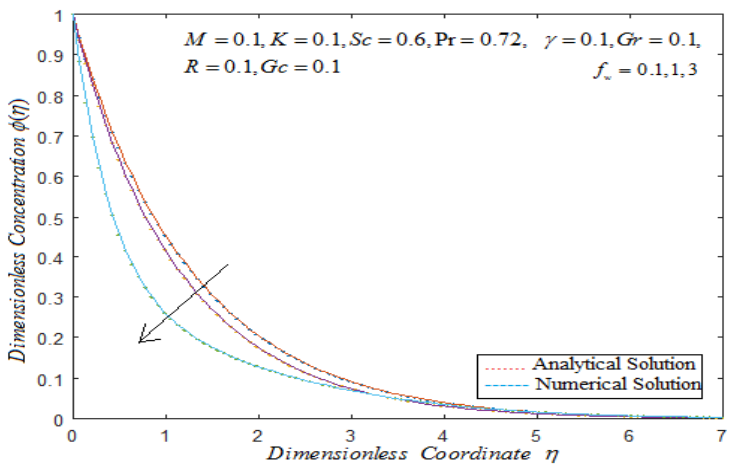

Fig:27. Dimensionless coordinate η versus concentrationprofile $\phi(\eta)$. The curve is plotted using (45) for fixed *R, Gr, Gc, K,* γ *, Pr ,Sc, M* and varying f_w=0.1, 1, 3.

Table.1 : Influence of theVelocity profile $f'(\eta)$ obtained in (43) on varying parameters.

M, Gr, Gc, Sc, γ *Pr, R,* $f_{w,}$	*K*	0.1	1	3
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.011055	0.023558	0.046818
	$\eta = 7$	0.000278	0.001367	0.003387
K, Gr, Gc, Sc, Pr, R, γ *,* f_w	*M*	0.1	1	3
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.008491	0.016049	0.018867
	$\eta = 7$	0.000055	0.000705	0.000959
M, K, Gc, Sc, Pr, R, γ *,* f_w	*Gr*	0.1	1	3
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.0183889	0.0329681	0.0924132
	$\eta = 7$	0.000918	0.0021885	0.007614
M, Gr, K, Sc, Pr, R, γ *,* f_w	*Gc*	0.1	1	3
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.0183889	0.02564189	0.012402
	$\eta = 7$	0.0009182	0.001550	0.014587
M, K, Gr, Gc, Pr, R, γ *,* f_w	*Sc*	0.24	0.60	0.78
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.018373	0.016916	0.014719
	$\eta = 7$	0.0009160	0.000789	0.000598
M, K, Gr, Gc, Sc, R, γ *,* f_w	*Pr*	0.54	0.72	1
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.018374	0.016516	0.014725
	$\eta = 7$	0.000917	0.000789	0.000601
M, K, Gr, Gc, Sc, Pr, γ *,* f_w	*R*	0.1	1	3
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.182691	0.016550	0.014726
	$\eta = 7$	0.000903	0.000758	0.000599
M, K, Gr, Gc, Sc, R, Pr, f_w	γ	0.1	1	3
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.018373	0.016916	0.014719

	$\eta = 7$	0.000916	0.000789	0.000598
M, K, Gr, Gc, Sc, R, γ *, Pr*	f_w	0.1	1	3
	$\eta = 0$	1.000022	1.000022	1.000022
	$\eta = 4$	0.007026	0.016834	0.031403
	$\eta = 7$	-0.0000702	0.000782	0.001043

Table.2 : Influence of the temperature profile $\theta(\eta)$ obtained in (44) on varying parameters.

M, Gr, Gc, Sc, Pr, R, γ *,* f_w	*K*	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.014397	0.0181472	0.020234
	$\eta = 7$	0.000575	0.000902	0.001083
K, Gr, Gc, Sc, Pr, R, γ *,* f_w	*M*	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.016685	0.018980	0.022316
	$\eta = 7$	0.000776	0.000974	0.001265
M, K, Gc, Sc, Pr, R, γ *,* f_w	*Gr*	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.022311	0.0189767	0.016641
	$\eta = 7$	0.001265	0.000974	0.000771
M, Gr, K, Sc, Pr, R, γ *,* f_w	*Gc*	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.016688	0.018977	0.022305
	$\eta = 7$	0.000775	0.000974	0.0012643

M, K, Gr, Gc, Pr, R, γ *,* f_w	*Sc*	0.24	0.60	0.78
	$\eta = 0$	1.000000	1.000000	1.000022
	$\eta = 4$	0.022019	0.0214775	0.020228
	$\eta = 7$	0.0012392	0.0011920	0.001083
M, K, Gr, Gc, Sc, R, γ *,* f_w	*Pr*	0.54	0.72	1
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.018150	0.0214775	0.020126
	$\eta = 7$	0.000926	0.0011920	0.001074
M, K, Gr, Gc, Sc, Pr, γ *,* f_w	*R*	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.024897	0.0214775	0.009727
	$\eta = 7$	0.001492	0.0011920	0.0001654
M, K, Gr, Gc, Sc, R, Pr, f_w	γ	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.001484	0.003567	0.005649
	$\eta = 7$	-0.000549	-0.000368	-0.000187
M, K, Gr, Gc, Sc, R, γ *, Pr*	f_w	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.015854	0.020373	0.022907
	$\eta = 7$	0.000702	0.0010893	0.001306

Table.3 : Influence of the concentration profile $\phi(\eta)$ obtained in (45) on varying parameters.

$M, Gr, Gc, Sc, Pr, R, \gamma, f_w$	K	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.018274	0.025141	0.029297
	$\eta = 7$	0.000908	0.001539	0.001921
$K, Gr, Gc, Sc, Pr, R, \gamma, f_w$	M	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.018760	0.023751	0.032073
	$\eta = 7$	0.00095	0.001411	0.002176
$M, K, Gc, Sc, Pr, R, \gamma, f_w$	Gr	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.036249	0.029313	0.018398
	$\eta = 7$	0.002559	0.001922	0.000919
$M, Gr, K, Sc, Pr, R, \gamma, f_w$	Gc	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.020980	0.023761	0.029322
	$\eta = 7$	0.001157	0.001412	0.001923
$M, K, Gr, Gc, Pr, R, \gamma, f_w$	Sc	0.24	0.60	0.78
	$\eta = 0$	1.000000	1.000000	1.000022
	$\eta = 4$	0.028469	0.023756	0.017975
	$\eta = 7$	0.001868	0.001412	0.000882
$M, K, Gr, Gc, Sc, R, \gamma, f_w$	Pr	0.54	0.72	1
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.033473	0.023756	0.018246
	$\eta = 7$	0.002304	0.001414	0.000905
$M, K, Gr, Gc, Sc, Pr, \gamma, f_w$	R	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.019731	0.018898	0.018218
	$\eta = 7$	0.001042	0.000965	0.000903
	γ	0.1	1	3

M, K, Gr, Gc, Sc, R, Pr, f_w	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.013643	0.004324	-0.003136
	$\eta = 7$	0.000471	-0.000372	-0.001017
M, K, Gr, Gc, Sc, R, γ, *Pr*	f_w	0.1	1	3
	$\eta = 0$	1.000000	1.000000	1.000000
	$\eta = 4$	0.039025	0.030504	0.034722
	$\eta = 7$	0.002814	0.002051	0.003222

Table.1 gives the numerical values of the derived analytical expression for velocity profile atη = 0, 4 and 7. From the values it is evident that $f'(\eta)$ increases with increase in *M, Gr, Gc*and$f_w$whereas it decreases with increase in *K, Sc,* γ, and *R.*

Table. 2 shows that the temperature raises as the parameters *M, K, Sc*, f_w and γ increases. While the value of temperature falls with increase in *Gr, Gc* and *R.*

Table.3 proves the fact that whenever the value of *K, M, R* andf_w are increased, the concentration profile also get increased. But the increase in *Gr, Gc, Sc* and γ decreases the concentration.

5.Conclusion.

In this paper Q-Homotopy analysis method is performed to study the effect of heat and mass transfer on a MHD flow of a electrically conducting fluid over a moving vertical porous plate. Analytical expression for velocity profile, temperature profile and concentration profile are determined and their effects on varying the governing parameters are discussed.

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Appendix: A

The differential equations are:

$$f''' + ff'' - f'^2 + Gr\theta + Gc\phi - (M+K)f' = 0 \quad \text{(A1)}$$

$$(1+\frac{16}{3R})\theta'' + \Pr(f\theta' - f'\theta) = 0 \text{ (A2)}$$

$$\phi'' + Sc(f\phi' - f'\phi) - Sc\gamma\phi = 0 \text{ (A3)}$$

with boundary conditions:

$$f' = 1, f = f_w, \theta = 1, \phi = 1 \; at \; \eta = 0$$
$$f' = 0, \theta = 0, \phi = 0 \;\; as \;\; \eta \to \infty \quad \text{(A4)}$$

Applying Q-Homotopy, we construct the Homotopy as:

$$(1-np)(f''' + f'') = hp(f''' + ff'' - f'^2 + Gr\theta + Gc\phi - (M+K)f') \text{ (A5)}$$

$$(1-nq)(\theta'' + \theta') = hq(1+\frac{16}{3R})\theta'' + \Pr(f\theta' - f'\theta) \text{ (A6)}$$

$$(1-nr)(\phi'' + \phi') = hr(\phi'' + Sc(f\phi' - f'\phi) - Sc\gamma\phi) \text{ (A7)}$$

The Analytical solution of (A1) to(A3) with boundary condition (A4) are given by:

$$f = \sum_0^\infty f_m \left(\frac{1}{n}\right)^m \text{ (A8)}$$

$$\theta = \sum_0^\infty \theta_m \left(\frac{1}{n}\right)^m \text{ (A9)}$$

$$\phi = \sum_0^\infty \phi_m \left(\frac{1}{n}\right)^m \text{ (A10)}$$

Substituting (A8), (A9) and (A10) in the equations (A5), (A6) ,(A7) and comparing the coefficients of the powers of p, q and r we get

$$p^0 : f_0''' + f_0'' = 0 \text{ (A11)}$$

$$p^1 : f_1''' + f_1'' - nf_0''' - nf_0'' + f_0''' + f_0 f_0'' - f_0'^2 + Gr\theta_0 + Gc\phi_0 - (M+K)f_0' = 0 \text{ (A12)}$$

$$p^2 : f_2''' + f_2'' - nf_1''' - nf_1'' + f_1''' + f_0 f_1'' + f_1 f_0'' - 2f_0' f_1' + Gr\theta_1 + Gc\phi_1 - (M+K)f_1' = 0 \text{ (A13)}$$

$$q^0 : \theta_0'' + \theta_0' = 0 \text{ (A14)}$$

$$q^1 : \theta_1'' + \theta_1' - n\theta_0'' - n\theta_0' + (1+\frac{16}{3R})\theta_0'' + \Pr(f_0\theta_0' - f_0'\theta_0) \text{ (A15)}$$

$$q^2 : \theta_2'' + \theta_2' - n\theta_1'' - n\theta_1' + (1+\frac{16}{3R})\theta_1'' + \Pr[(f_0\theta_1' + f_1\theta_0') - (f_0'\theta_1 + f_1'\theta_0)] = 0 \text{ (A16)}$$

$$r^0 : \phi_0'' + \phi_0' = 0 \text{ (A17)}$$

$$r^1 : \phi_2'' + \phi_2' - n\phi_1'' - n\phi_1' + \phi'' + Sc[(f_0\phi_1' + f_1\phi_0') - (f_0'\phi_1 + f_1'\phi_0) - Sc\gamma\phi_1 = 0 \text{ (A18)}$$

Solving the equations (A11) to (A18) with boundary conditions (A4) we get :

$$f_0 = 1 - f_w - e^{-\eta} \text{ (A19)}$$

$$f_1 = C_1 + C_2 e^{-\eta} - f_w \eta e^{-\eta} - (Gr + Gc)\eta e^{-\eta} + (M+K)\eta e^{-\eta} \text{ (A20)}$$

$$f_2 = C_3 + C_4 e^{-\eta} + P_1 \eta e^{-\eta} + P_2(\frac{\eta^2}{2} + 2\eta + 3)e^{-\eta} + P_3 \frac{e^{-2\eta}}{4} + P_4(\eta + 2)\frac{e^{-2\eta}}{4} \text{ (A21)}$$

$$\theta_0 = e^{-\eta} \text{ (A22)}$$

$$\theta_1 = (1+\frac{16}{3R})\eta e^{-\eta} - \Pr \eta e^{-\eta} + \Pr f_w \eta e^{-\eta} \text{ (A23)}$$

$$\theta_2 = C_5 e^{-\eta} - P_5 \eta e^{-\eta} - P_6 e^{-\eta}\left(\frac{\eta^2}{2} + \eta + 1\right) + P_7 \frac{e^{-2\eta}}{2} + P_8 \frac{e^{-2\eta}}{2}\left(\eta + \frac{3}{2}\right) \text{(A24)}$$

$$\phi_0 = e^{-\eta} \text{ (A25)}$$

$$\phi_1 = \eta e^{-\eta} - Sc\eta e^{-\eta} + Scf_w \eta e^{-\eta} - Sc\gamma\eta e^{-\eta} \text{ (A26)}$$

$$\phi_2 = C_6 e^{-\eta} - P_9 \eta e^{-\eta} - P_{10} e^{-\eta}\left(\frac{\eta^2}{2} + \eta + 1\right) + P_{11}\frac{e^{-2\eta}}{2} \text{ (A27)}$$

where,

$C_1 = f_w + (Gr + Gc) - (M + K)$ (A28)

$C_2 = -f_w - (Gr + Gc) + (M + K)$ (A29)

$C_3 = -P_1 - 2P_2 - \frac{1}{4}P_3 + \frac{1}{4}P_4$ (A30)

$C_4 = P_1 - P_2 - \frac{1}{2}P_3 - \frac{3}{4}P_4$ (A31)

$C_5 = P_6 - \frac{1}{2}P_7 - \frac{3}{4}P_8$ (A32)

$C_6 = P_{10} - \frac{1}{2}P_{11}$ (A33)

$$P_1 = -nf_w - n(Gr + Gc) + n(M + K) + f_w + 2(Gr + Gc) - (M + K) + C_2 f_w + 2 f_w^2 + 2 f_w (Gr + Gc) - 3 f_w (M + K) - C_2 (M + K) - (M + K)(Gr + Gc) + (M + K)^2 \quad \text{(A34)}$$

$$P_2 = -f_w^2 - f_w (Gr + Gc) - Gr\left(1 + \frac{16}{3R}\right) + Gr\Pr - Gr\Pr f_w - Gc + GcSc - GcScf_w + GcSc\gamma + 2(M + K) f_w + (M + K)(Gr + Gc) - (M + K)^2 \quad \text{(A35)}$$

$P_3 = -(M + K) + (Gr + Gc)$ (A36)

$P_4 = (M + K) - (Gr + Gc)$ (A37)

$$P_5 = -n\left(1 + \frac{16}{3R}\right) + n\Pr - n\Pr f_w - \left(1 + \frac{16}{3R}\right)^2 + 2\Pr f_w \left(1 + \frac{16}{3R}\right) + \Pr^2 - 2\Pr^2 f_w + C_1 \Pr + C_2 \Pr + \Pr^2 f_w^2 \quad \text{(A38)}$$

$$P_6 = \left(1 + \frac{16}{3R}\right) - 2 f_w \Pr\left(1 + \frac{16}{3R}\right) - \Pr^2 + 2\Pr^2 f_w + \Pr^2 f_w^2 \quad \text{(A39)}$$

$$P_7 = \left(1 + \frac{16}{3R}\right)\Pr - \Pr^2 + \Pr^2 f_w + C_2 + \Pr f_w + (Gr + Gc) - \Pr(M + K) \quad \text{(A40)}$$

$$P_8 = 2\Pr^2 f_w - 2\Pr f_w - 2(Gr + Gc)\Pr + 2(M + K)\Pr \quad \text{(A41)}$$

$$P_9 = nSc - nScf_w + nSc\gamma + n + 2 - 2Sc + 2Scf_w - 2Sc\gamma - Sc - f_w Sc^2 + Sc^2 f_w^2 - Sc^2 f_w \gamma + C_1 Sc \quad \text{(A42)}$$

$$P_{10} = -1 + 2Sc - 2Scf_w + 2Sc\gamma - Sc^2 + 2Sc^2 f_w - 2Sc^2\gamma - Scf_w - Sc^2 f_w^2 + 2Sc^2 f_w \gamma - Sc^2\gamma^2 \quad \text{(A43)}$$

$$P_{11} = -2n - Scf_w - (Gr + Gc)Sc + (M + K)Sc \quad \text{(A44)}$$

Substituting (A19) to (A44) in (A8), (A9) and (A10) we obtain the analytical expression for velocity profile, temperature profile and concentration profile.

Appendix B : Nomenclature

Symbol	Meaning
x, y	Cartesian coordinates
u, v	Dimensionless Cartesian coordinates
υ	Kinematics viscosity
B_0	Uniform magnetic field
T	Temperature inside the boundary layer

T_∞	Temperature far away from the plate
C	Concentration in the boundary layer
C_∞	Concentration of the ambient fluid
k_1	Rate constant
C_p	Specific heat constant
q_r	The relative heat flux
D	Molecular diffusity
B	Constant
a, b	Stratification rate
M	Magnetic parameter
K	Permeability parameter
Gr	Temperature Grashof number
Gc	Concentration Grashof number
Pr	Prandtl number
Sc	Schmict number
γ	Chemical reaction parameter
R	Radiation parameter
f'	Dimensionless velocity profile
θ	Dimensionless temperature profile
ϕ	Dimensionless concentration profile
f_w	Suction parameter
β, β^*	Thermal and concentration expansion parameter
σ	Electrical conductivity
ρ	Density
κ'	Permeability of the porous medium
α	Thermal diffusivity
η	Dimensionless coordinates