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MATHEMATICAL ANALYSIS ON THE EFFECT OF CHEMICAL REACTION ON MASS TRANSFER OVER A STRETCHING SURFACE EMBEDDED IN A POROUS MEDIUM

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Abstract: An approximate but very precise analytical solution is derived to analyse the effect of chemical reaction on mass transfer over a stretching surface embedded in a porous medium. Adopting similarity transformation technique, the governing partial differential equations are transformed intoordinary differential equations. In this paper, we used New Homotopyanalysis method(NHAM) to get accurate analytical expressionsof the dimensionless velocity profiles and the dimensionless concentration profiles.Using the dimensionless velocity profiles and dimensionless concentration profiles, we can also derive the approximate analytical expressions for the local skin friction coefficient and rate of change of mass transfer. This method is easy to carry out the analytical solution and the result is more precise. The numerical values and graphical representations of local skin friction coefficient and rate of change of mass transfer for various values of permeability parameter, Schmidt number, reaction rate parameter and the power law exponent have shown.The proposed analytical solution is in excellent agreement with the numerical results of previous study.

Keywords: Mass transfer – Stretching surface – Chemical reaction – New Homotopy Analysis method(NHAM)- Analytical solutions.

1. Introduction

Stretching of surface in porous medium enables boundary layer flow which a relevant process to a number of engineering tasks such as paper production, preparing plastic, metal sheets, etc. This phenomenon of boundary layer flow over a stretching surface is a ground breaking work of Sakiadis[1,2] in 1961 who studied the boundary layer flow over a continuous solid surface moving with stable speed. Further, Crane [3] analyzed the steady two dimensional boundary layer flow which results from stretching of elastic flat surface which moves with a speed which differs linearly with distance from a particular point. A lot of mathematical results can be reviewed in literatures by various authors who conducted a case steady state flow like: Ali [4] and [5], Elbashbeshy [6], Ishak et al. [7] and Elbashbeshy and Bazid [8]. The unsteady state problem over a stretching surface, which is stretched with a velocity that depends on time, is also considered by Anderson et al. [9], Elbashbeshy and Bazid [10] and Ishak et al. [11]. Chemically reactive solute distribution on fluid flow has an

impact because of a stretching surface, is equally essential for engineering professionals. The physical effects of chemical reaction were studied by a lot of research personnel's. The diffusion of a chemically reactive species in a laminar boundary layer flow over a flat surface plate was demonstrated by Chambre and Young [12]. The impact of transfer of chemically reactive species is also explained by Andersson et al. [13]. Takhar et al. [14] where analysis of the flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species with n-th order reacts. The mass transfer in boundary layer flows because of a stretching surface in porous medium, which contains essential applications in the industrial sector issues. Further it was investigated by Radwan and Elbashbeshy who analysed with magnetic field the flow and mass transfer on a stretching surface.[15, 16]. Later, a solution account for diffusion of chemically reactive species in a flow of a non- Newtonian fluid over a stretching sheet immersed in a porous medium was determined. El-Aziz [17] illuminated unsteady flow occurring as a result of a stretching sheet with mass and heat transfer. Recently, Krishnendu [18] studied the boundary layer flow with first order chemical reaction over a porous flat plate. Krishnendu [19], moreover studied the mass transfer on a consistent flat plate moving in a parallel or reversely to a free stream in the presence of a chemical reaction. Ferdows et al. [20] investigated the effects of order of chemical reaction on mass transfer over a linearly stretching surface. From the reviewed secondary studies on investigations and applications, the researcher opts on a paper on two-dimensional steady, incompressible, laminar boundary layer flow of a fluid over a linearly stretching surface. This study paper investigates statistically the effects of chemical reaction on the steady laminar two-dimensional boundary layer flow and mass transfer over a stretching surface embedded in porous medium. The method of solutions based on the well-known similarity analysis together with shooting method is employed to find numerical solution [22].
The main purpose of this analysis is to present analytical solution of mass transfer over a stretching surface embedded in a porous medium with the presence of chemical reaction. The governing system of partial differential equations reduced to ordinary differential equations which were solved analytically by New Homotopy Analysis Method. This method is effective and involves less computation to find analytical solution of strongly non-linear differential equations. The effect of Schmidt number, Reaction rate parameter, permeability number and power law exponent over a fluid flow and dimensionless concentration profiles are discussed in terms of graphical representation. We also calculate values of skin friction coefficient and mass transfer coefficient demonstrating excellent agreement between present analytical work and previous numerical work.

2. Mathematical Formulation of the problem

In this paper we have considered two dimensional steady, laminar boundary layer flow of a fluid over a linearly stretching surface (i.e stretched with a velocity proportional to x) embedded in porous medium with velocity u_w and concentration C_w moving axially through a stationary fluid. Influence of physical properties, the fluid is viscous incompressible. We have taken that the fluid far away from the surface is at rest and at concentration C_∞. The x- axis is the continuous surface in the direction of motion and y- axis is perpendicular to it. The continuity, momentum and reactive concentration equations for governing the flow and concentration distribution in the boundary layer region along the stretching surface may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k}u \tag{2}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R(C - C_\infty) \tag{3}$$

The boundary conditions are given by

$$y = 0: u = \lambda x, v = 0\ , C = C_\infty + Ax^n \tag{4}$$

$$y \to \infty:\ u = 0, C = C_\infty \ (5)$$

Where u and v are the velocity components in the x and y directions respectively, k is the permeability of porous medium, C is the fluid concentration, v is the kinematic viscosity, R is the constant of first-order chemical reaction rate, n is a power-law exponent, which signifies the change of amount of solute in the x-direction, D is the effective diffusion coefficients , A and λ are constants. If we have chosen a dimensionless stream function $\psi(x, y)$, then the equation of continuity is fulfilled.

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

In order to solve equ.,(1)-(4), we have to use similarity transformation and dimensionless concentration.

$$\eta = y\sqrt{\frac{n}{v}}, \qquad f(\eta) = \frac{\psi(x, y)}{x\sqrt{nv}}, \qquad \theta(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

When η is the similarity variable, $\theta(\eta)$ is the dimensionless concentration and $\psi(x, y)$ is the dimensionless stream function. Accordingly, equation(2)-(3) and the boundary conditions (4-5) are written in the following form

$$f''' + f\, f'' - f'^2 - K\, f' = 0 \tag{8}$$

$$\theta'' + Sc(f\theta' - n\, f'\theta) - Sc\, Rc\, \theta = 0 \tag{9}$$

When $\eta = 0$, then $f = 0,\ f' = 1$ and $\theta = 1$ (10)

When $\eta = \infty$, then $f' = 0$ and $\theta = 0$ (11)

Where prime denotes differentiation with respect to η. $K = \frac{v}{nk}$ is the permeability parameter, $Rc = \frac{R}{n}$ is a reaction rate parameter of the solute and $Sc = \frac{v}{D}$ is Schmidt parameter.

$$Sh = -\frac{x\left(\frac{\partial C}{\partial y}\right)}{C_w - C_\infty} = -x\sqrt{\frac{n}{v}}\ \ \theta'(0) \tag{12}$$

$$\frac{Sh}{\sqrt{R_e}} = \theta'(0) \ (13)$$

Where $R_e = \frac{u_w x}{v}$ is the local Reynolds number.

3. Approximate Analytical Solutions

(i)Homotopy Analysis Method (HAM)

Homotopy analysis method is a non-perturbative analytical method for finding series solutions to nonlinear equations and has been effectively applied to numerous problems in science engineering [24 – 34]. In comparison with other perturbative and non-perturbative analytical methods, HAMagreementstheabilitytoadjustandcontroltheconvergenceofa solution via the so – calledconvergence- control parameter. Because of this, HAM has showed to be the most effective method for obtaining analytical solutions to highly nonlinear differential equations. Previous applications of HAM have mainly concentrated on nonlinear differential equations in which the non-linearity is a polynomial in terms of the unknown function and itsderivatives.

Liao [23– 31] proposed this effective analytical method for nonlinear problems, namely the Homotopy analysis method. This method deals an analytical solution in terms of an infinite power series. Nevertheless, on that point is a pragmatic need to value this solution and to obtain numerical values from the infinite power series. In order to examine the accuracy of the Homotopy analysis method (HAM) solution in a finite number of terms, the system of differential equations was answered. The

Homotopy analysis method is aneffectivetechnique comparing to other perturbation methods. HomotopyperturbationmethodisanextraordinaryinstanceoftheHomotopyanalysismethod.Different from all stated perturbation and non-perturbative techniques, the Homotopy analysis method itself offersuswithaconvenientmeanstoholdandadjusttheconvergenceregionandrateofapproximation series, when required. Briefly speaking, the Homotopy analysis method has the following benefits. It is valid even if a given nonlinear problem does not hold in any small/large parameter at all; it can be used to powerfully approximate a non-linear problem by choosing different sets of basic functions. The Homotopy analysis method contains the auxiliary parameter, which delivers us with a simple means to adjust and hold the overlap area of the solution series. Recently, a new approach to HAM isintroduced to solve the nonlinear problem, in which one will get better simple approximate solution in the zeroth iteration. The basic concept of this method has been described in Appendix A. Detailed derivations of the dimensionless velocity profile and dimensionless concentration profile are described in AppendixB.

(ii) *Analytical expression of the concentration profiles of* $f(\eta)$ *and* $\theta(\eta)$ *in the stretching surface using New Homotopy Analysis Method*

The analytical solution of nonlinear differential equations is of great significance due to its wide application in scientific research. The New Homotopy Analysis approach is applied to find the approximate analytical expressions of non-linear differential Eqns. (8-9) with the boundary condition (10-11) are as follows.

$$f(\eta)=\frac{1}{\sqrt{1+K}}-\frac{e^{-\sqrt{1+K}\eta}}{\sqrt{1+K}}+h\left(\frac{1+K-(1+K)^2}{\sqrt{1+K}\,(K(1+K)-1)}-\frac{\sqrt{1+K}e^{-\sqrt{1+K}\eta}}{K(1+K)-1}+\frac{(1+K)^{3/2}e^{-\frac{\eta}{\sqrt{1+K}}}}{K(1+K)-1}\right) \quad (14)$$

$$\theta(\eta)=e^{-\sqrt{ScRc}\eta}+h\left(\begin{array}{l}\frac{Sc\sqrt{ScRc}e^{-\sqrt{ScRc}\eta}}{\sqrt{1+K}\,A}-\frac{nSce^{-\sqrt{ScRc}\eta}}{A}+\frac{nSce^{-(\sqrt{1+K}+\sqrt{ScRc})\eta}}{A}\\ -\frac{Sc\sqrt{ScRc}e^{-(\sqrt{1+K}+\sqrt{ScRc})\eta}}{\sqrt{1+K}\,A}+\frac{\eta Sce^{-\sqrt{ScRc}\eta}}{2K}\end{array}\right) \quad (15)$$

Where $A=1+K+2\sqrt{1+K}\sqrt{ScRc}$

4. Comparison of Analytical expressions with experimental work

The analytical expressions (14)-(15) of non-linear differential equations (8)-(9) are compared with previous numerical expressions and excellent agreement is noticed. Numerical expressions are obtained by shooting method along with fourth order classical Runge-Kutta method. Our analytical expressions of the boundary value problem are derived by New Homotopy Analysis Method. Fig., (1)-(6) and error tables show that NHAM is the one of the effective method.

5. Results and Discussion:

The system of non-linear differential equations (8) & (9) satisfying the boundary conditions(10)&(11) have been solved analytically using New Homotopy Method(NHAM). In table 1, we have shown that Mass transfer coefficient which is related with Schmidt parameter Sc, reaction rate parameter Rc with constant solute along the surface ($n=0$).From table 1, it is seen that constant solute along the surface $n=0$ and $K=0$, for the mass transfer coefficient $-\theta'(0)$ is compared with previous numerical results such as Elbashbeshy et al.[22],Takhar et al[14]., andersson et al.[9], and Uddin et al[21] and fabulous agreement is noted.

TABLE 1: Comparison of the analytical values of Mass transfer coefficient $-\theta'(0)$ with that of Numerical values for $n = 0$ and $K = 0$.

Sc (Schmidt parameter)	Rc (Chemical reaction rate parameter)	Present study (Analytical solution)	Elbashbeshy et al. (Numerical solution)	Takhar et al. [14]	Andersson et al.[13]	Uddin et al.[21]
0.1	0.1	0.149083	0.149083	0.15042	0.14900	0.15057
1	0.1	0.668758	0.668754	0.67044	0.66900	0.66873
1	1	1.176400	1.176401	1.17761	1.17700	1.17679
10	1	3.871337	3.871327	3.87469	3.88000	3.87347
10	10	10..241185	10..241185	10..25000	10..25000	10..24535

In order to evaluate the accuracy of present study, the values for skin friction $f''(0)$ and mass transfer $-\theta'(0)$ for various values of permeability parameter K are given in table 2 which shows excellent agreement with previous study. If increasing permeability value K increases the skin friction coefficient and decreases the rateof mass transfer.

TABLE 2: Values of skin friction coefficient $f''(0)$ and Mass transfer coefficient $-\theta'(0)$ for various values of permeability parameter K

K	$f''(0)$ (Analytical Values)	$f''(0)$ (Numerical Values)	ERROR (percentage)	$-\theta'(0)$ (Analytical Values)	$-\theta'(0)$ (Numerical Values)	ABSOLUTE ERROR (percentage)
0	-1.000001	-1.00000	0.99×10^{-8}	1.459422	1.45942	0.13×10^{-7}
1	-1.4142135	-1.414214	0.19×10^{-9}	1.4142017	1.414214	0.8×10^{-7}
4	-2.2360679	-2.236068	0.44×10^{-9}	1.3433	1..34331	0.7×10^{-7}
10	-3.3166247	-3.316625	0.9×10^{-9}	1..2781	1..27811	0.6×10^{-7}

TABLE 3: Comparison of the analytical values of Mass transfer coefficient $-\theta'(0)$ with that of the Numerical values for various values of Schmidt number Sc

Sc	$-\theta'(0)$ (Analytical values)	$-\theta'(0)$ (Numerical values)	ABSOLUTE ERROR (Percentage)
1	1.4142129	1.414214	0.71×10^{-6}
2	2.063934	2.063933	0.19×10^{-8}
5	3.3660921	3..366092	0.8×10^{-6}
11	5.0854758	5.085476	0.14×10^{-5}

TABLE 4: Comparison of the analytical values of Mass transfer coefficient $-\theta'(0)$ with that of the Numerical values for various values of Reaction rate parameter Rc

Rc	$-\theta'(0)$ (Analytical values)	$-\theta'(0)$ (Numerical values)	ABSOLUTE ERROR (Percentage)
0	0.892506	0.892503	0.33×10^{-6}
1	1.4142129	1.414214	0.71×10^{-6}
4	2.2692289	2.2692285	0.17×10^{-7}
12	3.641533	3.641534	0.27×10^{-8}

TABLE 5: Comparison of the analytical values of Mass transfer coefficient $-\theta'(0)$ with that of the Numerical values for various values of power law exponent n

n	$-\theta'(0)$ (Analytical values)	$-\theta'(0)$ (Numerical values)	ABSOLUTE ERROR (Percentage)
0	1.154941074	1.154956	0.12×10^{-5}
2	1.649915587	1.649948	0.19×10^{-6}
6	2.434264120	2.434269	0.2×10^{-7}
10	3.061960099	3.061963	0.2×10^{-6}

Table 3-5 summarize the calculated values of mass transfer coefficient $-\theta'(0)$ for the values $1\le Sc\le 11, 0\le n\le 0$ and $0\le Rc\le 12$. If we increase the values of Sc, n and Rc then the rate ofmass transfer also increasing. Since K does not occur explicitly in the diffusion equation, its effect on mass transfer is very small. Fig. 1 shows that the effect of permeability parameter on dimensionless velocity profile $f'(\eta)$. From this plot, it is noted that increasing value of permeability number K is to decrease the dimensionless velocity in the flow. Thus it is reveals that permeability parameter opposes the motion. As a result, the momentum boundary layer thickness reduces with the increasing value of K and this fact is also realised from wall shear stress behaviour. From fig. 2, concentration boundary layer thickness increases as permeability parameter increases. This suggests that permeability parameter performances to boost the distribution of the reaction solute on the stretching surface. Fig.3 shows that variation of solute curves for different values of Schmidt number Sc. The Schmidt number has most important effects on the distribution of solute. Increasing value of Sc, suddenly decreases the concentration boundary layer thickness. This is due to the fact that the rate of solute transfer from the surface increases when the Schmidt number increases. Figure 4 is the graphical representation of concentration profiles $\theta(\eta)$ for different values of chemical reaction rate parameter Rc. Reaction rate parameter is also affect the concentration profiles $\theta(\eta)$ in the same way as that of Schmidt number that is the increase value of reaction rate reduces solute boundary layer thickness. Figure 5 depicts dimensionless concentration $\theta(\eta)$ for several values of power law exponent $n\,(n>0)$. From the figure it is noticed that increasing value of positive n decreases the distribution of solute. Fig. 6 reveals that concentration profile $\theta(\eta)$ increases with increase in magnitude of $n\,(n<0)$ and for large negative values of n, the pass of solute is observed near the surface.

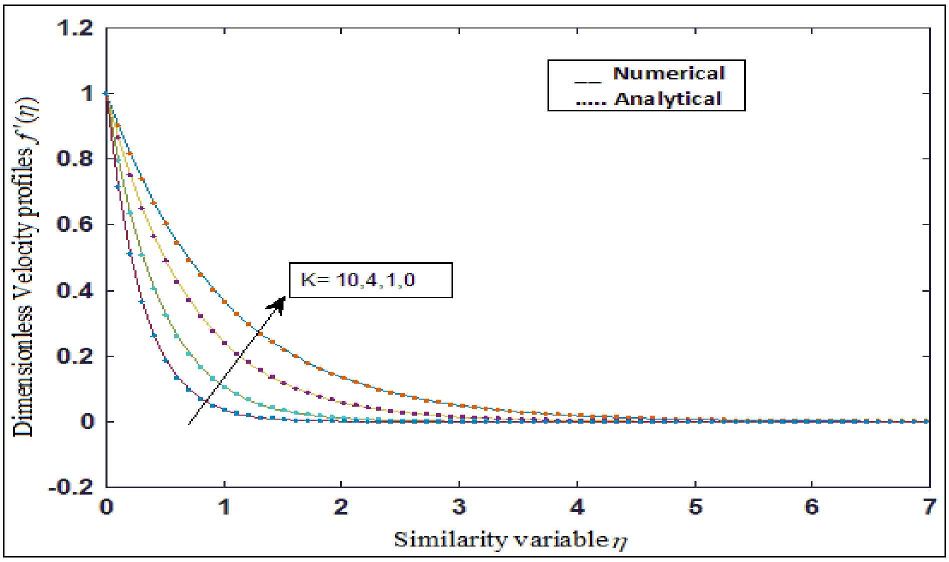

Figure1: The dimensionless velocity Profiles $f'(\eta)$ verses similarity variable η for the various values of K is plotted using the equation (14).

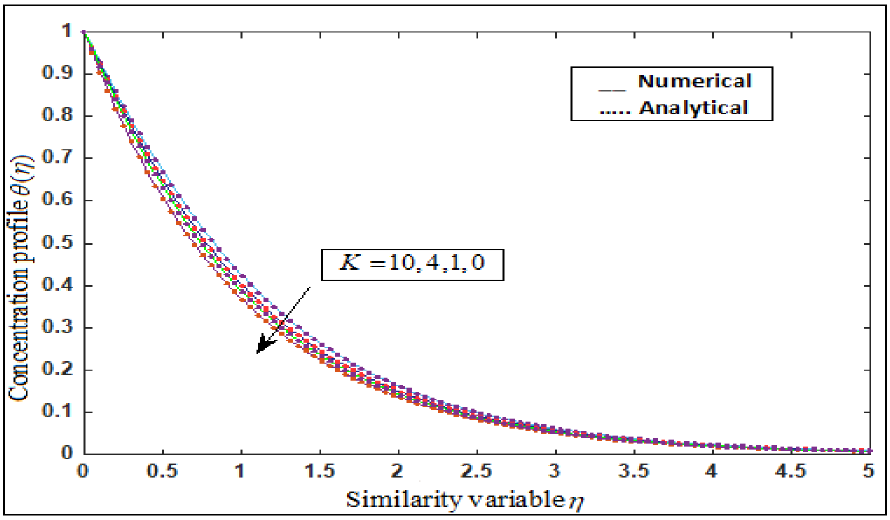

Figure 2: The dimensionless Concentration Profiles $\theta(\eta)$ verses similarity variable η for the various values of K and some fixed values as $Sc = 1,\ n = 1$ and $Rc = 1$ is plotted using the equation(15).

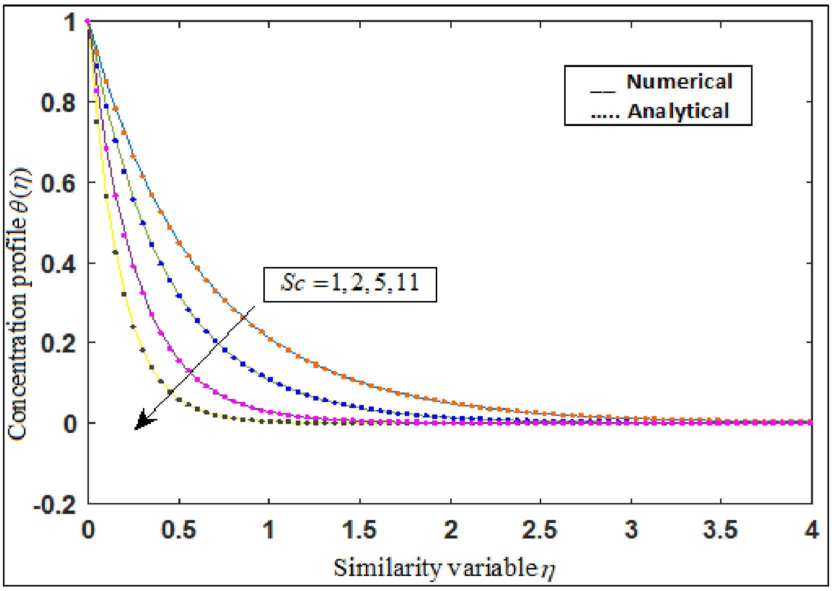

Figure 3: The dimensionless Concentration Profiles $\theta(\eta)$ verses similarity variable η for the various values of Sc and some fixed values as $K = 1,\ n = 1$ and $Rc = 1$ is plotted using the equation(15).

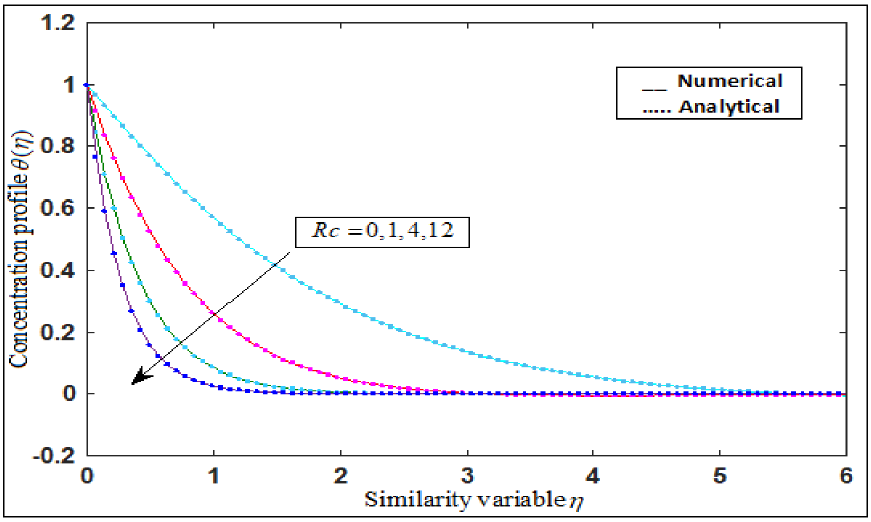

Figure 4: The dimensionless Concentration Profiles $\theta(\eta)$ verses similarity variable η for the various values of Rc and some fixed values as $Sc = 1$, $n = 1$ and $K = 1$ is plotted using the equation(15).

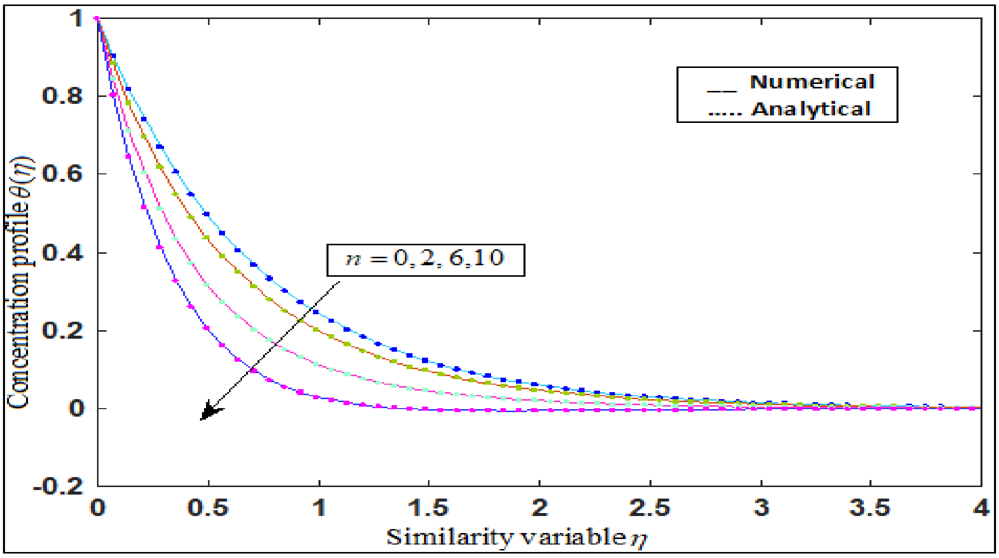

Figure 5: The dimensionless Concentration Profiles $\theta(\eta)$ verses similarity variable η for the various values of n (positive) and some fixed values as $Sc = 1$, $Rc = 1$ and $K = 1$ is plotted using the equation(15).

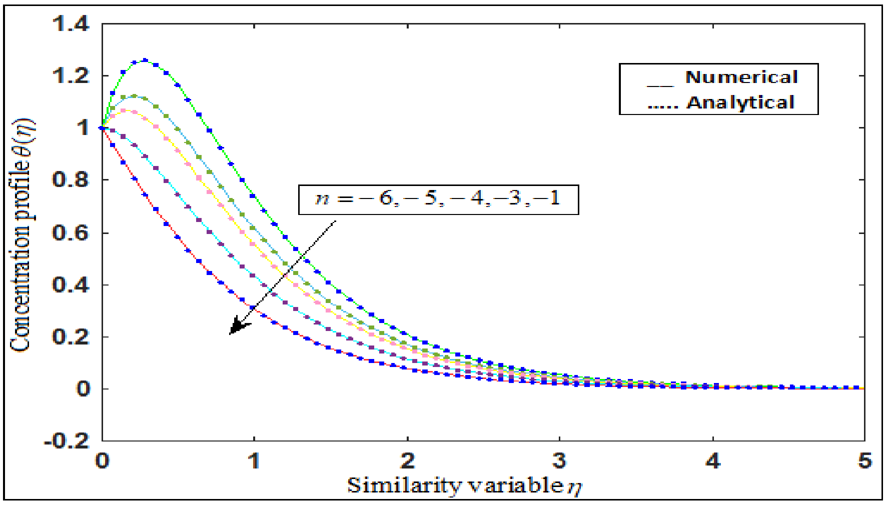

Figure 6: The dimensionless Concentration Profiles $\theta(\eta)$ verses similarity variable η for the various values of n (negative)and some fixed values as $Sc = 1$, $K = 1$ and $Rc = 1$ is plotted using the equation(15).

6. Conclusions

In this work, approximate analytical solution for non-linear differential equations has been presented using New Homotopy Analysis Method. We have also presented the analytical expression dimensionless velocity profile and concentration profile. Further, based on the outcome of this work, we can calculate the amount skin friction coefficient and mass transfer coefficient. Our analytical results are successfully compared with numerical results and excellent agreement is attained which is presented in tables. We have shown the variation of concentration profile for various values Schmidt number, power law exponent, Reaction rate parameter and permeability number in graphical representation. Under this investigation, an analysis is performed to ascertain the character of distribution of reactive solute which undergoes a first order reaction in steady boundary layer flow incompressible fluid over a stretching surface.

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