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MATHEMATICAL ANALYSIS OF HEAT AND MASS TRANSFER EFFECTS ON STEADY MHD FLOW

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Abstract. A study of heat and mass transfer effects on a steady MHD flow has been carried out. The non-linear differential equations governing the study are solved analytically using modified Homotopy analysis method. The effects of various parameters on velocity profile, temperature profile and concentration profile are discussed. The accuracy of the analytical solution while comparing with the previous results shows good agreement.

Keywords: Boundary layer flow; Heat generation parameter; Non-linear differential equations: Modified Homotopyanalysis method.

1. Introduction

The effects of heat and mass transfer on a steady MHD flow are inevitable in a wide range of industrial processes. Many investigationshave been carried out to analyze the hydromagnetic fluid flow on a continuous stretching sheet in the presence of uniform magnetic field .[2] was the first to study the boundary layer behavior of the flow in various dimensions.[3] and [4] observed the effects when the temperature difference between the surface and the fluid is proportional to a power of the distance from a fixed point.[5] Demonstrated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. [6] two[8] extended the study of fluid flow in the presence of uniform magnetic field. [9], [10] monitored the heat transfer effect of the MHD fluid in their work. Many other works addressing the thermal radiation on hydro-magnetic flow due to an exponential stretching heart were made [11] to [15]. The effects of viscous dissipation in natural convection are explained by [16] to [21].

With the knowledge of previous works done by [2] to [21], [1] studied the heat generation and radiation of a MHD fluid flow over an exponentially stretching surface. By means of similarity transformation [1] reduced the partial differential equations to non-linear differential equations and solved numerically. In this work, the system of non-linear differential equations obtained by [1] is solved analytically using modified HAM. The obtained results are compared with the numerical results and their effects on varying the governing parameters are discussed graphically.

2. Mathematical formulation of the problem

Consider the two-dimensional magnetohydrodynamic flow over a stretching sheet. The x-axis is taken along the stretching surface in the direction of motion and y-axis is perpendicular to it. The temperature and concentration are far away from the fluid and are assumed to be T_∞ and C_∞ respectively as shown in Fig:1.

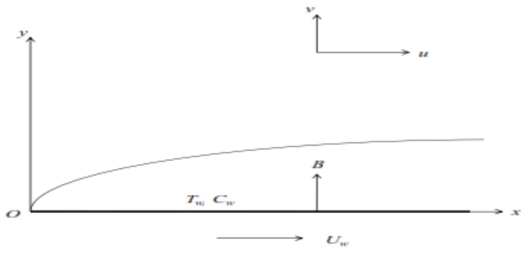

Figure 1 Schematics of the problem.

The equations governing the momentum, heat and mass transfers are given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=V\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{V}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2}{\rho C_p}u^2+\frac{Q_0}{\rho C_p}T-T_\infty \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where *u* and *v* are velocity components in the *x*, *y* directions respectively, *V* is the kinematic viscosity, ρ is the density, σ is the electrical conductivity of the fluid, *T* is the temperature, *C* is the concentration, *k* is the thermal conductivity, C_p is the specific heat at constant pressure, q_r is the radiative heat flux, Q_0 is the heat generation coefficient, *D* is the species diffusivity.

The boundary conditions for the velocity, temperature and concentration profiles are :

$$u=U_w=U_0e^{\frac{x}{L}},\ v=0,\ T=T_w=T_\infty+T_0e^{\frac{2x}{L}},C=C_w=C_\infty+C_0e^{\frac{2x}{L}}\quad at\quad y=0$$
$$u\to 0,\ T\to T_\infty,C\to C_\infty\quad as\quad y\to\infty \tag{5}$$

Introducing dimensionless quantities as follows:

$$u = U_0 e^{\frac{x}{L}} f'(\eta),\ v = \sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} f(\eta) + \eta f'(\eta)$$

$$T = T_\infty + T_0 e^{\frac{2x}{L}} \theta(\eta),\ C = C_\infty + C_0 e^{\frac{2x}{L}} \phi(\eta), \eta = \sqrt{\frac{U_0}{2\nu L}} y e^{\frac{x}{2L}} \quad (6)$$

Where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, $\phi(\eta)$ is the dimensionless concentration.

The differential equations become:

$$f''' - 2f'^2 + ff'' - Mf' = 0 \quad (7)$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + \Pr\left(f\theta' - f'\theta + Ec(f'')^2 + MEc(f')^2 + Q\theta\right) = 0 \quad (8)$$

$$\phi'' + Scf\phi' - Scf'\phi = 0 \quad (9)$$

With boundary conditions,

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) \to 0 \quad (10)$$

$$\theta(0) = 1, \theta(\infty) \to 0 \quad (11)$$

$$\phi(0) = 1, \phi(\infty) \to 0 \quad (12)$$

3. Solution of the nonlinear problem using the Modified Homotopy analysis method ([22] - [31])

Consider a differential equation $N[f(\eta)] = 0$ (13)

WhereN is a non-linear operator, η denotes independent variable and $f(\eta)$ is an approximate analytical solution of (11) which is an unknown function. Let $f_0(\eta)$ denote an initial approximation of $f(\eta)$, $H(\eta)$ is known as auxiliary function and L denotes an auxiliary linear operator, h is a non-zero embedding parameter lies between -1 and 1. Then the homotopy is given by:

$$(1-p)L[f(\eta; p, h, H(\eta) - f_0(\eta)] - phH(\eta)N[f(\eta; p, h, H(\eta))] \quad (14)$$

where $p \in [0,1]$ is an embedding parameter.

By means of analyzing the boundary conditions of the non-linear differential problem, we can know an appropriate base functions to represent the solution, even without solving the given non-linear problem.

In view of the boundary conditions (10), (11) and (12),

$$f_0(\eta) = \frac{1}{2} - \frac{1}{2}e^{-2\eta}, \theta_0(\eta) = e^{-2\eta},\ \phi_0(\eta) = e^{-2\eta} \quad (15)$$

And the linear operators L_f, L_θ, L_ϕ are defined as:

$$L_f = f''' + 2f'' \quad (16)$$

$$L_\theta = \theta'' + 2\theta' \quad (17)$$

$$L_\phi = \phi'' + 2\phi' \quad (18)$$

Applying modified Homotopy analysis method,

$$f = \frac{1}{2} - \frac{1}{2}e^{-2\eta} + \left(\frac{M}{16} - \frac{11}{32}\right) + \left(\frac{M}{16} + \frac{3}{8}\right)e^{-2\eta} + \left(\frac{5-M}{8}\right)\eta e^{-2\eta} - \frac{e^{-4\eta}}{32} \quad (19)$$

$$\theta = e^{-2\eta} + \frac{(M+4)}{12}\Pr Ec\, e^{-2\eta} - \frac{1}{4}\left(\Pr - \Pr Q - \frac{16R}{3} - 4\right)\eta e^{-2\eta} - \frac{(M+4)}{12} Ec \Pr e^{-4\eta} \quad (20)$$

$$\phi = e^{-2\eta} - \frac{Sc}{4}e^{-2\eta} + 2\eta e^{-2\eta} - \frac{Sc}{2}\eta e^{-2\eta} + \frac{Sc}{4}e^{-4\eta} \tag{21}$$

The Velocity profile is given as:

$$f' = e^{-2\eta} - \left(\frac{M}{8} + \frac{3}{4}\right)e^{-2\eta} + \frac{(5-M)}{8}\left(e^{-2\eta} - 2\eta e^{-2\eta}\right) + \frac{e^{-4\eta}}{8} \tag{22}$$

4. Results & Discussion

The effects of the velocity profile and temperature on increasing the magnetic parameter *M* is observed in Fig:2 and Fig:3. It is evident that the velocity decreases with increase in *M* whereas the temperature increases. The influence of *Pr* on temperature is inspected in Fig:4.There is a reduction in temperature with increasing prandtl number *Pr*. From Fig:5, it is noticed that the temperature increases with increase in *R*.In Fig:6 and Fig:7 it is realized that the temperature is directly proportional to Eckert number *Ec* and heat generation parameter *Q*. The influence of *M* and *Sc* on concentration profile is depicted in Fig:8 and Fig:9. It is prominent that Concentration increases with *M* whereas, it decreases with increase in *Sc*.

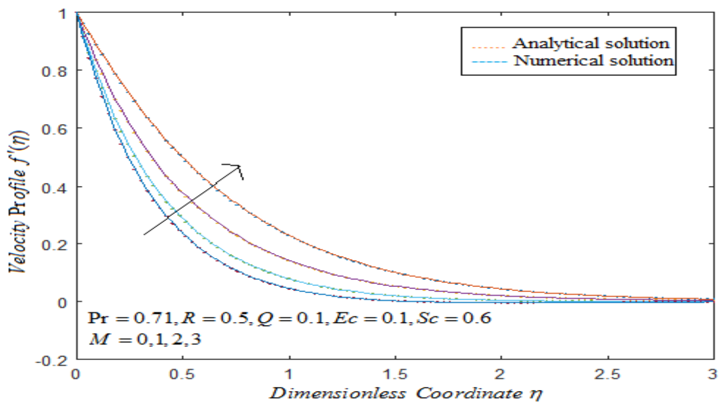

Fig:.2: Dimensionless coordinate η versus Velocity profile $f'(\eta)$. The curve is plotted using (22) for fixed *Pr* ,*R*, *Q*, *Sc* and *Ec* and varying *M*=0,1,2,3.

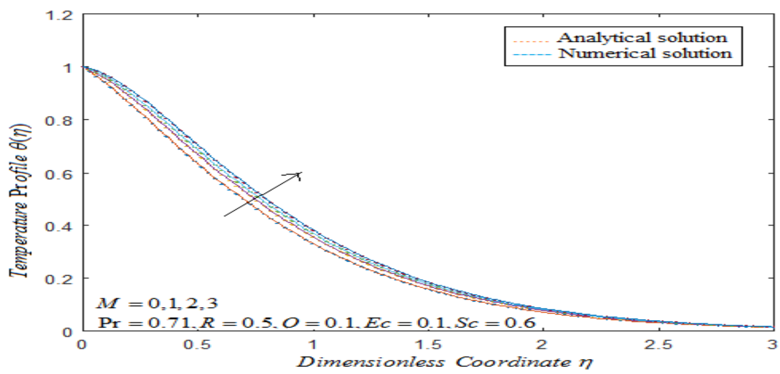

Fig.3: Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (20) for fixed *Pr* ,*R*, *Q*, *Sc* and *Ec* and varying *M*=0,1,2,3.

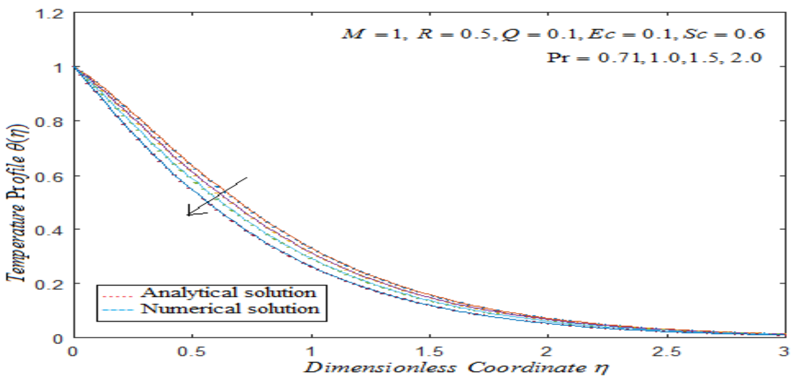

Fig.4: Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (20) for fixed *M* ,*R*, *Q*, *Sc* and *Ec* and varying *Pr*=0.71, 1.0, 1.5, 2.

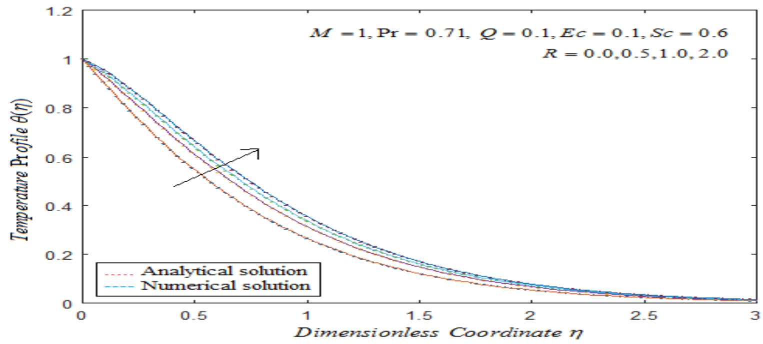

Fig.5. Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (20) for fixed *Pr* ,*M*, *Q*, *Sc* and *Ec* and varying *R*=0.0, 0.5, 1.0,2.0.

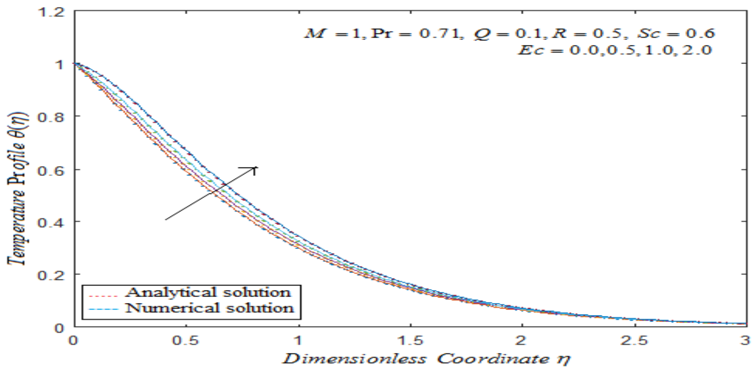

Fig.6: Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (20) for fixed *Pr* ,*M*, *Q*, *Sc* and *R* and varying *Ec*=0.0, 0.5, 1.0, 2.0.

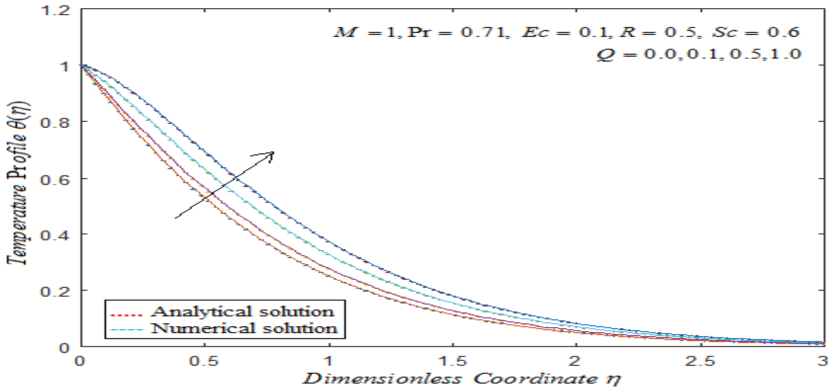

Fig.7: Dimensionless coordinate η versus temperature profile $\theta(\eta)$. The curve is plotted using (20) for fixed *Pr* ,*M*, *Ec*, *Sc* and *R* and varying *Q*=0.0, 0.1, 0.5, 1.0.

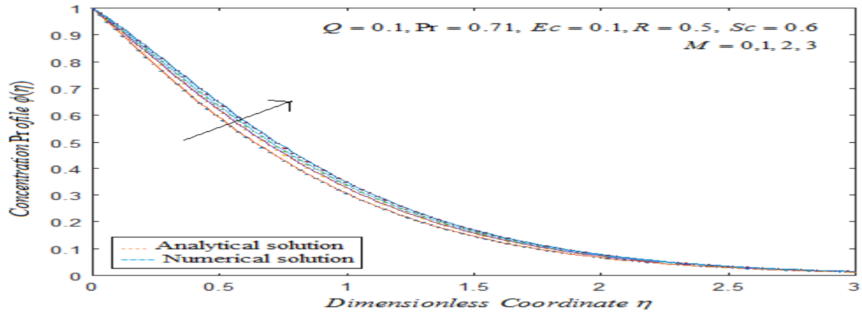

Fig.8: Dimensionless coordinate η versus concentration profile $\phi(\eta)$. The curve is plotted using (21) for fixed *Pr* ,*R*, *Q*, *Sc* and *Ec* and varying *M*=0,1,2,3.

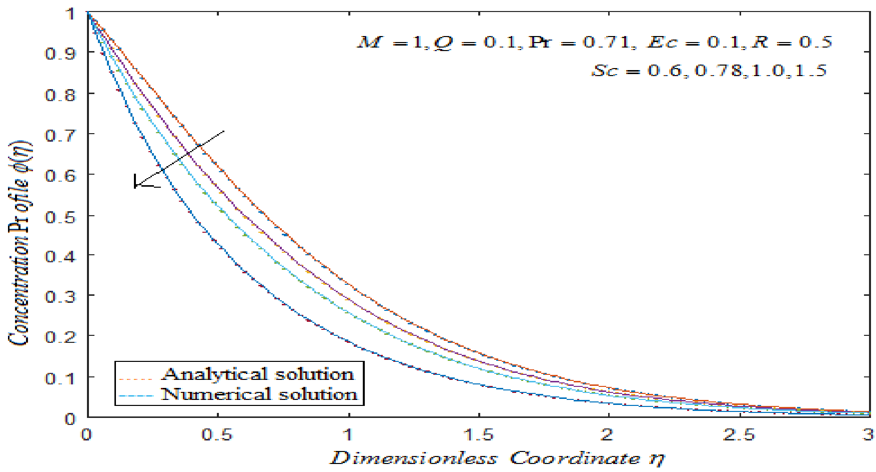

Fig.9: Dimensionless coordinate η versus concentration profile $\phi(\eta)$. The curve is plotted using (21) for fixed *Pr* ,*R*, *Q*,*M* and *Ec* and varying *Sc*=0.6, 0.78, 1.0,1.5.

5. Conclusion

The mathematical analysis of the non-linear boundary value problem The dimensionless velocity profile $f'(\eta)$, the dimensionless temperature profile $\theta(\eta)$ and the dimensionless concentration profile $\phi(\eta)$ are obtained analytically and their effects on varying the governing parameters are discussed graphically. The results are successfully compared with the previous work.

References

[1] S.Mohammed Ibrahim, Heat and mass transfer effects on steady MHD flow over an exponentially stretching surface with viscous dissipation, heat generation and radiation, Journal of Global Research in Mathematical Archives,Vol.1, pp. 67-77, (2013).

[2] Sakiadis, Boundary layerbehaviour oncontinuoussolid surfaceII,Boundary layerbehavior on continuousflat surface, AIChEJournal, Vol.7, pp.221–235, (1961).

[3] CraneLJ,Flowpastastretchingplate, ZeitschriftF¨urAngewandteMathematikundPhysikZAMP,vol.21,no.4,pp.645-647, (1970).

[4] CarragherPandCraneLJ,Heattransferonacontinuousstretchingsheet,Z.Angew.Math.Mech.,Vol.62 pp.564-573, (1982).

[5] MagyariEandKellerB(1999),Heatandtransferintheboundarylayersonanexponentiallystretchingcontinuoussurface,J. Phys. D:Appl.Phys.,Vol.32, pp. 577-585, (1999).

[6] Liu, I.C.,A noteon heatand mass tranfer for ahydromagneticflow overastretching sheet. International CommunicationsinHeatandMass Transfer, Vol.32, pp. 1075-1084, (2005).

[7] Chen, C.H., Laminarmixedconvectionadjacentto vertical, continuouslystretching sheets.Heat andMassTransfer, Vol.33, pp.471476, (1998).

[8] Ishak, A.,Nazar, R., & Pop, I, Hydromagneticflowandheat transfer adjacent toastretching verticalsheet, HeatMass Transfer, Vol.44, pp. 921-927, (2008).

[9] Seddeek, M.A., 2002, Effectsof radiationandvariable viscosityona MHDfreeconvection flow pasta semi-infinite flatplate with analignedmagneticfield inthe caseof unsteadyflow, Int.J. HeatMass Transfer, Vol. 45, pp. 931-935,(2002).

[10] Raptis,A,Radiation and viscoelastic flow,Int. Commun. HeatMassTransf. Vol.26(6), pp. 889-895 (1999).

[11] Raptis,A, Perdikis,C,Viscoelasticflow by thepresenceof radiation. Z. Angew,Math.Mech.,Vol. 78, pp. 277-279 (1998).

[12] Siddheshwar,PG,Mahabaleswar,US,EffectsofradiationandheatsourceonMHDflowofaviscoelastic liquidandheat transfer overastretchingsheet,Int.J.Non-Linear Mech. Vol.40, pp.807-820 (2005).

[13] Bidin. B,Nazar.R, Numericalsolutionof theboundarylayer flow overanexponentially stretching sheetwith thermal radiation,Eur.J.Sci.Res. Vol.33(4), pp.710-717 (2009).

[14] Ishak,A,MHDboundarylayerflowduetoanexponentiallystretchingsheetwithradiationeffect,Sains Malays. Vol.40(4),pp.391-395, (2011).

[15] Reddy,PBA,Reddy,NB,Thermalradiationeffectsonhydromagneticflowduetoanexponentiallystretchingsheet, Int.J. Appl. Math.Comput, Vol. 3(4), pp.300-306 (2011).

[16] Sanjayanand,E,Khan,SK,Onheatandmasstransferinaviscoelasticboundarylayerflowoveranexponentiallystretching sheet, Int.J. Therm. Sci.Vol. 45, pp.819-828(2006).

[17] KameswaranPK.,NarayanaM.,SibandaP,andMakandaG,OnradiationeffectsonhydromagneticNewtonianliquid flow duetoan exponentialstretching sheet, Boundary Value Problems,Vol. 105,pp. 1-16,(2012).

[18] TaniaS.KhalequeandSamadM.A.(2010),Effectsofradiation,heatgenerationandviscousdissipation onMHDfree convectionflowalongastretchingsheet, ResearchJ ofAppl.Sci., Eng. and Tech.,Vol.2, No. 4, pp.368-377,(2010).

[19] JatRNandGopiChand,MHDflowandheattransferoveranexponentiallystretchingsheetwithviscous dissipation and radiation effects, AppliedMathematicalSciences,Vol.7, No.4, pp. 167-180, (2013).

[20] M.K.Partha,P.V.S.N.Murthy,andG.P.Rajasekhar,Effectofviscousdissipationonthemixedconvectionheattransfer fromanexponentially stretching surface, Heat andMass Transfer, Vol. 41, no. 4,pp. 360–366, (2005).

[21] MohammedIbrahimSandBhaskarReddyN,RadiationandMasstransfereffectsonMHDfreeconvectionflow alongastretching surfacewithviscous dissipation and heatgeneration,Int. J. of Appl. Math and Mech. Vol.8,No.8, pp. 1-21, (2012).

[22] S.J.Liao, Anapproximatesolutiontechniquewhichdoesnotdependuponsmall parameters: A

special example,International Journal ofNon-Linear Mech, Vol. 30, pp. 371-380, (1995)
[23] S.J.Liao, OntheHomotopyanalysismethodfornon-linearproblems, Appl.Math. Computations. Vol.147, pp. 499-513, (2004).
[24] S. J. Liao, An optimal Homotopy-analysis approach for strongly non-linear differential equations, Communications in Non-linear Science and Numerical Simulation, Vol. 15, pp. 2003-2016, (2010).
[25] S.J.Liao, An explicit totally analytic approximation of basis viscous flow problems, Int.NonlinearMach; Vol. 34, pp. 759–78, (1999).
[26] S.J.Liao, Ontheanalyticsolution of magneto hydrodynamicflowsnon- newtonianfluids over astretching sheet, J FluidMach, Vol. 488, pp. 189–212, (2003).
[27] S. J. Liao, Anewbranchofboundarylayerflowsoverapermeablestretching plate,Int.JNonlinear Mech,Oscillatory flowandheattransferina HorizontalComposite Porous Medium Channel, Vol. 42, pp. 19–30, (2007).
[28] M. Subha, V.Ananthaswamy and L.Rajendran, Analytical solution of a boundary value problem for the electro hydrodynamic flow equation, International Journal of Automation and ControlEngineering, Vol.3,No. 2, pp. 48-56, (2014).
[29] K.Saravanakumar, V.Ananthaswamy, Subha M., and Rajendran L., Analytical Solution ofnonlinearboundaryvalueproblemfor inefficiency of convective straight Fins with temperature-dependent thermal conductivity, ISRN Thermodynamics,Article I282481, pp. 1-18, 2013.
[30] V.Ananthaswamy,M.Subha, Analyticalexpressionsforexothermicexplosionsin a slab, International Journal ofResearch– Granthaalayah, Vol.1, No. 2, pp. 22-37, (2014).
[31] V.Ananthaswamy,S.UmaMaheswari, Analyticalexpression forthe hydrodynamic fluidflow througha porous medium, InternationalJournalof Automation and Control Engineering, Vol. 4, No.2, pp. 67-76, (2015).

Appendix:A

Solution of the differential equation using modified HAM:
The non-linear differential equations are:

$$f''' - 2f'^2 + ff'' - Mf' = 0 \quad \text{(A1)}$$

$$\left(1+\frac{4}{3}R\right)\theta'' + \Pr\left(f\theta' - f'\theta + Ec(f'')^2 + MEc(f')^2 + Q\theta\right) = 0 \quad \text{(A2)}$$

$$\phi'' + Scf\phi' - Scf'\phi = 0 \quad \text{(A3)}$$

with boundary conditions,

$$f(0)=0,\ f'(0)=1,\ f'(\infty)\to 0 \quad \text{(A4)}$$

$$\theta(0)=1, \theta(\infty)\to 0 \quad \text{(A5)}$$

$$\phi(0)=1, \phi(\infty)\to 0 \quad \text{(A6)}$$

Construct Homotopy as :

$$(1-p)[f''' + 2f''] + p\ [f''' - 2f'^2 + ff'' - Mf'] = 0 \quad \text{(A7)}$$

$$(1-q)[\theta'' + 2\theta'] + q[\left(1+\frac{4}{3}R\right)\theta'' + \Pr\left(f\theta' - f'\theta + Ec(f'')^2 + MEc(f')^2 + Q\theta\right)] = 0 \quad \text{(A8)}$$

$$(1-r)[\phi'' + 2\phi'] + r\left[\phi'' + Scf\phi' - Scf'\phi\right] = 0 \quad \text{(A9)}$$

The analytical solution of (A1) , (A2) and (A3) with boundary conditions (A4), (A5) and (A6) are:

$$f = f_0 + pf_1 + p^2 f_2 + \dots\dots\dots\dots \quad \text{(A10)}$$

$$\theta = \theta_0 + q\theta_1 + q^2\theta_2 + \dots\dots\dots\dots\dots \quad \text{(A11)}$$

$$\phi = \phi_0 + r\phi_1 + r^2\phi_2 + ... \quad (A12)$$

Substituting (A10), (A11) and (A12) in the equations (A7), (A8) ,(A9) and comparing the coefficients of the powers of p, q and r we get

$$p^0 : f_0''' + 2 f_0'' = 0 \quad (A13)$$

$$p^1 : f_1''' + 2 f_1'' - 2 f_0'' - 2 f_0''^2 + f_0 f_0'' - Mf_0' = 0 \quad (A14)$$

$$q^0 : \theta_0'' + 2\theta_0' = 0 \quad (A15)$$

$$q^1 : \theta_1'' + 2\theta_1' - 2\theta_0' + \left(\frac{4}{3}R\right)\theta_0'' + \Pr\left(f_0\theta_0' - f_0'\theta_0 + Ec(f_0'')^2 + MEc(f_0')^2 + Q\theta_0\right)] = 0$$

(A16) $r^0 : \phi_0'' + 2\phi_0' = 0$ (A17)

$$r^1 : \phi_1'' + 2\phi_1' - 2\phi_0' + Scf_0\phi_0' - Scf_0'\phi_0 = 0 \quad (A18)$$

Solving the equations

$$f_0(\eta) = \frac{1}{2} - \frac{1}{2}e^{-2\eta} \quad (A19)$$

$$f_1(\eta) = \left(\frac{M}{16} - \frac{11}{32}\right) + \left(\frac{M}{16} + \frac{3}{8}\right)e^{-2\eta} + \left(\frac{5-M}{8}\right)\eta e^{-2\eta} - \frac{e^{-4\eta}}{32} \quad (A20)$$

$$\theta_0(\eta) = e^{-2\eta} \quad (A21)$$

$$\theta_1(\eta) = \frac{(M+4)}{12}\Pr Ec\, e^{-2\eta} - \frac{1}{4}\left(\Pr - \Pr Q - \frac{16R}{3} - 4\right)\eta e^{-2\eta} - \frac{(M+4)}{12}Ec\Pr e^{-4\eta} \quad (A22)$$

$$\phi_0(\eta) = e^{-2\eta} \quad (A23)$$

$$\phi_1(\eta) = -\frac{Sc}{4}e^{-2\eta} + 2\eta e^{-2\eta} - \frac{Sc}{2}\eta e^{-2\eta} + \frac{Sc}{4}e^{-4\eta} \quad (A24)$$

According to HAM, for $-1 \le h \le 1$, As $p \to 1,\ q \to 1\ and\ r \to 1$,

$$f = f_0 - h f_1 \quad (A25)$$

$$\theta = \theta_0 - h\theta_1 \quad (A26)$$

$$\phi = \phi_0 - h\phi_1 \quad (A27)$$

Substituting (A19) to (A24) in (A25), (A26) and (A27) we obtain the result in the text (19), (20) and (21).