Advances in Mathematics: Scientific Journal **9** (2020), no.1, 93–100
ADV MATH SCI JOURNAL
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.1.9

A STUDY ON TRIPLE CONNECTED TOTAL PERFECT DOMINATION IN FUZZY GRAPHS

K. ELAVARASAN AND T. GUNASEKAR[1]

ABSTRACT. In this paper, we introduce the concept of triple connected total perfect domination in fuzzy graph. We have defined and derived some results related to the triple connected total perfect domination number with examples. Finally the triple connected total perfect dominating set and number are obtained.

1. INTRODUCTION

The idea of fuzzy relation and fuzzy set are initiated in 1965 by L.A.Zadeh [8]. In 1975, Rosenfeld [6] initiated the idea of fuzzy graph and theoretical ideas such as paths, loops and connectivity. In 1998, the theory of dominance in fuzzy graphs started with A.Somasundaram and S.Somasundaram. The idea of perfect domination and total dominating set initiated by Cokayne et al [1]. Revathi et al [4, 5], Sarala et all [7] and Nagoorganiet al [2, 3] initiated about the idea of connected total perfect domination of fuzzy graph. The motive of the present paper is to initiate the concept of triple connected total perfect dominating set, triple connected total perfect domination number in a fuzzy graph and interpret some results of fuzzy graph.

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 03E72, 05C69, 05C72.
Key words and phrases. Fuzzy graphs, Total perfect domination, connected total perfect domination, Triple connected total perfect domination.

2. Preliminaries

Definition 2.1. *The fuzzy set of a base set or reference set V is specified by its function of membership σ, where $\sigma : V[0,1]$ assigning to each $u \in V$ the degree or grade to which u belongs to σ.*

Definition 2.2. *There are two fuzzy sets σ and τ of a set V, then the set σ is called a fuzzy subset of τ, if $\sigma(u) \leq \tau(u)$ each $u \in V$.*

Definition 2.3. *Let $G = (\sigma, \mu)$ is called a fuzzy graph, if there exist a set of functions of membership $\sigma : V[0,1]$ and $\mu : V \times V \to [0,1]$ such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.*

Definition 2.4. *If $\tau(u) \leq \sigma(u)$ where $u \in V$ and $\rho(u,v) \leq \mu(u,v)$ for every $u, v \in V$, then $H = (\tau, \rho)$ is said to be a fuzzy subgraph of a fuzzy graph G.*

Definition 2.5. *If $\tau(u) = \sigma(u)$ where $u \in V$ and $\rho(u,v) \leq \mu(u,v)$ for every $u, v \in V$, then H is called a spanning fuzzy subgraph of a fuzzy graph G.*

Definition 2.6. *Order $p = \sum_{(u \in V)} \sigma(u)$ and size $q = \sum_{((u,v) \in E)} \mu(u,v)$.*

Definition 2.7. *If $\mu^{\infty}(u,v) \leq \mu(u,v)$ for every $u, v \in V$, then $arc(u,v)$ is called a strong arc. Where $\mu^{\infty}(u,v)$ be the strongest path strength and the vertex u is said to be a strong neighbor to v, otherwise it is called weak arc. The vertex u is called a isolated in G if $\mu(u,v) = 0$ every $v \neq u, v \in V$.*

Definition 2.8. $d_N(v) = \sum_{(u \in N_s(v))} \sigma(u)$, $\delta_N(G) = min\{d_N(u) : u \in V(G)\}$ *and* $\Delta_N(G) = max\{d_N(u) : u \in V(G)\}$.

Definition 2.9. *If $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for every $u, v \in V$, then the fuzzy graph G is called a complete fuzzy graph. It is described by K_σ.*

Definition 2.10. *There is a bipartition V_1 and V_2 of G. If every vertex in V_1 has a strong neighbor in V_2 and also V_2 has a strong neighbor in V_1, then the bipartition (V_1, V_2) is called a complete bipartite fuzzy graph of G. It is identified by $K_{(m,n)}$.*

Definition 2.11. *If (u,v) be a strong arc, then the node u dominates the node v for every node $u, v \in V$ of G. If for every node v not in a subset P of V which dominated by absolutely a node of P, then P is called a perfect dominating set of G. It is identified by P_D.*

Definition 2.12. *If there is a subgraph P_C of G which is connected and induced by P_D of G, then P_C is said to be connected P_D.*

Definition 2.13. *If for each node of G be dominates to at lest a node of P_t of G, then P_t is called a total P_D of G*

Definition 2.14. *A total P_D of G is called a connected total P_D if the induced subgraph total P_D is connected. It is identified by $ctp(G)$.*

Definition 2.15. *A ctp of G is called a minimal $ctp(G)$ if for all node in, $ctp - v$ is not $ctp(G)$.*

Definition 2.16. $\gamma_{ctp}(G) = min\{ctp(G)\}$ *and* $\Gamma_{ctp}(G) = max\{ctp(G)\}$

Definition 2.17. *If there are three nodes connected and lying on a path T_C of G, then $T_C(G)$ called triple connected fuzzy graph.*

3. Main Result

In the present section, we initiate the new concept of triple connected total perfect dominating set(Tctp) in fuzzy graph and define the concept of minimal triple connected total perfect dominating set as well as introducing a triple connected total perfect dominating number (γ_{Tctp}).

Definition 3.1. *A ctp(G) is called a triple connected total perfect dominating set if the induced subgraph<ctp(G)> is triple connected. It is denoted by Tctp(G).*

Definition 3.2. *A Tctp of G is called a minimal $Tctp(G)$ if for all node in, $Tctp - v$ is not $Tctp(G)$.*

Definition 3.3.

$$\gamma_{Tctp}(G) = min\{Tctp(G)\} \quad \textit{and} \quad \Gamma_{Tctp}(G) = max\{Tctp(G)\}\,.$$

Example 1. *The first example is given on Figure 1.*

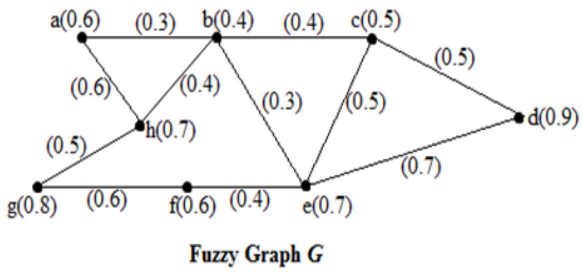

FIGURE 1. $Tctp(G)$ are $\{b, e, f\}$ and $\{e, b, h\}$,
Minimal $Tctp(G) = \{b, e, f\}, \gamma_{Tctp}(G) = 1.6$, $\Gamma_{Tctp}(G) = 1.7$

Remark 3.1. (1) *There is no $Tctp(G)$ if Gis a K_σ .*
(2) *There is no $Tctp(G)$ if G is a $K_{(m,n)}$.*

Example 2. *The second example is given on Figure 2. Here $\{a, e\}$ be a connected total perfect dominating set, but not $Tctp(G)$.*

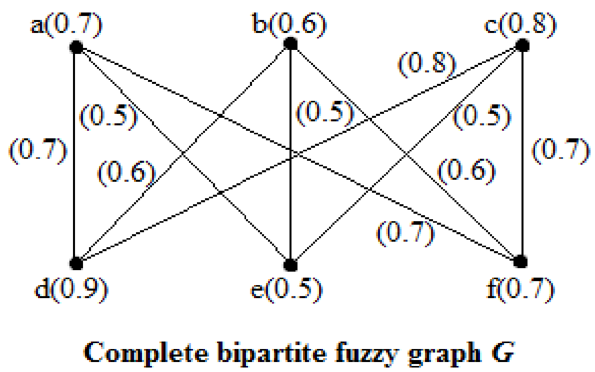

FIGURE 2

Remark 3.2. (1) *There is a $\gamma_{Tctp}(G)$, then $\gamma_{Tctp}(G) \leq p - 1$.*
(2) *Every $Tctp(G)$ is a P_D of a fuzzy graph G.*
(3) *Every $Tctp(G)$ is a total P_D of a fuzzy graph G.*
(4) *Every $Tctp(G)$is a ctp of a fuzzy graph G.*

Note that the converse of the observation in the Remark 3.2 are not true.

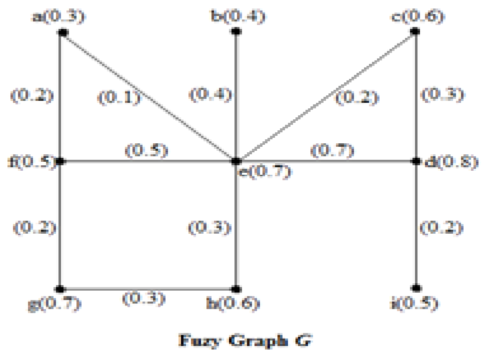

FIGURE 3. $Tctp(G) = \{d, e, f\}$, which is $P_D(G)$, total $P_D(G)$ and $ctp(G)$, $\gamma_{Tctp}(G) = 2$.

Example 3. *The third example is given on Figure 3.*

Theorem 3.1. *If there is $\gamma_{Tctp}(G)$ of G, then $\gamma_{tp}(G) \leq \gamma_{ctp}(G) \leq \gamma_{Tctp}(G)$.*

Proof. Since every $Tctp(G)$ of a fuzzy graph G is a $ctp(G)$ in a fuzzy graph G and every $ctp(G)$ is a total P_D. Hence $\gamma_{tp}(G) \leq \gamma_{ctp}(G) \leq \gamma_{Tctp}(G)$ for any $\gamma_{Tctp}(G)$ in a fuzzy graph G. □

Example 4. *The following example is given on Figure 4.*
$Tctp(G) = \{a, d, e\}; \quad \gamma_{Tctp}(G) = 1.5$
$ctp(G) = \{a, d, e\}; \gamma_{ctp}(G) = 1.5$
Total $P_D(G) = \{a, d, e\}; \quad Total \gamma_{tp}(G) = 1.5$
Therefore, $\gamma_{tp}(G) \leq \gamma_{ctp}(G) \leq \gamma_{Tctp}(G)$.

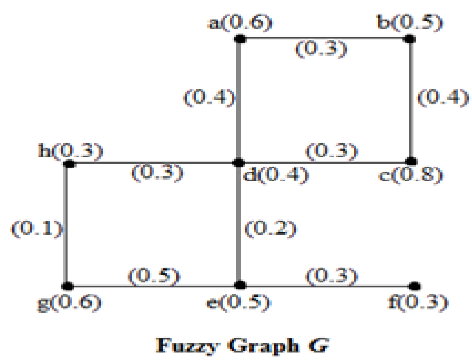

FIGURE 4

Theorem 3.2. *If there is $\gamma_{Tctp}(G)$ in a fuzzy graph G with maximum degree Δ , then*

$$\frac{p}{2(\Delta+1)} \leq \gamma_{Tctp}(G) \leq 2q - p + 1\,.$$

Proof. For the lower bound, each membership values of $Tctp(G)$ of a fuzzy graph G can dominate almost maximum degree Δ membership values and itself. Hence:

$$\frac{p}{2(\Delta+1)} \leq \gamma_{Tctp}(G)\,. \tag{3.1}$$

Now we consider the upper bound, If there is a $\gamma_{Tctp}(G)$ in a fuzzy graph G, then:

$$\begin{aligned} \gamma_{Tctp}(G) &\leq p-1 \\ \gamma_{Tctp}(G) &\leq 2(p-1)-p+1 \\ \gamma_{Tctp}(G) &\leq 2q-p+1 \qquad (3.2) \end{aligned}$$

From (3.1) and (3.2), we have $\frac{p}{2(\Delta+1)} \leq \gamma_{Tctp}(G) \leq 2q-p+1$. □

Example 5. *In the following example we have:* $Tctp(G) = \{c, d, f\}$.
Here, $p = 2.9; q = 1.8$, $d(a) = 1.2; d(b) = 1.1; d(c) = 1.2; d(d) = 1.5; d(e) = 0.8; d(f) = 1.3; d(g) = 0.4\Delta = 1.5;$ $\frac{p}{2(\Delta+1)} = 0.05;$ $\gamma_{Tctp}(G) = 1;$ $2q-p+1 = 1.9$. *Hence,* $\frac{p}{2(\Delta+1)} \leq \gamma_{Tctp}(G) \leq 2q-p+1$.

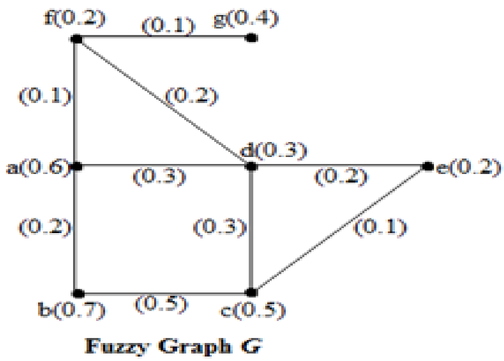

FIGURE 5

Remark 3.3. *If there is a $\gamma_{Tctp}(G)$ in a fuzzy graph G with order p, then* $\gamma_{Tctp}(G) \leq p - \Delta(G) + 1$.

Example 6. $Tctp(G) = \{a, e, d\}, \gamma_{Tctp}(G) = 1.6, p = 2.9; d(a) = 1; d(b) = 1.4; d(c) = 1.1; d(d) = 1.1; d(e) = 1.2; \Delta(G) = 1.4 p - \Delta(G) + 1 = 2.9 - 1.4 + 1 = 2.5$. *Hence* $\gamma_{Tctp}(G) \leq p - \Delta(G) + 1$.

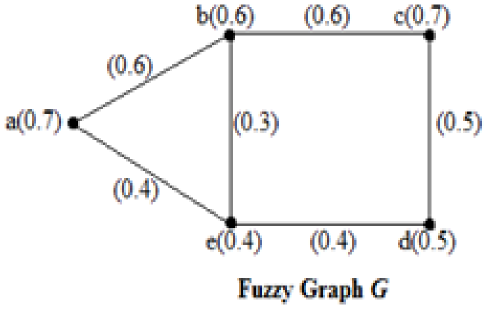

FIGURE 6

Theorem 3.3. *If there isH be a triple connected spanning sub graph of a fuzzy graph G, then* $\gamma_{Tctp}(G) \leq \gamma_{Tctp}(H)$.

Proof. Let G be a triple connected total perfect dominating set which induced the subgraph $< P_{Tct} >$ is triple connected and let H be a triple connected spanning subgraph of G if $\tau(u) = \sigma(u)$ every $u \in H$ and $\rho(u, v) \leq \mu(u, v)$ for every $u, v \in V$. Since every $Tctp(G)$ of H is also the $Tctp(G)$ of G. Hence $\gamma_{Tctp}(G) \leq \gamma_{Tctp}(H)$. □

Example 7. $Tctp(G) = \{b, e, h\}$; $\gamma_{Tctp}(G) = 1.7$, $Tctp(H) = \{d, e, f\}$; $\gamma_{Tctp}(H) = 2$. *Hence* $\gamma_{Tctp}(G) \leq \gamma_{Tctp}(H)$.

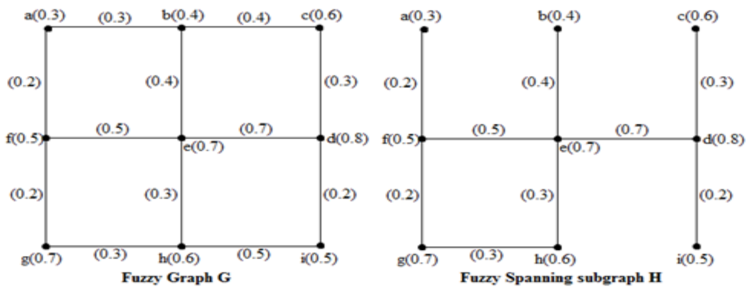

FIGURE 7

4. Conclusions

We investigated the idea of triple connected total perfect domination and triple connected perfect domination in fuzzy graph. We interpreted some results related to the triple connected total perfect domination number for standard theorems with examples. Finally we conclude that the triple connected total perfect dominating set and number are obtained. Further these results can be extended to the fields of a bipolar fuzzy graph.

References

[1] E.J. Cockayne, S.T.Hedetniemi: *Towards a theory of domination in graphs,* Networks, **7** (1977), 247–261.

[2] A. Nagoorgani. M. Basheer Ahamed: *Order and Size in fuzzy graph*, Bulletin of Pure and Applied Sciences, **22** (2003), 145– 148.

[3] A. Nagoorgani, V.T. Chandrasekaran: *Domination in fuzzy graph*, Advances in fuzzy and system, **1** (2006), 17–26.

[4] S. Revathi, C.V.R.Harinarayanan, R.Muthuraj: *Connected perfect domination in fuzzy Graph,* Golden Research thoughts, **5** (2015), 1–5.

[5] S. Revathi, C.V.R.Harinarayanan, R.Muthuraj: *Connected total perfect dominating set In fuzzy graph,* International journal of computational and applied Mathematics, **12**(1) (2017), 84–98.

[6] A. Rosenfeld, L.A. Zadeh, K.S.Fu, M. Shimura: *Fuzzy sets and their Applications,* Academic press, New York, (1975), 77–95.

[7] N. Sarala, T. Kavitha: *Triple connected domination number of fuzzy graph*, International Journal of Applied Engineering Research, **10**(51) (2015), 914–917.

[8] L.A. Zadeh: *Fuzzy sets,* Information sciences, **8** (1965), 338–353.

Department of Mathematics
VelTech Rangarajan Dr.Sagunthala
R&D Institute of Science and Technology
Chennai - 600062, Tamil Nadu, India
E-mail address: `elavarasank2@gmail.com`

Department of Mathematics
VelTech Rangarajan Dr.Sagunthala
R&D Institute of Science and Technology
Chennai - 600062, Tamil Nadu, India
E-mail address: `tguna84@gmail.com`