INTUITIONISTIC FUZZY TRANSLATION ON INK-ALGEBRA

M.KAVIYARASU, K.INDHIRA, AND V.M.CHANDRASEKARAN

ABSTRACT. In this article, we introduced intuitionistic fuzzy translation and intuitionistic fuzzy multiplication on INK-algebras and derived some results to get the structure of intuitionistic fuzzy INK-subalgebra.

1. INTRODUCTION

The idea of fuzzy translations trendy fuzzy subalgebras in addition ideals in BCK/BCI-algebras has been deliberated respectively by Lee et al. and Jun. They studied relatives’ mid fuzzy translations, fuzzy extensions and fuzzy multiplications. Inspired by this, Senapati, T and M. Bhowmik, M. Pal presented fuzzy translations of fuzzy H-ideals in BCK/BCI-algebras. They likewise outspread this learning from fuzzy translations to intuitionistic fuzzy translations in BCK/BCI-algebras. In this paper, intuitionistic fuzzy translations, intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of intuitionistic fuzzy INK-subalgebra in INK-algebras be situated conversed. Relatives between intuitionistic fuzzy translations, intuitionistic fuzzy extensions too intuitionistic fuzzy multiplications of intuitionistic fuzzy INK-algebra in INK-algebras are as well studied in [2–7] and [1].

1corresponding author

2010 Mathematics Subject Classification. 06F35, 03G25.

Key words and phrases. INK-Algebra,Fuzzy INK-Subalgebra, Intuitionistic fuzzy INK-Subalgebra, Intuitionistic fuzzy translation, Intuitionistic fuzzy multiplication.
2. PRELIMINARIES

Definition 2.1. An algebra \((E, \circ, 0)\) is called a INK-algebra if you meet the ensuing conditions for every \(p, q, r \in E\).

INK-1: \(((p \circ q) \circ (p \circ r)) \circ (r \circ q) = 0\)

INK-2: \(((p \circ r) \circ (q \circ r)) \circ (p \circ q) = 0\)

INK-3: \(p \circ 0 = p\)

INK-4: \(p \circ q = 0\) and \(q \circ p = 0\) imply \(p = q\).

Definition 2.2. Let \(Y \subseteq E\) is called a INK-subalgebra of \(E\). If \(p \circ q \in Y\).

Definition 2.3. Let \(E\) be a INK-algebra. Then a fuzzy set \(A\) is defined as \(A = \{(p, v_A(p))|p \in E\}, 0 \leq v_A(p) \leq 1\), for all \(p \in E\).

Definition 2.4. A FS \(v\) in a INK-algebra \(E\) is called a fuzzy INK ideal of \(E\), if:

i) \(v(0) \geq v(p)\)

ii) \(v(p) \geq \min \{(v(q \circ p) \circ (q \circ r)), v(q)\}\) for all \(p, q, r \in E\).

Definition 2.5. A fuzzy set \(v\) in a INK-algebra \(E\) is named a fuzzy INK-subalgebra of \(E\) if \(v_A(p \circ q) \geq \min \{v_A(p), v_A(q)\}\), for all \(p, q \in E\).

Definition 2.6. An intuitionistic fuzzy set(IFS) \(A\) in a non-empty set \(E\) is an object having the form \(A = \{(p, v_A(p), w_A(p))|p \in E\}\), where the function:

\(v_A : E \rightarrow [0, 1]\) and \(w_A : E \rightarrow [0, 1]\), denote the degree of membership and the degree of non-membership of each element \(p \in E\) to the set \(A\) respectively, and \(0 \leq v_A(p) + w_A(p) \leq 1\), for all \(p \in E\). Then denoted by \(A = (p, v_A, w_A)\) for the intuitionistic fuzzy set \(A = \{(p, v_A(p), w_A(p))|p \in E\}\).

Definition 2.7. An IFS \(A = (p, v_A, w_A)\) is named an IF-subalgebra of \(E\) if it satisfies:

i) \(v_A(p \circ q) \geq \min \{v_A(p), v_A(q)\}\),

ii) \(w_A(p \circ q) \leq \max \{w_A(p), w_A(q)\}\), for all \(p, q \in E\).

3. TRANSLATION OF INTUITIONISTIC FUZZY SUBALGEBRA

For the sake of straightforwardness, we mean to usage the symbol \(A = (p, v_A, w_A)\) for the IFS \(A = \{(p, v_A(p), w_A(p)|p \in E\}\).

We consider \(T = 1 - \inf \{w_A(p)|p \in E\}\) for any \(A = (p, v_A, w_A)\) of \(E\).
Definition 3.1. Let $A = (p, v_A, w_A)$ be an IFS of $E$ and let $\alpha \in [0, T]$. An object having the form $A^\alpha_T = ((v_A)^T, (w_A)^T)$ is named an IF-\(\alpha\)-translation of $A$ if $(v_A)^T(p) = v_A(p) + \alpha$ and $(w_A)^T(p) = v_A(p) - \alpha$ for all $p \in E$.

Theorem 3.1. Let $A$ be an IF-INK subalgebra of $E$ and $\alpha \in [0, T]$. Then the IF-\(\alpha\)-translation $A^\alpha_T$ of $A$ is an IF-INK subalgebra of $E$.

Proof. Let $p, q \in E$. Then $v(p \odot q) \geq \min \{v(p), v(q)\}$.

Now

$$v^\alpha_T(p \odot q) = v(p \odot q) + \alpha$$

$$\geq \min \{v(p), v(q)\} + \alpha$$

$$= \min \{v(p) + \alpha, v(q) + \alpha\}$$

$$= \min \{v^\alpha_T(p), v^\alpha_T(q)\},$$

and

$$w^\alpha_T(p \odot q) = w(p \odot q) - \alpha$$

$$\leq \max \{w(p), w(q)\} - \alpha$$

$$= \max \{w(p) - \alpha, w(q) - \alpha\}$$

$$= \max \{w^\alpha_T(p), w^\alpha_T(q)\}.$$

\[\square\]

Theorem 3.2. Let $A$ be an IFS of $E$ such that the IF-\(\alpha\)-translation $A^\alpha_T$ of $A$ is an IF-INK subalgebra of $E$ for some $\alpha \in [0, T]$. Then $A$ is an IF-INK subalgebra of $E$.

Proof. Let $A^\alpha_T$ is an IF-subalgebra of $E$ for some $\alpha \in [0, T]$. Then

$$v(p \odot q) + \alpha = v^\alpha_T(p \odot q)$$

$$\geq \min \{v^\alpha_T(p), v^\alpha_T(q)\}$$

$$= \min \{v(p) + \alpha, v(q) + \alpha\}$$

$$= \min \{v(p), v(q)\} + \alpha$$

$$v(p \odot q) \geq \min \{v(p), v(q)\}.$$
and

\[ w(p \odot q) - \alpha = w^T_\alpha(p \odot q) \]
\[ \leq \max \{ w^T_\alpha(p), w^T_\alpha(q) \} \]
\[ = \max \{ w(p) - \alpha, w(q) - \alpha \} \]
\[ = \max \{ w(p), w(q) \} - \alpha \]
\[ w(p \odot q) \leq \max \{ w(p), w(q) \}. \]

This implies that \( v(p \odot q) \geq \min \{ v(p), v(q) \} \) and \( w(p \odot q) \leq \max \{ w(p), w(q) \} \). Hence \( A \) is an IF-INK subalgebra of \( E \).

**Definition 3.2.** Let \( A \) be an IFS of \( E \) and \( e \in [0, 1] \). An object having the form \( A_e^M = ((v_A)_e^M, (w_A)_e^M) \) is called an IF \( e \)-multiplication of \( A \) if \( (v_A)_e^M(p) = v_A(p) \cdot e \) and \( (w_A)_e^M(p) = w_A(p) \cdot e \) for all \( p \in E \).

**Theorem 3.3.** If \( A = (v_A, w_A) \) be an IF-INK subalgebra of \( E \), then the IF \( e \)-multiplication of \( A \) is an IF-INK subalgebra of \( E \) for all \( e \in [0, 1] \).

**Proof.** Assume that \( A = (v_A, w_A) \) is a IF-INK subalgebra of \( E \). Then \( e \in [0, 1] \).

\[ (v_A)_e^M(p \odot q) = e \cdot v_A(p \odot q) \]
\[ \geq e \cdot \min \{ v_A(p), v_A(q) \} \]
\[ \geq \min \{ (v_A)_e^M(p), (v_A)_e^M(p) \} , \]

and

\[ (w_A)_e^M(p \odot q) = e \cdot w_A(p \odot q) \]
\[ \leq e \cdot \max \{ w_A(p), w_A(q) \} \]
\[ \leq \max \{ (w_A)_e^M(p), (w_A)_e^M(p) \} . \]

Hence \( (v_A)_e^M \) and \( (w_A)_e^M \) is an IF-INK subalgebra of \( E \).

**Theorem 3.4.** If \( A = (v_A, w_A) \) be an IF subset of \( E \), then the following assertions are equivalent.

i) \( v_A \) is an IF-INK subalgebra of \( E \).

ii) \( (v_A)_e^M \) is an IF-INK-subalgebra of \( E \).
Proof. Necessity follows from Theorem 3.3. For the sufficiency part, let $e \in [0,1]$ be such that $A_e^M$ is an IF-INK subalgebra of $A$. Then for all $p, q \in E$ we have:

$$v_A(p \odot q) \cdot e = (v_A)_e^M(p \odot q)$$

$$\geq \min \{(v_A)_e^M(p), (v_A)_e^M(q)\}$$

$$\geq \min \{v_A(p) \cdot e, v_A(q) \cdot e\}$$

$$v_A(p \odot q) \cdot e = \min \{v_A(p), v_A(q)\} \cdot e,$$

$$w_A(p \odot q) \cdot e = (w_A)_e^M(p \odot q)$$

$$\leq \max \{(w_A)_e^M(p), (w_A)_e^M(q)\}$$

$$\leq \max \{w_A(p) \cdot e, w_A(q) \cdot e\}$$

$$w_A(p \odot q) \cdot e = \max \{w_A(p), w_A(q)\} \cdot e.$$ 

Hence, $A$ is an IF-INK subalgebra of $E$. 

\[ \square \]

4. INTUITIONISTIC FUZZY EXTENSION ON INK-SUBALGEBRA

**Definition 4.1.** Let $A = (p, v_A, w_A)$ and $B = (p, v_B, w_B)$ be IFS of $E$. If $A \geq B$, $v_A(p) \leq v_B(p)$ and $w_A(p) \geq w_B(p)$ for all $p \in E$. Then we call $B$ is an IF-extension of $A$.

**Definition 4.2.** Let $A = (p, v_A, w_A)$ and $B = (p, v_B, w_B)$ be IFS of $E$. Then $B$ is named an IF-S-extension of $A$. If the ensuing assertions are valid:

i) $B$ is an IF-extension of $A$.

ii) If $A$ is an IF-INK subalgebra of $E$, then $B$ is an IF-INK subalgebra of $E$.

**Theorem 4.1.** Let $A$ be an IF-INK subalgebra of $E$ and $\alpha \in [0, T]$. Then the IF $\alpha$-translation $v^\alpha_A$ of $A$ is an IF-S-extension of $A$.

The converse of Theorem 4.1 is not true in general as seen in the following example.
Example 1. Consider a INK-algebra $E = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

\[
\begin{array}{c|ccccc}
\circ & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 2 & 0 \\
3 & 3 & 1 & 3 & 0 & 1 \\
4 & 4 & 4 & 4 & 4 & 0 \\
\end{array}
\]

Define $A = (p, v_A, w_A)$ be an IF subset of $E$ by

\[
\begin{array}{c|ccc}
E & 0 & 1 & 2 \\
\hline
v_A & 0.8 & 0.5 & 0.3 \\
w_A & 0.36 & 0.45 & 0.55 \\
\end{array}
\]

Then $A$ is an IF-INK subalgebra of $E$. Let $B = (p, v_B, w_B)$ be an IF subset of $E$ given by

\[
\begin{array}{c|c|ccc}
E & 0 & 1 & 2 & 3 \\
\hline
v_B & 0.84 & 0.56 & 0.38 & 0.67 \\
w_B & 0.10 & 0.35 & 0.40 & 0.32 \\
\end{array}
\]

Then $B$ is an IF-S-extension of $A$. But it is not an IF-$\alpha$-translation of $A^T$ of $A$, for all $\alpha \in [0, T]$. Clearly, the intersection of IF S-extension of an IF-INK subalgebra $A$ of $E$ is an IF S-extension of $A$. But the union of IF S-extensions of an IF-INK subalgebra $A$ of $E$ is not an IF S-extension of $A$ as seen in the following example.

Example 2. Consider a INK-algebra $A = \{0, 1, 2, 3, 4\}$ with the following Cayley table:

\[
\begin{array}{c|ccccc}
\circ & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
2 & 2 & 1 & 0 & 0 & 0 \\
3 & 3 & 1 & 1 & 0 & 0 \\
4 & 4 & 3 & 3 & 1 & 0 \\
\end{array}
\]
Define \( A = (p, v_A, w_A) \) be an IF subset of \( E \) by

\[
\begin{array}{c|cccc}
E & 0 & 1 & 2 & 3 & 4 \\
v_A & 0.7 & 0.4 & 0.6 & 0.3 & 0.3 \\
w_A & 0.2 & 0.3 & 0.4 & 0.4 & 0.6 \\
\end{array}
\]

Then \( A \) is an IF-INK subalgebra of \( E \). Let \( B = (p, v_B, w_B) \) be an IF subset of \( E \) given by

\[
\begin{array}{c|cccc}
E & 0 & 1 & 2 & 3 & 4 \\
v_B & 0.8 & 0.6 & 0.8 & 0.4 & 0.4 \\
w_B & 0.2 & 0.3 & 0.2 & 0.3 & 0.2 \\
\end{array}
\]

and

\[
\begin{array}{c|cccc}
E & 0 & 1 & 2 & 3 & 4 \\
v_C & 0.9 & 0.6 & 0.6 & 0.6 & 0.7 \\
w_C & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\
\end{array}
\]

Then \( B \) and \( C \) are IF-S-extensions of \( A \). But the union \( B \cup C \) is an IF-S-extension of \( A \) since, but it is not an IF S-extension of \( A \), since,

\[
v_{B \cup C}(4 \odot 2) = 0.3 \neq 0.5 = \min \{v_{B \cup C}(4), v_{B \cup C}(2)\}
\]

For an IF-subset \( A = (v_A, w_A) \) of \( E, \alpha \in [0, T] \) and \( t, s \in [0, 1] \) by \( t \geq \alpha \). Let \( U_\alpha(v_A; t) = \{p \in E | v_A(p) \geq t - \alpha \} \) and \( L_\alpha(w_A; s) = \{p \in E | w_A(p) \geq s + \alpha \} \). If \( A \) is an IF-INK subalgebra of \( E \), then it is clear that \( U_\alpha(v_A; t) \) and \( L_\alpha(w_A; s) \) are subalgebras of \( E \), for all \( t \in \text{Im}(v_A) \) and \( s \in \text{Im}(w_A) \) with \( t \geq \alpha \). But, if we do not give a condition that \( A \) is an IF-INK subalgebra of \( E \), then \( U_\alpha(v_A; t) \) and \( L_\alpha(w_A; s) \) are not INK-subalgebras of \( E \) as seen in the following example.

**Example 3.** Let \( A = \{0, 1, 2, 3, 4\} \) be a INK-algebra which is given in Example 4.1. Define an IF-subset \( v \) of \( E \) by

\[
\begin{array}{c|cccc}
E & 0 & 1 & 2 & 3 & 4 \\
v_A & 0.7 & 0.4 & 0.6 & 0.3 & 0.5 \\
w_A & 0.2 & 0.3 & 0.4 & 0.7 & 0.6 \\
\end{array}
\]

Then \( v \) is not an IF-INK subalgebra of \( E \). Then

\[
v_A(4 \odot 2) = 0.3 \neq 0.5 = \min \{v_A(4), v_A(2)\}
\]
and

\[ w_A(4 \odot 1) = 0.7 \neq 0.6 = \max \{ w_A(4), w_A(1) \} \]

\( A = (v_A, w_A) \) is not an IF-INK subalgebra of \( E \). For \( \alpha = 0.1 \) and \( t = 0.5 \), we obtain \( U_\alpha(v_A; t) = \{ 0, 1, 2, 4 \} \), which is not an INK-subalgebra of \( E \). For \( \alpha = 0.1 \) and \( s = 0.6 \), we obtain \( L_\alpha(w_A; s) = \{ 0, 1, 2, 4 \} \) which is not a INK-subalgebra of \( E \).

**Theorem 4.2.** Let \( A = (v_A, w_A) \) be an IF-INK subalgebra of \( E \) and let \( \alpha, \beta \in [0, T] \). If \( \alpha \geq \beta \) then the IF translation \( A^T_\alpha = ((v_A)^T_\alpha, (w_A)^T_\alpha) \) of \( A \) is an IF S-extension of the IF \( \beta \)-translation \( A^T_\beta = ((v_A)^T_\beta, (w_A)^T_\beta) \) of \( A \).

For every IF-INK subalgebra \( A \) of \( E \) and \( \beta \in [0, T] \), the IF \( \beta \)-translation \( A^T_\beta \) of \( A \) is an IF-INK subalgebra of \( E \). If IF S-extension of \( A^T_\beta \), then there exists \( \alpha \in [0, T] \) such that \( \alpha \geq \beta \) and \( B \geq A^T_\beta \), that is \( v_A(p) \geq (v_A)^T_\alpha \) and \( w_A(p) \leq (w_A)^T_\alpha \), for all \( \alpha \in E \). Hence, we have the following theorem.

**Theorem 4.3.** Let \( A \) be an IFINK-subalgebra of \( E \) and let \( \beta \in [0, T] \) for every IF S-extension \( B = (v_B, w_B) \) of the IF \( \beta \)-translation \( A^T_\beta \) of \( A \), there exists \( \alpha \in [0, T] \) such that \( \alpha \geq \beta \) and \( B \) is an IF S-extension of the IF \( \alpha \)-translation \( (v_A)^T_\alpha \) of \( A \).

Let us illustrate the Theorem 4.3 using the following example.

**Example 4.** Let \( A = \{ 0, 1, 2, 3, 4 \} \) be a INK-algebra and \( A = (v_A, w_A) \) be an IFS of \( E \). Take \( T = 0.3 \). If we take \( \beta = 0.15 \), then the IF \( \beta \)-translation \( A^T_\beta \) of \( A \) is given by

<table>
<thead>
<tr>
<th>( v_A^T_\beta )</th>
<th>0.85</th>
<th>0.55</th>
<th>0.75</th>
<th>0.45</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_A^T_\beta )</td>
<td>0.05</td>
<td>0.25</td>
<td>0.25</td>
<td>0.15</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Let \( B = (v_B, w_B) \) be an IFS of \( E \) defined by

<table>
<thead>
<tr>
<th>( v_B )</th>
<th>0.90</th>
<th>0.68</th>
<th>0.88</th>
<th>0.58</th>
<th>0.78</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_B )</td>
<td>0.12</td>
<td>0.22</td>
<td>0.32</td>
<td>0.32</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Then \( B \) is clearly an IF INK subalgebra of \( E \) which is an IF S-extension of the IF \( \beta \)-translation \( A^T_\beta \) of \( A \). But \( B \) is not an IF translation of \( A \), for all \( \alpha \in [0, T] \). If
we take $\alpha = 0.18$ then $\alpha = 0.18 > 0.15 = \beta$ and the IF translation $A^T_\alpha = ((v_A)^T_\alpha, (w_A)^T_\alpha)$, of $A$ is given as follows,

<table>
<thead>
<tr>
<th>$E$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(v_A)^T_\alpha$</td>
<td>0.90</td>
<td>0.68</td>
<td>0.88</td>
<td>0.58</td>
<td>0.78</td>
</tr>
<tr>
<td>$(w_A)^T_\alpha$</td>
<td>0.12</td>
<td>0.22</td>
<td>0.32</td>
<td>0.32</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Note that $B(p) \geq (v_A)^T_\alpha(p)$ that is $v_B(p) \geq (v_A)^T_\alpha$ and $w_B(p) \leq (w_A)^T_\alpha$, for all $p \in E$, and hence, $B$ is an IF $S$-extension of the IF $\alpha$-translation $(v_A)^T_\alpha$ of $A$.

REFERENCES