TRANSLATIONS OF INTUITIONISTIC FUZZY SUBALGEBRAS IN BF-ALGEBRAS

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ABSTRACT. This research article explores on, the concepts of IFT to IFS in BF-
algebras. The phenomenon of IF-extensions and IF-multiplications of IFS is
proposed and several related properties are investigated. In this paper, the
interaction between IFTs and IF-extensions of IFSs are investigated.

1. INTRODUCTION AND PRELIMINARIES

Iseki et al. proposed two classes of abstract algebras BCI-algebras and BCK-
algebras [3]. It is evident that the class of BCK-algebras is a proper subclass of
the class of BCI-Algebras. H.S. Kim et al. [24] proposed a new notion known
as a B-algebras, which is a simplification of BCK-algebra. Walendziak [1] de-
defined BF-algebras. In 1965, the notion of fuzzy sets, an extraordinary idea in
mathematics, was proposed by Zadeh [25]. Saeid and Rezvani [2] proposed
BF-subalgebras based on the above concepts. Atanassov [21,22] was the first
researcher who introduced the new idea of “IF-set”, which is depicted as gen-
eralized idea of fuzzy set. Satyanarayana et al. [4,6-8]. Proposed fuzzy BF-
subalgebras and IFS. Fuzzy Translation of BCK/BCI-algebras, worked out by
many researchers such as Lee and Jun [23]. Further some more researchers

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(IFT), intuitionistic fuzzy-multiplication(IFM).
used the concept of fuzzy and fuzzy functions on time scales [9-20]. The aim of this article is applying the notion of the IFTs, IF-extensions and IF-multiplications of IFSs in BF-algebras are investigated.

**Definition 1.1.** [1] A BF-algebra is a non-empty set $Y$ with a constant $0$ and a binary operation satisfying the following axioms:

(i) $\alpha_1 \ast \alpha_1 = 0$,

(ii) $\alpha_1 \ast 0 = \alpha_1$,

(iii) $0 \ast (\alpha_1 \ast \alpha_2) = \alpha_2 \ast \alpha_1$ for all $\alpha_1, \alpha_2 \in Y$.

$X$ is considered a BF-algebra in the following conversation.

**Example 1.** [6] $R$ = The set of real numbers and $A = (R, \ast, 0)$ be an Algebra given by

$$\alpha_1 \ast \alpha_2 = \begin{cases} 
\alpha, & \text{if } \alpha_2 = 0, \\
\alpha_2, & \text{if } \alpha_1 = 0, \\
0, & \text{otherwise}
\end{cases}$$

Then $A$ will become a BF-algebra.

**Example 2.** [5] The set $X = \{0, p, q, s, t\}$, $\ast$ is given by the Table: $I$ is a BF-Algebra

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**Definition 1.2.** [1] $I \subseteq Y$ is know to be subalgebra of $Y$, if

(i) $0 \in I$,

(ii) $y_1 \in I$ and $y_2 \in I \Rightarrow y_1 \ast y_2 \in I$.

**Definition 1.3.** An IF-set $A = (X, R_A, J_A)$ is supposed to be IFSs of $X$ if

(i) $R_A(x_1 \ast x_2) \geq \min\{R_A(x_1), R_A(x_2)\}$

(ii) $J_A(x_1 \ast x_2) \leq \max\{J_A(x_1), J_A(x_2)\}$

for all $x_1, x_2 \in X$. 
2. Translations of IFSs in BF-Algebras

The following discussion is on the notation of IFT on $X$. It evident that, $X$ stands a BF-algebra, and for any IF-set $A = (R_A, J_A)$ of $X$, $T = 1 - \inf\{R_A(x_1) : x_1 \in X\} = 1 - \inf\{J_A(x_1) : x_1 \in X\}$.

**Definition 2.1.** $A = (R_A, J_A)$ is an IF-subset of $X$. Let $\alpha \in [0, T]$. An object having the form $A_\alpha = ((R_A)_\alpha^T, (J_A)_\alpha^T)$ is called IF-$\alpha$-translation of $A$ if $(R_A)_\alpha^T(x_1) = R_A(x_1) + \alpha$ and $(J_A)_\alpha^T(x_1) = J_A(x_1) - \alpha$ for all $x_1 \in X$.

**Definition 2.2.** Let $A = (R_A, J_A)$ be an IF-Subset of $X$ and let $\alpha \in [0, 1]$. An object having the form $A_\alpha^m = ((R_A)_\alpha^m, (J_A)_\alpha^m)$ is called an IF-$\alpha$-multiplication of $A$ if $(R_A)_\alpha^m(x_1) = \alpha R_A(x_1)$ and $(J_A)_\alpha^m(x_1) = \alpha J_A(x_1)$ for all $x_1 \in X$. For any IF-set $A = (R_A, J_A)$ of $X$, an IF-$0$-multiplication $A_0^m = ((R_A)_0^m, (J_A)_0^m)$ of $A$ is an IFS of $X$.

**Example 3.** Consider the BF-algebra $X = \{0, p, q, s\}$ in Example 2. Define a IF-subset $A = (R_A, J_A)$ of $X$ by

$$R_A(r) = \begin{cases} 0.4; r \neq q \\ 0.1; r = q \end{cases} \quad \text{and} \quad J_A(r) = \begin{cases} 0.4; r \neq q \\ 0.7; r = q \end{cases},$$

then $A = (R_A, J_A)$ is IFS of $X$. Here $T = 1 - \sup\{R_A(r) : r \in X\} = 1 - 0.4 = 0.6 = 1 - \inf\{J_A(r) : r \in X\} = 1 - 0.4 = 0.6$. Choose $\alpha = 0.3 \in [0, T]$ and $\beta = 0.2 \in [0, 1]$. Then the mapping $(R_A)^T_{0.3} : X \to [0, 1]$ $(J_A)^T_{0.3} : X \to [0, 1]$ are defined by

$$(R_A)^T_{0.3}(r) = \begin{cases} 0.4 + 0.3 = 0.7; r \neq q \\ 0.1 + 0.3 = 0.4; r = q \end{cases}$$

and

$$(J_A)^T_{0.3}(r) = \begin{cases} 0.4 - 0.3 = 0.1; r \neq q \\ 0.7 - 0.3 = 0.4; r = q \end{cases},$$

which satisfies

$$A^T_{0.3} = ((R_A)^T_{0.3}, (J_A)^T_{0.3}) = (R_A(r) + 0.3, J_A(r) - 0.3) \text{ for all } r \in X$$

is IF-$0.3$-Translation.

The mappings $(R_A)^m_{0.2} : X \to [0, 1]$ and $(J_A)^m_{0.2} : X \to [0, 1]$ are defined by

$$(R_A)^m_{0.2}(r) = \begin{cases} (0.4)(0.2) = 0.08; r \neq q \\ (0.1)(0.2) = 0.02; r = q \end{cases}$$
and

\[
(J_A)^m_{0.2}(r) = \begin{cases} 
(0.4)(0.2) = 0.08; & r \neq q \\
(0.7)(0.2) = 0.14; & r = q
\end{cases}
\]

which satisfies \((R_A)^m_{0.2}(r) = 0.2. R_A (J_A)^m_{0.2}(r) = 0.2. J_A\) for all is IF-0.2-multiplication.

**Theorem 2.1.** For all IFS \(A = (R_A, J_A)\) of \(X \land \alpha \in [0, T] the 'IF-\alpha\text{-translation}'. \(A^\alpha_T = ((R_A)^T_\alpha, (J_A)^T_\alpha)\) of A = (R_A, J_A) is a IFS of \(X\).

**Proof.** Let \(r, s \in X\) and \(\alpha \in [0, T]\). Then \(R_A(r * s) \geq \min\{R_A(r), R_A(s)\}\). Now,

\[
(R_A)^T_\alpha(r * s) = R_A(r * s) + \alpha \geq \min\{R_A(r), R_A(s)\} + \alpha
\]

\[
= \min\{R_A(r) + \alpha, R_A(s) + \alpha\} = \min\{(R_A)^T_\alpha(r), (R_A)^T_\alpha(s)\}
\]

and

\[
(J_A)^T_\alpha(r * s) = J_A(r * s) - \alpha \leq \max\{J_A(r), J_A(s)\} - \alpha
\]

\[
= \max\{J_A(r) - \alpha, J_A(s) - \alpha\} = \max\{(J_A)^T_\alpha(r), (J_A)^T_\alpha(s)\}.
\]

Hence the theorem follows. \(\square\)

**Theorem 2.2.** For all IF-subset \(A = (R_A, J_A)\) of \(X\) \land \(\alpha \in [0, T]\) if the IF-\(\alpha\) -Translation \(A^\alpha_T = ((R_A)^T_\alpha, (J_A)^T_\alpha)\) of A = (R_A, J_A) is a IFS of \(X\) then A = (R_A, J_A) is IFS of \(X\).

**Proof.** Suppose that \(A^\alpha_T = ((R_A)^T_\alpha, (J_A)^T_\alpha)\) is an IFSs of \(X\) and \(\alpha \in [0, T]\). Let \(r, s \in X\), we have

\[
R_A(r * s) + \alpha = (R_A)^T_\alpha(r * s) \geq \min\{(R_A)^T_\alpha(r), (R_A)^T_\alpha(s)\}
\]

\[
\geq \min\{R_A(r) + \alpha, R_A(s) + \alpha\} = \min\{R_A(r), R_A(s)\} + \alpha
\]

and

\[
J_A(r * s) - \alpha = (J_A)^T_\alpha(r * s) \leq \max\{(J_A)^T_\alpha(r), (J_A)^T_\alpha(s)\}
\]

\[
\leq \max\{J_A(r) - \alpha, J_A(s) - \alpha\} = \max\{J_A(r) - \alpha, J_A(s) - \alpha\},
\]

which implies that \(R_A(r * s) \geq \min\{R_A(r), R_A(s)\}\) and \(J_A(r * s) \leq \max\{J_A(r), J_A(s)\}\) for all \(r, s \in X\). Hence A = (R_A, J_A) is IFS of \(X\). \(\square\)

**Theorem 2.3.** \((A_1)^T_\alpha\) and \((A_2)^T_\alpha\) are two IFS of \(X\) \Rightarrow \((A_1 \cap A_2)^T_\alpha\) is also a IFS of \(X\).

**Proof.** \((A_1)^T_\alpha\) and \((A_2)^T_\alpha\) are two IFS of \(X\). Then

\[
(R_{A_1 \cap A_2})^T_\alpha(x_1 \times x_2) = \min\{(R_{A_1})^T_\alpha(x_1 \times x_2), (R_{A_2})^T_\alpha(x_1 \times x_2)\}
\]

\[
= \min\{(R_{A_1})(x_1 \times x_2) + \alpha, (R_{A_2})(x_1 \times x_2) + \alpha\}
\]
\[ \begin{align*}
\geq & \min \{ \min \{ R_A(x_1), R_A(x_2) \} + \alpha, \min \{ R_A(x_1), R_A(x_2) \} + \alpha \} \\
= & \min \{ \min \{ R_A(x_1) + \alpha, R_A(x_2) + \alpha \}, \min \{ R_A(x_1) + \alpha, R_A(x_2) + \alpha \} \} \\
= & \min \{ \min \{ (R_A)_\alpha^T(x_1), (R_A)_\alpha^T(x_2) \}, \min \{ (R_A)_\alpha^T(x_1), (R_A)_\alpha^T(x_2) \} \} \\
= & \min \{ \min \{ (R_A)_\alpha^T(x_1), (R_A)_\alpha^T(x_2) \}, \min \{ (R_A)_\alpha^T(x_2), (R_A)_\alpha^T(x_2) \} \} \\
= & \min \{ (R_A)_\alpha^T(x_1), (R_A)_\alpha^T(x_2) \} (R_A_1 \cap A_2)_\alpha^T(x_1 * x_2) \geq \min \{ (R_A_1 \cap A_2)_\alpha^T(x_1), (R_A_1 \cap A_2)_\alpha^T(x_2) \}
\end{align*} \]

\[(J_{A_1 \cap A_2})_\alpha^T(x_1 * x_2) = \max \{ (J_{A_1})_\alpha^T(x_1 * x_2), (J_{A_2})_\alpha^T(x_1 * x_2) \}
= \max \{ (J_{A_1}(x_1) - \alpha, (J_{A_2}(x_1) - \alpha \}
\leq \max \{ \max \{ J_{A_1}(x_1), J_{A_2}(x_2) \} - \alpha, \max \{ J_{A_2}(x_1), J_{A_2}(x_2) \} - \alpha \}
= \max \{ \max \{ (J_{A_1}(x_1) - \alpha, (J_{A_2}(x_2) - \alpha \}, \max \{ J_{A_2}(x_1) - \alpha, J_{A_2}(x_2) - \alpha \} \}
= \max \{ \max \{ (J_{A_1})_\alpha^T(x_1), (J_{A_2})_\alpha^T(x_2) \}, \max \{ (J_{A_2})_\alpha^T(x_1), (J_{A_2})_\alpha^T(x_2) \} \}
= \max \{ (J_{A_1})_\alpha^T(x_1), (J_{A_2})_\alpha^T(x_2) \}
\]

Hence \((A_1 \cap A_2)_\alpha^T\) is an IFS of \(X\). \(\square\)

**Theorem 2.4.** Let \(\{ A_i / i = 1, 2, 3, \ldots \}\) be a family of IFS of \(X\). Then \((\bigcap A_i)_\alpha^T\) is also an IFS of \(X\), where \((\bigcap A_i)_\alpha^T = \min \{ (A_i)_\alpha^T(x) \}\).

**Theorem 2.5.** For any IFS \(A = (R_A, J_A)\) of \(X\), \(\alpha\) is an element in \([0, 1]\), the IF-\(\alpha\)-multiplication \(A_A^m = (R_A)_\alpha^m, (J_A)_\alpha^m)\) of \(A = (R_A, J_A)\) is an IFS of \(X\).

**Proof.** Let \(r, s \in X \& \alpha \in [0, 1]\). Then \(R_A(r * s) \geq \min \{ R_A(r), R_A(s) \}\). Now
\[ \begin{align*}
(R_A)_\alpha^m(r * s) &= \alpha R_A(r * s) \geq \alpha \min \{ R_A(r), R_A(s) \} \\
= & \min \{ \alpha R_A(r), \alpha R_A(s) \} = \min \{ \{ (R_A)_\alpha^m(r), (R_A)_\alpha^m(s) \}, (J_A)_\alpha^m(r * s) \} \\
= & \alpha J_A(r * s) \leq \alpha \max \{ J_A(r), J_A(s) \} = \max \{ \alpha J_A(r), \alpha J_A(s) \} \\
= & \max \{ (J_A)_\alpha^m(r), (J_A)_\alpha^m(s) \}.
\end{align*} \]

Hence the theorem follows. \(\square\)

**Theorem 2.6.** For any IF-subset \(A = (R_A, J_A)\) of \(X\) and \(\alpha \in [0, 1]\), if the IF-\(\alpha\)-multiplication \(A_A^m = ((R_A)_\alpha^m, (J_A)_\alpha^m)\) of \(A = (R_A, J_A)\) is an of \(X\) then \(A = (R_A, J_A)\) is IFS of \(X\).
**Proof.** Assume $A^m_\alpha = ((\mu A)^m_\alpha, (\lambda A)^m_\alpha)$ is an IFS of $X$ where $\alpha \in [0, T], x_1, x_2 \in X$. One can have

$$\begin{align*}
\alpha.R_A(x_1 \ast x_2) &= (R_A^m_\alpha(x_1 \ast x_2) = \min\{\alpha.R_A(x_1), \alpha.R_A(x_2)\} \\
&= \alpha.J_A(x_1 \ast x_2)
\end{align*}$$

which implies that $R_A(x_1 \ast x_2) \geq \min\{R_A(x_1), R_A(x_2)\}$ and $J_A(x_1 \ast x_2) \leq \max\{J_A(x_1), J_A(x_2)\}$ for all $x_1, x_2 \in X$ since $\alpha \neq 0$. Hence $A = (R_A, J_A)$ is IFS of $X$. □

**Theorem 2.7.** $(A_1)^m_\alpha$ and $(A_2)^m_\alpha$ be two IFS of $X \Rightarrow (A_1 \cap A_2)^m_\alpha$ is also IFS of $X$.

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