SINGLE VALUED LINEAR OCTAGONAL NEUTROSOPHIC NUMBER AND ITS APPLICATION IN MINIMAL SPANNING TREE

A.S. RICHARD$^1$ AND A. RAJKUMAR$^2$

**Abstract.** In this paper Single Valued Linear Octagonal Neutrosophic Number (SVLONN) is introduced. The De-Neutrosophication method of neutrosophic number is found using Removal area method, and numerical example is given to help to understand the concept, also this paper finds the better way to find the Minimal Spanning Tree of the given graph using single valued linear octagonal neutrosophic number.

1. **Introduction**

In 1998, (Smarandache, 1998)$^1$, introduced a new theory called Neutrosophy, which is basically a branch of philosophy that focus on the origin, nature, and scope of neutralities and their interactions with different ideational spectra. Based on the neutrosophy, Smarandache$^1$ defined the concept of neutrosophic set which is characterized by a degree of truth membership $T$, a degree of indeterminate-membership $I$ and a degree false-membership $F$. The concept of neutrosophic set theory is a generalization of the concept of classical sets, fuzzy sets (Zadeh, 1965)$^3$, intuitionistic fuzzy sets (Atanassov, 1986)$^6$, Neutrosophic sets is mathematical tool used to handle problems like imprecision, indeterminacy and inconsistency of data. Specially, the indeterminacy presented in

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the neutrosophic sets is independent on the truth and falsity values. To easily apply the neutrosophic sets to real scientific and engineering areas, (Smarandache, 1998) proposed the single valued neutrosophic sets as subclass of neutrosophic sets. Later on, (Wang et al., 2010) provided the set-theoretic operators and various properties of single valued neutrosophic sets. The concept of neutrosophic sets and their extensions such as bipolar neutrosophic sets, complex neutrosophic sets, bipolar complex neutrosophic sets (Broumi et al. 2017) and so on have been applied successfully in several fields.

Graphs are the most powerful tool used in representing information involving relationship between objects and concepts. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from [0, 1]. The concept fuzzy graphs and their extensions have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

\section{Preliminaries}

\textbf{Definition 2.1. (Fuzzy Set[11])} A set \( \bar{A} \) is denoted as \( \bar{A} = \{(x, \mu_{\bar{A}}(x)) : x \in X, \mu_{\bar{A}}(X) \in [0, 1]\} \) represented by \((x, \mu_{\bar{A}}(x))\), where \( x \in \text{the crisp set} \ X \) and \( \mu_{\bar{A}}(x) \in \text{the interval} \ [0, 1] \), the set \( \bar{A} \) is called fuzzy set.

\textbf{Definition 2.2. (Single Valued Linear Neutrosophic Set (SVLNS) [1])} A Neutrosophic set \( \bar{n}A \) is said to be a single-Valued linear Neutrosophic Set \( (S_{\bar{n}A}) \) if \( x \) is a single-valued independent variable.

\[ S_{\bar{n}A} = \{x ; (\rho_{\bar{n}A}(x), \sigma_{\bar{n}A}(x), \omega_{\bar{n}A}(x)) : x \in X\} , \text{ where } \rho_{\bar{n}A}(x), \sigma_{\bar{n}A}(x), \omega_{\bar{n}A}(x) \text{ denoted the concept of trueness, indeterminacy and falsity memberships function respectively.} \]

\textbf{Definition 2.3. (Single Valued Linear Pentagonal Neutrosophic Number (SVLPNN) [2])} A single valued linear pentagonal neutrosophic number \( \bar{S} \) is defined and described as

\[ \bar{S} = \langle ([g^1, h^1, i^1, j^1, k^1] ; \rho) , ([g^2, h^2, i^2, j^2, k^2] ; \sigma) , ([g^3, h^3, i^3, j^3, k^3] ; \omega) \rangle , \]

where \( \rho, \sigma, \omega \in [0, 1] \). The truth membership function \( (\theta_s) : R \rightarrow [0, \rho] \), the indeterminacy membership function \( (\emptyset_s) : R \rightarrow [\sigma, 1] \) and the falsity membership function \( (\varphi_s) : R \rightarrow [\omega, 1] \) are given as:
\[ \theta_S(x) = \begin{cases} 
\theta_{s1}(x), & g^1 \leq x < h^1 \\
\theta_{s2}(x), & h^1 \leq x < i^1 \\
\rho, & x = i^1 \\
\theta_{s3}(x), & i^1 \leq x < j^1 \\
\theta_{s4}(x), & j^1 \leq x < k^1 \\
0, & \text{otherwise} 
\end{cases} \]

\[ \varphi_S(x) = \begin{cases} 
\varphi_{s1}(x), & g^3 \leq x < h^3 \\
\varphi_{s2}(x), & h^3 \leq x < i^3 \\
\omega, & x = i^3 \\
\varphi_{s3}(x), & i^3 \leq x < j^3 \\
\varphi_{s4}(x), & j^3 \leq x < k^3 \\
1, & \text{otherwise} 
\end{cases} \]

\[ \phi_S(x) = \begin{cases} 
\phi_{s1}(x), & g^2 \leq x < h^2 \\
\phi_{s2}(x), & h^2 \leq x < i^2 \\
\sigma, & x = i^2 \\
\phi_{s3}(x), & i^2 \leq x < j^2 \\
\phi_{s4}(x), & j^2 \leq x < k^2 \\
1, & \text{otherwise} 
\end{cases} \]

**Definition 2.4.** (Single Valued Linear Octagonal Neutrosophic Number (LSVONN))

A single valued linear octagonal neutrosophic number \( S \) is defined and described as

\[ \tilde{S} = < [(a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1); \rho] , [(a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2); \sigma] , [(a_3, b_3, c_3, d_3, e_3, f_3, g_3, h_3); \omega] > , \]

where \( \rho, \sigma, \omega \in [0, 1] \). The truth membership function \( \theta_S : R \rightarrow [0, \rho] \), the indeterminacy membership function \( \theta_S : R \rightarrow [\sigma, 1] \), the falsity membership function \( \varphi_S : R \rightarrow [\omega, 1] \) are given as

\[ \theta_S(x) = \begin{cases} 
x - a_1/a_1 & a_1 \leq x < b_1 \\
x - b_1 & b_1 \leq x < c_1 \\
x - c_1/d_1 & c_1 \leq x < d_1 \\
1 & x = d_1 \\
1 & x = e_1 \\
x - e_1/f_1 & e_1 \leq x < f_1 \\
x - f_1/g_1 & f_1 \leq x < g_1 \\
ge_1 - h_1 & g_1 \leq x < h_1 \\
0 & \text{otherwise} 
\end{cases} \]

\[ \varphi_S(x) = \begin{cases} 
x - a_2/a_2 & a_2 \leq x < b_2 \\
x - b_2 & b_2 \leq x < c_2 \\
x - c_2/d_2 & c_2 \leq x < d_2 \\
0 & x = d_2 \\
x - e_2/f_2 & e_2 \leq x < f_2 \\
x - f_2/g_2 & f_2 \leq x < g_2 \\
e_2 - h_2 & g_2 \leq x < h_2 \\
1 & \text{otherwise} 
\end{cases} \]
$\varphi_S(x) = \begin{cases} 
\frac{x-a_3}{b_3-a_3} & a_3 \leq x < b_3 \\
\frac{x-b_3}{c_3-b_3} & b_3 \leq x < c_3 \\
\frac{x-c_3}{d_3-c_3} & c_3 \leq x < d_3 \\
0 & x = d_3 \\
\frac{e_3-x}{e_3-f_3} & e_3 \leq x < f_3 \\
\frac{f_3-x}{f_3-g_3} & f_3 \leq x < g_3 \\
\frac{g_3-x}{g_3-h_3} & g_3 \leq x < h_3 \\
1 & \text{otherwise} 
\end{cases}$

$A_{neu} = (a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1; a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2; a_3, b_3, c_3, d_3, e_3, f_3, g_3, h_3)$

**Figure 1**

Graphical representation of single valued linear heptagonal neutrosophic number where $-0 \leq \theta_S(x) + \phi_S(x) + \varphi_S(x) \leq 3^+$. 
3. De-Neutrosophication of single valued linear octagonal neutrosophication number

Research developed some of the de-neutrosophication method to change a neutrosophic fuzzy number to crisp number some of them are
1. BADD (Basic defuzzification distribution)
2. BOA (Bisector of area)
3. CDD (Constraint decision defuzzification)
4. COA (Centre of area)
5. COG (Centre of gravity)

To transform a neutrosophic number to crisp number “REMOVAL AREA METHOD” is proposed in this paper.

Considering octagonal Neutrosophic number
\[ \bar{A}_{neu} = (a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1; a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2; a_3, b_3, c_3, d_3, e_3, f_3, g_3, h_3) \]

\[ \bar{A}_{neu}(\hat{P}, 0) = \frac{1}{2}[\bar{A}_{neu}(\hat{P}, 0) + \bar{A}_{neur}(\hat{P}, 0)] \]
\[ = \frac{1}{4}[(a_1 + 2b_1 + c_1 + f_1 + 2g_1 + h_1)\delta + (c_1 + d_1 + e_1 + f_1)(1 - \delta) + 2(d_1 + e_1)] \]

comparably
\[ \bar{A}_{neu}(\hat{Q}, 0) = \frac{1}{2}[\bar{A}_{neu}(\hat{Q}, 0) + \bar{A}_{neur}(\hat{Q}, 0)] \]
\[ = \frac{1}{4}[(a_2 + 2b_2 + c_2 + f_2 + 2g_2 + h_2)(1 - \delta) + (c_2 + d_2 + e_2 + f_2)\delta] \]
\[ \bar{A}_{neu}(\hat{R}, 0) = \frac{1}{2}[\bar{A}_{neu}(\hat{R}, 0) + \bar{A}_{neur}(\hat{R}, 0)] \]
\[ = \frac{1}{4}[(a_3 + 2b_3 + c_3 + f_3 + 2g_3 + h_3)(1 - \delta) + (c_3 + d_3 + e_3 + f_3)\delta] \]

\[ \bar{A}_{neu}(\hat{D}, 0) = \frac{1}{2}[\bar{A}_{neu}(\hat{D}, 0) + \bar{A}_{neur}(\hat{D}, 0)] \]
\[ = \frac{1}{4}[(a_1 + 2b_1 + c_1 + f_1 + 2g_1 + h_1 + c_2 + d_2 + e_2 + f_2 + c_3 + d_3 + e_3 + f_3)\delta + (c_1 + d_1 + e_1 + f_1 + a_2 + 2b_2 + c_2 + f_2 + 2g_2 + h_2 + a_3 + 2b_3 + c_3 + f_3 + 2g_3 + h_3)(1 - \delta) + 2(d_1 + e_1)] \]

4. Numerical Example

Algorithm:

1. Construct an adjacency matrix of the graph
2. Utilize De-neutrosophication technique of single valued heptagonal neutrosophication number Construct an crisp matrix
3. Select the least weight and if there is a tie in selection of least weight then take any one edge from the given graph.
(4) From the graph the remaining edge containing the least edge that doesn’t form a loop with previous figure Continue this process until all vertices are covered

(5) Stop.

To acquire minimum Spanning tree of the given single valued linear octagonal neutrosophic number

<table>
<thead>
<tr>
<th>Edges</th>
<th>Single Valued Linear Heptagonal Neutrosophic Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>e_{12}</td>
<td>&lt; 0.5, 1, 1.2, 2.5, 0.4, 0.6, 0.5, 0.3, 0.7, 1.2, 1.6, 2.2, 1.4, 0.5, 0.3, 0.8, 1.3, 1.8, 2.3, 3, 0.4, 0.3, 0.5 &gt;</td>
</tr>
<tr>
<td>e_{13}</td>
<td>&lt; 0.7, 1.2, 1.8, 2.4, 3, 1.3, 0.7, 0.5, 0.6, 1.1, 1.4, 2, 2.5, 0.5, 0.7, 0.5, 1.1, 1.5, 2, 2.5, 3.5, 0.5, 0.3, 0.5 &gt;</td>
</tr>
<tr>
<td>e_{15}</td>
<td>&lt; 0.3, 0.8, 1.4, 2, 2.6, 0.5, 0.9, 0.5, 0.2, 0.7, 1.2, 1.8, 2, 2, 0.5, 0.3, 0.5, 1.1, 1.5, 2.4, 3, 0.7, 0.9, 0.5 &gt;</td>
</tr>
<tr>
<td>e_{16}</td>
<td>&lt; 1.1, 5, 2.2, 5, 3, 1.3, 0.7, 0.5, 0.7, 1.2, 1.8, 2.2, 2, 6, 0.9, 0.5, 0.3, 1.2, 1.6, 2, 2.8, 3.5, 0.9, 0.6, 0.5 &gt;</td>
</tr>
<tr>
<td>e_{24}</td>
<td>&lt; 0.9, 1.4, 2, 2.6, 3, 2, 0.7, 0.5, 0.3, 0.6, 1.2, 1.6, 2, 2.4, 0.7, 0.1, 0.5, 1.2, 1.5, 2.3, 2, 8, 3.5, 0.9, 0.5, 0.3 &gt;</td>
</tr>
<tr>
<td>e_{25}</td>
<td>&lt; 0.8, 1.4, 2, 2.6, 3, 2, 0.7, 0.5, 0.3, 0.6, 1.2, 1.8, 2, 2, 2, 8, 0.9, 0.5, 0.3, 1, 1, 2, 2, 2, 4, 3, 0.9, 0.5, 0.3 &gt;</td>
</tr>
<tr>
<td>e_{26}</td>
<td>&lt; 0.6, 1.1, 5, 2, 2.5, 0.9, 0.5, 0.3, 0.4, 0.8, 1.2, 1.8, 2, 2, 2, 0.9, 0.7, 0.5, 0.8, 1.4, 2, 2, 4, 3, 0.9, 0.3, 0.1 &gt;</td>
</tr>
<tr>
<td>e_{34}</td>
<td>&lt; 1.1, 1.5, 1.9, 2, 2, 3, 2, 7, 0.9, 0.3, 0.1, 0.8, 1.2, 1.7, 2, 1.7, 2.1, 2, 4, 0.7, 0.5, 0.3, 1.4, 1.8, 2, 2, 2, 6, 3, 0.9, 0.5, 0.3 &gt;</td>
</tr>
<tr>
<td>e_{35}</td>
<td>&lt; 0.8, 1.2, 1.5, 1.8, 2, 4, 0.9, 0.5, 0.3, 0.5, 0.9, 1.3, 1.7, 2, 1, 0.9, 0.3, 0.1, 1.4, 1.8, 2, 2, 2, 6, 0.5, 0.9, 0.3 &gt;</td>
</tr>
<tr>
<td>e_{46}</td>
<td>&lt; 0.7, 1, 1.3, 1.6, 2, 0.5, 0.9, 0.5, 0.6, 0.9, 1.2, 1.5, 1.8, 0.9, 0.5, 0.3, 1.1, 1.4, 1.8, 2, 2, 2, 5, 0.7, 0.3, 0.1 &gt;</td>
</tr>
<tr>
<td>e_{56}</td>
<td>&lt; 1.2, 1.5, 1.8, 2, 4, 2, 6, 0.9, 0.3, 0.1, 0.9, 1.3, 1.7, 2, 2, 3, 0.7, 0.5, 0.3, 1.4, 1.8, 2, 2, 2, 5, 2, 8, 0.3, 0.5, 0.1 &gt;</td>
</tr>
</tbody>
</table>

**Figure 2**

**Step 1:** The Associated adjacency matrix of Figure 2

\[
A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}
\]
Step 2: Using De-neutrosophication the associative matrix becomes

\[
A = \begin{pmatrix}
0 & 1.1812 & 1.3979 & - & 0.9937 & 1.525 \\
1.1812 & 0 & - & 1.433 & 1.5166 & 1.225 \\
1.3979 & - & 0 & 1.402 & 1.183 & - \\
- & 1.433 & 1.402 & 0 & - & 1.0833 \\
0.9937 & 1.5166 & 1.183 & - & 0 & 1.366 \\
1.525 & 1.225 & - & 1.0833 & 1.366 & 0
\end{pmatrix}
\]
Step 3 and 4:

![Figure 3](image3.png) ![Figure 4](image4.png)

![Figure 5](image5.png) ![Figure 6](image6.png)

![Figure 7](image7.png) ![Figure 8](image8.png)

Least weight of the graph is \( (0.9957 + 1.0833 + 1.1812 + 1.183 + 1.225) = 5.6682 \) units

Step 5: End
5. Conclusion

In this article, the concept of single valued linear octagonal neutrosophic number has been developed in a different aspect. This result is applied in the field of graph theory to evaluate the minimal spanning tree of a graph. This method is more fast, accurate and exact results after the total computation. Future work, this neutrosophic number can be extended for the better results and some of the algorithm can be introduced to get a new idea of getting the accurate result.

References

[9] [Florentin Smarandache1: Definition of neutrosophic logic - A generalization of the intuitionistic fuzzy logic, Proceedings of 3rd Conference of the European Society for Fuzzy Logic and Technology, Zittau, Germany, September 10-12, 2003, 141-146.

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