ON NANO GENERALIZED PRE c-CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACES

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ABSTRACT. The purpose of this paper is to introduce a new class of continuous functions called Nano generalized pre c-continuous functions in Nano Topological Spaces and derive their characterizations in terms of nano generalized pre c-closure, nano generalized pre c-interior, nano generalized pre c-kernel and nano generalized pre c-surface.

1. INTRODUCTION

Continuous functions play a major role in general topology. Several authors have studied different types of generalization of continuous functions. Lellis Thivagar and Carmel Richard [1] introduced the notion of Nano Topology with respect to a subset $X$ of a universe which is defined in terms of approximations and boundary region. They defined nano closed sets, nano interior and nano closure. They [2] also introduced nano continuous functions, nano open maps, nano closed maps and nano homeomorphisms in nano topological spaces. Padmavathi and Nithyakala [4] introduced nano generalized pre c-closed sets.

In this paper we have introduced a new class of continuous functions called nano generalized pre c continuous functions and established some of their representations in terms of nano interior, nano closure, nano kernel, nano generalized pre c-interior, nano generalized pre c-closure, nano generalized pre c-kernel and nano generalized pre c-surface of sets.

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2. Preliminaries

Definition 2.1. [1] Let $U$ be a non empty finite set of objects called the universe and $R$ be an equivalence relation on $U$ named as indiscernibility relation. Then $U$ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be approximation space. Let $X \subseteq U$. Then

(1) The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and is denoted by $L_R(X)$. $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $L_R(X)$.

(2) The upper approximation of $X$ with respect to $R$ is the set of all objects which can be possibly classified as $X$ with respect to $R$ and is denoted by $U_R(X)$. $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

(3) The boundary region of $X$ with respect to $R$ is the set of all objects which can be classified neither as $X$ nor as not-$X$ with respect to $R$ and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.1. [1] If $(U, R)$ is an approximation space and $X, Y \subseteq U$, then

(1) $L_R(X) \subseteq X \subseteq U_R(X)$
(2) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$
(3) $L_R(U) = U_R(U) = U$
(4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(9) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(10) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
(11) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.2. [1] Let $U$ be the universe, $R$ be an equivalence relation on $U$ as $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.
- $U$ and $\emptyset \in \tau_R(X)$.
- The union of all the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$. 
The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is a topology on $U$ called the nano topology on $U$ with respect to $X$. We call $(U, \tau_R(X))$ as a nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. The complement of the nano open sets are called nano closed sets.

Remark 2.1. [1] If $\tau_R(X)$ is a nano topology on $U$ with respect to $X$, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.3. [1] If $(U, \tau_R(X))$ is a nano topological space with respect to $X$ where $X \subseteq U$ and if $A \subseteq U$ then

1. The nano interior of $A$ is defined as the union of all nano open subsets contained in $A$ and is denoted by $Nint(A)$.
2. The nano closure of $A$ is defined as the intersection of all nano closed sets containing $A$ and is denoted by $Ncl(A)$.

Definition 2.4. [3] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The set $Nker(A) = \cap\{U : A \subseteq U, U \in \tau_R(X)\}$ is called the nano kernel of $A$ and is denoted by $Nker(A)$.

3. Nano Generalized pre c-Continuous Functions

In this section, we define nano generalized pre c-continuous function and study its characterization with Ngpc-int, Ngpc-cl, Ngpc-ker and Ngpc-surf of sets.

Definition 3.1. Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two nano topological spaces. The function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is said to be Nano generalized pre c-continuous (briefly Ngpc-continuous) on $U$ if the inverse image of every nano open set in $V$ is a Ngpc-open set in $U$.

Example 1. Let $U = \{a, b, c, d\}$ with $L_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology on $U$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology on $V$. 
Let $\tau^C_R(X) = \{ \emptyset, U, \{a\}, \{a, b\}, \{a, c, d\} \}$ and $\tau^C_R(Y) = \{ \emptyset, V, \{z\}, \{x, z\}, \{y, z, w\} \}$ are the complements of $\tau_R(X)$ and $\tau_R(Y)$ respectively.

Define $f : (U, \tau_R(X)) \rightarrow (V, \tau^R_R(Y))$ as $f(a) = z, f(b) = y, f(c) = x, f(d) = w$. Then $f^{-1}(\{x\}) = \{c\}, f^{-1}(\{y, w\}) = \{b, d\}, f^{-1}(\{x, y, w\}) = \{b, c, d\}$. Thus the inverse image of every nano open set in $V$ is Ngpc-open in $U$.

**Definition 3.2.** The Nano generalized pre c-surface of $A$ is defined as the union of all Ngpc-closed sets of $U$ contained in $A$ and it is denoted by $\text{Ngpc-surf}(A)$.

**Example 2.** Let $U = \{a, b, c, d\}$ with $U^c_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology with respect to $X$ and the complement $\tau_R(X)^c = \{\emptyset, U, \{a\}, \{a, b\}, \{a, c, d\}\}$.

Then $\text{Ngpc-surf}(\{a\}) = \{a\}, \text{Ngpc-surf}(\{b\}) = \emptyset$ and $\text{Ngpc-surf}(\{a, c\}) = \{a, c\}$.

**Theorem 3.1.** A function $f : (U, \tau_R(X)) \rightarrow (V, \tau^R_R(Y))$ is Ngpc-continuous iff the inverse image of every nano open set in $V$ is Ngpc-closed in $U$.

**Proof.** Let $f$ be Ngpc-continuous. Let $A$ be a nano closed set in $V$. Then $V - A$ is nano open in $V$. Since $f$ is Ngpc-continuous, $f^{-1}(V - A)$ is Ngpc-open in $U$. That is $U - f^{-1}(A)$ is Ngpc-open in $U$. Therefore $f^{-1}(A)$ is Ngpc closed in $U$. Thus the inverse image of every nano closed set in $V$ is Ngpc-closed in $U$ if $f$ is Ngpc-continuous. Conversely, let the inverse image of every nano closed set in $V$ is Ngpc-closed in $U$. Let $B$ be a nano open set in $V$. Then $V - B$ is nano closed in $V$. Then $f^{-1}(V - B)$ is Ngpc-closed in $U$. That is $U - f^{-1}(B)$ is Ngpc-closed in $U$. Therefore $f^{-1}(B)$ is Ngpc open in $U$. Hence $f$ is Ngpc-continuous.

**Theorem 3.2.** Let $f : (U, \tau_R(X)) \rightarrow (V, \tau^R_R(Y))$ be a function. Then the following statements are equivalent.

(i) $f$ is Ngpc-continuous.
(ii) For every subset $A$ of $U$, $f(\text{Ngpc-cl}(A)) \subseteq \text{Ncl}(f(A))$.
(iii) For every subset $B$ of $V$, $\text{Ngpc-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{Ncl}(B))$.

**Proof.**

(i) $\iff$ (ii). Let $f$ be Ngpc-continuous and $A \subseteq U$. Then $f(A) \subseteq V$. $\text{Ncl}(f(A))$ is nano closed in $V$. Since $f$ is Ngpc-continuous, $f^{-1}(\text{Ncl}(f(A)))$ is Ngpc-closed in $U$. Since $f(A) \subseteq \text{Ncl}(f(A))$, $A \subseteq f^{-1}(\text{Ncl}(f(A)))$. $f^{-1}(\text{Ncl}(f(A)))$ is a Ngpc-closed set containing $A$. But Ngpc-cl$(A)$ is the smallest Ngpc-closed set containing $A$. Therefore Ngpc-cl$(A) \subseteq f^{-1}(\text{Ncl}(f(A)))$. That is $f(\text{Ngpc-cl}(A)) \subseteq \text{Ncl}(f(A))$. 

Conversely let $f(N_{gpc} - cl(A)) \subseteq Ncl(f(A))$ for every subset $A$ of $U$. Let $G$ be a nano closed set in $U$. Since $f^{-1}(G) \subseteq U, f(N_{gpc} - cl(f^{-1}(G))) \subseteq Ncl(f(f^{-1}(G))) = Ncl(G)$. That is $N_{gpc}-cl(f^{-1}(G)) \subseteq f^{-1}(Ncl(G)) = f^{-1}(G)$ since $G$ is nano closed. Hence $N_{gpc}-cl(f^{-1}(G)) \subseteq f^{-1}(G)$. But $f^{-1}(G) \subseteq N_{gpc} - cl(f^{-1}(G))$. Therefore $f^{-1}(G) = N_{gpc} - cl(f^{-1}(G))$. This implies $f^{-1}(G)$ is $N_{gpc}$-closed in $U$. Thus the inverse image of every nano closed set in $V$ is $N_{gpc}$-closed in $U$. Hence $f$ is $N_{gpc}$-continuous.

(ii) $\Leftrightarrow$ (iii). Assume (ii) holds. Let $B$ be any subset of $V$. Then replacing $A$ by $f^{-1}(B)$ in (ii) we have $f(N_{gpc} - cl(f^{-1}(B))) \subseteq Ncl(f(f^{-1}(B))) = Ncl(B)$. That is $N_{gpc}-cl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$.

Conversely suppose (iii) holds. Let $A$ be any subset of $U$. Then $f(A) \subseteq V$. Let $B = f(A)$. Then we have $N_{gpc}-cl(A) = N_{gpc} - cl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B)) = f^{-1}(Ncl(f(A)))$. This implies $N_{gpc}-cl(A) \subseteq f^{-1}(Ncl(f(A)))$. Thus $f(N_{gpc} - cl(A)) \subseteq Ncl(f(A))$. □

**Theorem 3.3.** Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a function. Also let $A \subseteq U$ and $B \subseteq V$. Then

(i) $f$ is $N_{gpc}$-continuous $\Leftrightarrow$ $Nint f(A) \subseteq f(N_{gpc} - int(A)) \Leftrightarrow f^{-1}(Nint(\tilde{B})) \subseteq N_{gpc} - int(f^{-1}(\tilde{B}))$.

(ii) $f$ is $N_{gpc}$-continuous $\Leftrightarrow$ $A \subseteq N_{gpc} - int(f^{-1}Nker(f(A))) \Leftrightarrow f^{-1}(B) \subseteq N_{gpc} - int(f^{-1}(Nker(B)))$.

**Proof.** Proof is similar as theorem 3.2 □

**Theorem 3.4.** Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be $N_{gpc}$-continuous. Then for every subset $A$ of $U$ we have

(i) $f(N_{gpc} - surf(A)) \subseteq Ncl(f(A)).$

(ii) $Nint f(A) \subseteq f(N_{gpc} - ker(A)).$

(iii) $f(N_{gpc} - ker(A)) \subseteq Nker(f(A)).$

**Proof.** (i) Let $f$ be $N_{gpc}$-continuous and $A \subseteq U$. Then $f(A) \subseteq V.Ncl(f(A))$ is nano closed in $V$. Since $f$ is $N_{gpc}$-continuous, $f^{-1}(Ncl(f(A)))$ is $N_{gpc}$-closed in $U$. Therefore $N_{gpc}$-surf($f^{-1}(Ncl(f(A)))$) = $f^{-1}(Ncl(f(A)))$. But we know that $f(A) \subseteq Ncl(f(A)) , A \subseteq f^{-1}(Ncl(f(A)))$ which implies $N_{gpc}$-surf $A \subseteq N_{gpc} - surf(f^{-1}(Ncl(f(A))))$. Hence $N_{gpc}$-surf $A \subseteq f^{-1}(Ncl(f(A)))$. That is $f(N_{gpc} - surf(A)) \subseteq Ncl(f(A))$. Proof of (ii) and (iii) are similar. □
4. Nano Contra Generalized pre c-Continuous functions

In this section, we define nano contra generalized pre c-continuous function and study its characterization with Ngpc-int and Ngpc-cl of sets.

Definition 4.1. Let \((U, \tau_R(X))\) and \((V, \tau'_R(Y))\) be two nano topological spaces. The function \(f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))\) is said to be Nano contra generalized pre c-continuous (briefly Ncgpc-continuous) on \(U\) if the inverse image of every nano open set in \(V\) is Ngpc-closed in \(U\).

Example 3. In Example 1, Define \(f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))\) as \(f(a) = w, f(b) = y, f(c) = x, f(d) = z\). Then \(f^{-1}\{x\} = \{c\}, f^{-1}\{y, w\} = \{a, b\}, f^{-1}\{x, y, w\} = \{a, b, c\}\). The inverse image of every nano open set in \(V\) is Ngpc-closed in \(U\).

Theorem 4.1. A function \(f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))\) is Ncgpc-continuous iff the inverse image of every nano closed set in \(V\) is Ngpc-open in \(U\).

Proof. Let \(f\) be Ncgpc-continuous. Let \(A\) be a nano closed set in \(V\). Then \(V - A\) is nano open in \(V\). Since \(f\) is Ncgpc-continuous, \(f^{-1}(V - A) = U - f^{-1}(A)\) is Ngpc-closed in \(U\). Therefore \(f^{-1}(A)\) is Ngpc-open in \(U\). Thus the inverse image of every nano closed set in \(V\) is Ngpc-open in \(U\). Conversely, assume that the inverse image of every nano closed set in \(V\) is Ngpc-open in \(U\). Let \(B\) be a nano open set in \(V\). Then \(V - B\) is nano closed in \(V\). By our assumption \(f^{-1}(V - B) = U - f^{-1}(B)\) is Ngpc-open in \(U\). Therefore \(f^{-1}(B)\) is Ngpc-closed in \(U\). That is the inverse image of every nano open set in \(V\) is a Ngpc-closed set in \(U\). Hence \(f\) is Ncgpc-continuous.

Theorem 4.2. Let \(f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))\) be a function. Then the following statements are equivalent.

(i) \(f\) is Ncgpc-continuous.

(ii) For every subset \(A\) of \(U\), \(f(Ngpc - cl(A)) \subseteq Nker(f(A))\).

(iii) For every subset \(B\) of \(V\), \(Ngpc - cl(f^{-1}(B)) \subseteq f^{-1}(Nker(B))\).

Proof.

(i) \(\Rightarrow\) (ii). Let \(f\) be Ncgpc-continuous and \(A \subseteq U\). Then \(f(A) \subseteq V\). \(Nker(f(A))\) is nano open in \(V\). Since \(f\) is Ncgpc-continuous, \(f^{-1}(Nker(f(A)))\) is Ngpc-closed in \(U\). Since \(f(A) \subseteq Nker(f(A))\), \(A \subseteq f^{-1}(Nker(f(A)))\). \(f^{-1}(Nker(f(A)))\)
is a Ngpc-closed set containing $A$. But $Ngpc - cl(A)$ is the smallest Ngpc-closed set containing $A$. Therefore $Ngpc - cl(A) \subseteq f^{-1}(Nker(f(A)))$. That is $f(Ngpc - cl(A)) \subseteq Nker(f(A))$.

(ii) $\Rightarrow$ (iii). Assume (ii) holds. Let $B$ be any subset of $V$. Then $f^{-1}(B) \subseteq U$. By our assumption $f(Ngpc - cl(f^{-1}(B))) \subseteq Nker(f(f^{-1}(B))) = Nker(B)$. That is $Ngpc-cl(f^{-1}(B)) \subseteq f^{-1}(Nker(B))$.

(iii) $\Rightarrow$ (i). Suppose (iii) holds. Let $G$ be a nano open subset of $V$. Then by our assumption $Ngpc-cl(f^{-1}(G)) \subseteq f^{-1}(Nker(G)) = f^{-1}(G)$. But we know that $f^{-1}(G) \subseteq Ngpc - cl(f^{-1}(G))$. That is $Ngpc-cl(f^{-1}(G)) = f^{-1}(G)$ implies $f^{-1}(G)$ is Ngpc-closed in $U$. Therefore $f$ is Ngcpc continuous.

\[\square\]

**Theorem 4.3.** Let $f : U \to V$ be a function. Also let $A \subseteq U$ and $B \subseteq V$. Then we have

1. $f$ is Ncgpc-continuous $\iff$ $Ngpc-cl(f^{-1}(Nint(f(A)))) \subseteq A$ 
   $\iff$ $Ngpc-cl(f^{-1}(Nint(B))) \subseteq f^{-1}(B)$.

2. $f$ is Ncgpc-continuous $\iff$ $A \subseteq Ngpc-int f^{-1}(Ncl f(A)))$ 
   $\iff$ $f^{-1}(B) \subseteq Ngpc-int (f^{-1}(Ncl(B)))$.

**Proof.** Proof is similar as theorem 4.2. \[\square\]

**References**


