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ON $(\in, \in \vee q)$ INTUITIONISTIC FUZZY IDEAL OF N-GROUP

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ABSTRACT. We present the idea of N-group's $(\in, \in \vee q)$- intuitionistic fuzzy ideal and some associated properties as the content material of this paper.It's shown with the help of example that every intuitionistic fuzzy ideal of N-group is although an $(\in, \in \vee q)$-intuitionistic fuzzy ideal,but the converse isn't true and hence a necessary and sufficient condition is introduced in this purpose.The usage of the idea of the level set we provide an essential and sufficient circumstance for a level set to be an ideal of N-group.Discussions on image and pre-image of a $(\in, \in \vee q)$-intuitionistic fuzzy ideal under N-homomorphism also are a part of our research.

1. INTRODUCTION

Rosenfield [2] in 1971 utilize the notion of the fuzzy set by way of Zadeh [10] in 1965 to define fuzzy subgroups, which were studied in detail through the numerous researchers for various algebraic systems.Liu in [19] discussed apropreatly the fuzzy ideal of a ring and Abou Zaid in [15] added about the fuzzy sub near ring and fuzzy ideals of near rings. Moreover Davvaz [3] additionally mentioned some properties of fuzzy ideals of the same. In [18,7], the perception of fuzzy ideals and their numerous natures are brought. Using the concept of fuzzy point and its belongingness to a fuzzy set, Bhakat and Das in [16] define (α, β) fuzzy subgroups where α and β are members of the

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collection $\{\in, q, \in \wedge q, \in \vee q\}, \alpha \neq\in \wedge q$ and using this in [17] they introduced $(\in, \in \vee q)$-fuzzy near ring's subrings and ideals.A loop of researchers like Davvaz [4], Narayanan, and Manikantan [1], Zhan et al.[8] introduced $(\in, \in \vee q)$-fuzzy subnear rings and ideals of the near ring.

In 1986, Atanassov [9] presented the model of intuitionistic fuzzy sets by way of a simplification of a fuzzy set. Considering that then many researchers executed this belief to look at the intuitionistic fuzzy group [13], intuitionistic fuzzy near ring and about its ideal in [14]. Coker and Demirci [6] delivered the intuitionistic fuzzy point and which become by Jun [20] to define (ϕ, ψ)-intuitionistic fuzzy subgroup with ϕ and ψ are any dual of $\{\in, q, \in \wedge q, \in \vee q\}, \alpha \neq\in \wedge q$. In our studies, we present the idea of $(\in, \in \vee q)$-intuitionistic fuzzy ideal of N-group and explore associated matters.

2. PRELIMINARIES

For a non void set n the triplet $(N, +, .)$ wherein the group $(N, +)$is not necessarily abelian and wherein only one distributive law holds. A near ring is referred to as 0 symmetric if $0.k = 0$for all k in N. Again if 1 is in N such that $1.k = k$ for all k in N,then N is known as near ring with unity. In our discussion, we prefer zero symmetric near ring with unity. Again for near ring N and additive group E, E is stated to be a left N-group if there exist a mapping $N \times E \to E, (n, e) \to ne$ such that

(i) $(n + m)e = ne + me$.
(ii) $(nm)e = n(me)$
(iii) $1.e = e, \forall n, m \in N, e \in E$

We denote the zero element of E via 0.We note that N may be taken into cosideration as a left N-group indicated throughN^N.A non empty subset S of an N-group E remains known as an ideal of E when (i) $(S, +)$ is a normal subgroup of E,(ii)$NS \subseteq S$ and (iii) $n(y+x)-ny \in S$ for all $x \in S, y \in E, n \in N$. Moreover for any two N-groups E and F a mapping $f : E \to F$ is called an N-homomorphism if

(i) $f(x + y) = f(x) + f(y)$.
(ii) $f(nx) = nf(x), \forall x, y \in E, n \in N$.

Definition 2.1. *[10] Assuming X be a non empty set. A function $\mu : X \rightarrow [0,1]$ is called a fuzzy subset of X. It's far characterized as μ. The complement of the fuzzy set μ is denoted by $\bar{\mu}$ and is defined as $\bar{\mu}(x) = 1 - \mu(x), \forall x \in X$.*

Definition 2.2. *[9] The intuitionistic fuzzy set (in quick IFS) are characterized on a non empty set X as devices taking the shape $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ in which $\mu_A : X \rightarrow [0,1], \nu_A : X \rightarrow [0,1]$ signify the amount of participation and non-participation of separately constituent $x \in X$ to the set A correspondingly and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*

Definition 2.3. *[11] For any IFS $A =< \mu_A, \nu_A >$ the collection $A_{(s,t)} = \{x \in X | \mu_A(x) \geq s, \nu_A(x) \leq t\}$ is called (s,t)-cut of A or level subset of A where $s, t \in [0,1], s + t \leq 1$.*

Definition 2.4. *[12] Assuming P and Q be two non-empty sets and $f : P \rightarrow Q$ be a mapping.At that time for any IFS $A =< \mu_A, \nu_A >$ and $B =< \mu_B, \nu_B >$ of P and Q respectively the image set $f(A) =< \mu_{f(A)}, \nu_{f(A)} >$ of A is IFS defined as*

$$\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in f^{-1}(y)\} \\ 0; otherwise \end{cases}$$

and

$$\nu_{f(A)}(y) = \begin{cases} \wedge\{\nu_A(x) : x \in f^{-1}(y)\} \\ 1; otherwise \end{cases} .$$

Similarly, pre-image of B under f is the IFS $f^{-1}(B) =< \mu_{f^{-1}(B)}, \nu_{f^{-1}(B)} >$ defined as $\mu_{f^{-1}(B)}(x)\mu_B(f(x)), \nu_{f^{-1}(B)}(x) = \nu_B(f(x))$.

Definition 2.5. *[6] Taking c to be a point in a non-empty set X.Uncertainity $\alpha \in (0,1]$ and $\beta \in [0,1)$ are double real records such that $0 \leq \alpha + \beta \leq 1$, then the IFS $c_{(\alpha,\beta)} =< x, c_\alpha, 1 - c_{1-\beta} >$ termed as an intuitionistic fuzzy point in X,where α(resp.β)is the amount of attachement (resp. non attachment) of $c_{(\alpha,\beta)}$and $c \in X$is entitled the sustenance of $c_{(\alpha,\beta)}$. Assuming $A =< \mu_A, \nu_A >$ be an IFS in X. Then an intuitionistic fuzzy point $c_{(\alpha,\beta)}$ is supposed to fit into A, written as $c_{(\alpha,\beta)} \in A$ if $\mu_A(x) \geq \alpha, \nu_A(x) \leq \beta$. We believe that $c_{(\alpha,\beta)}$ is quasi-cincident with A, inscribed as $c_{(\alpha,\beta)} qA$ if $\mu_A(c) + \alpha > 1, \nu_A(c) + \beta < 1$.By $c_{(\alpha,\beta)} \in \vee qA$ it meant that $c_{(\alpha,\beta)} \in A$ or $c_{(\alpha,\beta)} qA$ and by $c_{(\alpha,\beta)} \overline{\in \vee} qA$ it intended that $c_{(\alpha,\beta)} \in \vee qA$ does not hold.*

Definition 2.6. *[5] Assuming μ be a fuzzy subclass of an N-group E. Then μ is called a $(\in, \in \vee q)$fuzzy ideal of E if for all $x, y \in E, n \in N$*

(i) $x_t, y_r \in \mu \Rightarrow (x-y)_{(t\wedge r)} \in \vee q\mu$.
(ii) $x_t \in \mu \Rightarrow (nx)_t \in \vee q\mu$
(iii) $x_t \in \mu \Rightarrow (y+x-y)_t \in \vee q\mu$
(iv) $x_t \in \mu \Rightarrow (n(y+x)-ny)_t \in \vee q\mu$

Definition 2.7. *[14] An IFS $A =< \mu_A, \nu_A >$ in N-group E is named as the intuitionistic fuzzy ideal of E uncertainity it fulfils for all $x, y \in E, n \in N$*

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$.
(ii) $\mu_A(y+x-y) \geq \mu_A(x)$
(iii) $\mu_A(nx) \geq \mu_A(x)$
(iv) $\mu_A(n(x+y)-nx) \geq \mu_A(y)$
(v) $\nu_A(x-y) \leq \nu_A(x) \wedge \nu_A(y)$.
(vi) $\nu_A(y+x-y) \leq \nu_A(x)$
(vii) $\nu_A(nx) \leq \nu_A(x)$
(viii) $\nu_A(n(x+y)-nx) \leq \nu_A(y)$

Lemma 2.1. *[14] Assuming $f: E \to F$ be an N-epimorphism and A and B are the intuitionistic fuzzy ideal of E and F,individually. Formerly $f(A)$ is an intuitionistic fuzzy ideal of F and $f^{-1}(B)$ is an intuitionistic fuzzy ideal of A.*

3. $(\in, \in \vee q)$-INTUITIONISTIC FUZZY IDEAL

In this segment we represent $(\in, \in \vee q)$ intuitionistic fuzzy ideal and express a number of its assets.

Definition 3.1. *An IFS $A =< \mu_A, \nu_A >$ of an N-group E of a near ring N is supposed to be an $(\in, \in \vee q)$ intuitionistic fuzzy ideal of E if for all $x, y \in E$ and $s_1, s_2 \in (0,1], t_1, t_2 \in [0,1)$, the following hold:*

(IE1) $x_{(s_1,t_1)} \in A, y_{(s_2,t_2)} \in A \Rightarrow (x-y)_{(s_1\wedge s_2, t_1\vee t_2)} \in \vee qA$.
(IE2) $x_{(s_1,t_1)} \in A \Rightarrow (y+x-y)_{(s_1,t_1)} \in \vee qA$
(IE3) $x_{(s_1,t_1)} \in A, n \in N \Rightarrow (nx)_{(s_1,t_1)}$
(IE4) $x_{(s_1,t_1)} \in A \Rightarrow (n(y+x)-ny)_{(s_1,t_1)} \in \vee qA$

Theorem 3.1. *If I is an ideal of E then an IFS $A =< \mu_A, \nu_A >$ of E satisfies the followings:*

(i) $\mu_A(x) \geq 0.5$ *and* $\nu_A(x) \leq 0.5$.

(ii) $\mu_A(x) = 0$ *and* $\nu_A(x) = 1$ *otherwise.*

is an $(\in, \in \vee q)$*-intuitionistic fuzzy ideal of E.*

Proof.

(IE1): Let $x, y \in E$ and $s_1, s_2 \in (0,1], t_1, t_2 \in [0,1)$ such that $x_{(s_1,t_1)} \in A$ and $y_{(s_2,t_2)} \in A$. Then $\mu_A(x) \geq s_1, \nu_A(x) \leq t_1$ and $\mu_A(x) \geq s_2, \nu_A(x) \leq t_2$. Thus $x, y \in I$ and as I is an ideal so $x-y \in I$, which implies $\mu_A(x-y) \geq 0.5, \nu_A(x-y) \leq 0.5$. Now if $s_1 \wedge s_2 \leq 0.5$ and $t_1 \vee t_2 \geq 0.5$ when $(x-y)_{(s_1\wedge s_2, t_1 \vee t_2)} \in A$and if $s_1 \wedge s_2 > 0.5$ and $t_1 \vee t_2 < 0.5$ then $\mu_A(x-y) + (s_1 \wedge s_2) > 1$ and $\nu_A(x-y) + (t_1 \vee t_2) < 1$ which means $(x-y)_{(s_1\wedge s_2, t_1 \vee t_2)} \in \vee qA$.

(IE2): If $x_{(s_1,t_1)} \in A, n \in N$ then $\mu_a(x) \geq s_1, \nu_A(x) \leq t_1$ gives $x \in I$ and as I is an ideal so $nx \in I$. Thus $\mu_A(x) \geq 0.5, \nu_A(x) \leq 0.5$. Now if $s_1 \leq 0.5, t_1 \geq 0.5$ then $(nx)_{(s_1,t_1)} \in A$ and if $s_1 > 0.5, t_1 < 0.5$ then $\mu_A(nx) + s_1 > 1; \nu_A(x) + t_1 < 1$ implies $(nx)_{(s_1,t_1)} qA$.

(IE3): If $x_{s_1,t_1)} \in A, y \in E$ then $\mu_A(x) \geq s_1, \nu_A(x) \leq t_1$ gives $x \in I$. Now as I is an ideal of E so $y + x - y \in I$ and hence $\mu_A(y+x-y) \geq 0.5, \nu(y+x-y) \leq 0.5$. Now, if $s_1 \leq 0.5, t_1 \geq 0.5$ then $\mu_A(y+x-y) \geq s_1, \nu(y+x-y) \leq t_1$implies $(y+x-y)_{(s_1,t_1)} \in A$ and if $s_1 > 0.5, t_1 < 0.5$then $\mu_A(y+x-y) + s_1 > 1; \nu_A(y+x-y) + t_1 < 1$ suggests $(y+x-y)_{(s_1,t_1)} qA$.

(IE4): If $x_{(s_1,t_1)} \in A$then $\mu_A(x) \geq s_1, \nu_A(x) \leq t_1$ gives $x \in I$. Now as I is an ideal of E so $n \in N, y \in E$ we have $y + x - y \in I$ and hence $\mu_A(n(y+x) - ny) \geq 0.5, \nu(n(y+x) - ny) \leq 0.5$. Now, if $s_1 \leq 0.5, t_1 \geq 0.5$ then $\mu_A(n(y+x) - ny) \geq s_1, \nu(n(y+x) - ny) \leq t_1$ implies $(n(y+x) - ny)_{(s_1,t_1)} \in A$ and if $s_1 > 0.5, t_1 < 0.5$ then $\mu_A(n(y+x) - ny) + s_1 > 1; \nu_A(n(y+x) - ny) + t_1 < 1$ suggests $(n(y+x) - ny)_{(s_1,t_1)} qA$. Thus $A =< \mu_A, \nu_A >$ is $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E. □

Example 1. *Let the near ring* $N = \{0, a, b, c\}$ *whenever addition and multiplication is defined as,*

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Assumme $A =< \mu_A, \nu_A >$ *be IFS on N so that* $\mu_A(0) > \mu_A(a) > \mu_A(b) > \mu_A(c)$ *and* $\nu_A(0) < \nu_A(a) < \nu_A(b) < \nu_A(c)$. *Then it can be understood that* $A =< \mu_A, \nu_A >$ *is a* $(\in, \in \vee q)$*-intuitionistic fuzzy ideal of N-group* N^N. *However it is not an intuitionistic fuzzy ideal of* N^N. *Thus concluding that* $(\in, \in \vee q)$ *iintuitionistic fuzzy ideal stays an overview of the intuitionistic fuzzy ideal of an N-group.*

Remark 3.1. *From the above example, it acknowledged that all intuitionistic fuzzy ideal of N-group is a* $(\in, \in \vee q)$*-intuitionistic fuzzy ideal. However the reverse is not valid.*

Theorem 3.2. *Let* $A =< \mu_A, \nu_A >$ *be an IFS of N-group E. At that time A is* $(\in, \in \vee q)$ *intuitionistic fuzzy ideal of E if and only if for all* $x, y \in E, n \in N$.

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ *and* $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y) \vee 0.5$
(ii) $\mu_A(y+x-y) \geq \mu_A(x) \wedge 0.5$ *and* $\nu_A(y+x-y) \leq \nu_A(x) \vee 0.5$
(iii) $\mu_A(nx) \geq \mu_A(x) \wedge 0.5$ *and* $\nu_A(nx) \leq \nu_A(x) \vee 0.5$
(iv) $\mu_A(n(y+x)-ny) \geq \mu_A(x) \wedge 0.5$ *and* $\nu_A(n(y+x)-ny) \leq \nu_A(x) \vee 0.5$

Proof. Let $A =< \mu_A, \nu_A >$ be $\in, \in \vee q)$-intuitionistic fuzzy ideal of E.

(i) Let $x, y \in E$ be such that$\mu_A(x-y) < \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\nu_A(x-y) > \nu_A(x) \vee \nu_A(y) \vee 0.5$

Case(a): If $\mu_A(x-y) < \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\nu_A(x-y) > \nu_A(x) \vee \nu_A(y) \vee 0.5$ then allow us to pick out s and t such that $\mu_A(x-y) < s < \mu_A(x) \wedge \mu_A(y) \wedge 0.5 = \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x-y) > t > \nu_A(x) \vee \nu_A(y) \vee 0.5 = \nu_A(x) \vee \nu_A(y)$,which gives $x_{(s,t)}, y_{(s,t)} \in A \Rightarrow (x-y)_{(s,t)} \bar{\in} A$. Also $\mu_A(x-y) + s < 0.5 + 0.5 = 1$ and $\nu_A(x-y) + t > 0.5 + 0.5 = 1$ gives $(x-y)_{(s,t)} \bar{q} A$,which is a contradiction.

Case(b): If $\mu_A(x) \wedge \mu_A(y) \geq 0.5$ and $\nu_A(x) \vee \nu_A(y) \leq 0.5$,then $\mu_A(x-y) < \mu_A(x) \wedge \mu_A(y) \wedge 0.5 = 0.5$ and $\nu_A(x-y) < \nu_A(x) \vee \nu_A(y) \vee 0.5 = 0.5$. Which gives $x_{(0.5,0.5)}, y_{(0.5,0.5)} \in A \Rightarrow (x-y)_{(0.5,0.5)} \bar{\in} A$. Also $\mu_A(x-y) + 0.5 > 0.5 + 0.5 = 1$ gives $(x-y)_{(0.5,0.5)} \bar{q} A$, which is a contradiction.

(ii) Let $\mu_A(y+x-y) < \mu_A(x) \wedge 0.5$ and $\nu_A(y+x-y) > \nu_A(x) \vee 0.5$. Then allow us to pick out s and t such that $\mu_A(y+x-y) < s < \mu_A(x) \wedge 0.5$ and $\nu_A(y+x-y) > t > \nu_A(x) \vee 0.5$. If $\mu_A(x) < 0.5, \nu_A(x) > 0.5$ then we have $x_{(s,t)} \in A$ whereas $(y+x-y)_{(s,t)} \bar{\in} A$ and also $\mu_A(y+x-y) + s < 0.5 + 0.5 = 1$ and $\nu_A(y+x-y) + t > 0.5 + 0.5 = 1$ gives $(y+x-y)_{(s,t)} \bar{q} A$. Again if $\mu_A(x) \geq 0.5, \nu_A(x) \leq 0.5$ then we can show that $x_{(0.5,0.5)} \in A$ whereas $(y+x-y)_{(0.5,0.5)} (\in \bar{\vee} q) A$.

(iii) Let $x \in E, n \in N$. Suppose $\mu_A(nx) < \mu_A(x) \wedge 0.5$ and $\nu_A(x) > \nu_A(x) \vee 0.5$.Let us assume s and t be such that $\mu_A(nx) < s < \mu_A(x) \wedge 0.5$ and $\nu_A(nx) >$

$t > \nu_A(x) \vee 0.5$.Now if $\mu_A(x) < 0.5, \nu_A(x) > 0.5$ then $x_{(s,t)} \in A$ whereas $(nx)_{(s,t)}(\overline{\in \vee q})A$ and if $\mu_A(x) \geq 0.5, \nu_A(x) \leq 0.5$ then $(nx)_{(0.5,0.5)}(\overline{\in \vee q})A$.

(iv) Let $x, y \in E, n \in N$ such that $\mu_A(n(y+x)ny) < \mu_A(x) \wedge 0.5 = \mu_A(x)$ or 0.5 and $\nu_A(n(y+x)ny) > \nu_A(x) \vee 0.5 = \nu_A(x)$ or 0.5. If $\mu_A(n(y+x)ny) < s < \mu_A(x) \wedge 0.5$ and $\nu_A(n(y+x)ny) > t > \nu_A(x) \vee 0.5 = \nu_A(x)$ then according to $\mu_A(x) < 0.5, \nu_A(x) > 0.5$ and $\mu_A(x) \leq 0.5, \nu_A(x) \geq 0.5$ we have $x_{(s,t)} \in A$ whereas $(n(y+x) - ny)_{(s,t)}(\overline{\in \vee q})A$ and also $x_{(0.5,0.5)} \in A$ whereas $(n(y+x) - ny)_{(0.5,0.5)}(\overline{\in \vee q})A$. Conversely,let $x_{(s_1,t_1)}, y_{(s_2,t_2)} \in A$. Then since (i) hold so $\mu_A(x-y) \geq s_1 \wedge s_2$ or $\mu_A(x-y) \geq 0.5$ and $\nu_A(x-y) \leq t_1 \vee t_2$ or $\nu_A(x-y) \leq 0.5$ which implies $x_{(s_1 \wedge s_2, t_1 \vee t_2)} \in \vee qA$. Again for $x_{(s_1,t_1)} \in A, y \in E$, since (ii) holds so $\mu_A(y+x-y) \geq s_1$ or $\mu_A(y+x-y) \geq 0.5$ and $\nu_A(y+x-y) \leq t_1$ or $\nu_A(y+x-y) \leq 0.5$, which means $(y+x-y)_{(s-1,t_1)} \in \vee qA$. Also, for $x_{(s_1,t_1)} \in A, n \in N$ since (iii) holds so $\mu_A(nx) \geq s_1$ or $\mu_A(nx) \geq 0.5$ and $\nu_A(nx) \leq t_1$ or $\nu_A(nx) \leq 0.5$, which implies $(nx)_{(s_1,t_1)} \in \vee qA$. Lastly, $x_{(s_1,t_1)} \in A, y \in E, n \in N$ since (iv) holds so $\mu_A(n(y+x) - ny) \geq s_1$ or $\mu_A(n(y+x) - ny) \geq 0.5$ and $\nu_A(n(y+x) - ny) \leq t_1$ or $\nu_A(n(y+x) - ny) \leq 0.5$, which means $(n(y+x) - ny)_{(s-1,t_1)} \in \vee qA$. Hence $A =< \mu_A, \nu_A >$ is a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E. □

Theorem 3.3. *An IFS $A =< \mu_A, \nu_A >$ of E is a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E if and only if the level set $A_{(s,t)}$ with $s \in (0, 0.5], t \in [0.5, 1)$ is an ideal of E.*

Proof. Assume $A =< \mu_A, \nu_A >$ be $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E. Assuming $s \in (0, 0.5], t \in [0.5, 1)$ and $n \in N$. Then for $x, y \in A_{(s,t)}$.

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 \geq s \wedge 0.5 = s$ and $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y) \vee 0.5 \leq t \vee 0.5 = t$, which implies $x - y \in A_{(s,t)}$.

(ii) $\mu_A(y+x-y) \geq \mu_A(x) \wedge 0.5 \geq s \wedge 0.5 = s$ and $\nu_A(y+x-y) \leq \nu_A(x) \vee 0.5 \leq t \vee 0.5 = t$, which implies $y + x - y \in A_{(s,t)}$.

(iii) $\mu_A(nx) \geq \mu_A(x) \wedge 0.5 \geq s \wedge 0.5 = s$ and $\nu_A(nx) \leq \nu_A(x) \vee 0.5 \leq t \vee 0.5 = t$, which implies $nx \in A_{(s,t)}$.

(iv)$\mu_A(n(y+x) - ny) \geq \mu_A(x) \wedge 0.5 \geq s \wedge 0.5 = s$ and $\nu_A(n(y+x) - ny) \leq \nu_A(x) \vee 0.5 \leq t \vee 0.5 = t$, which implies $n(y+x) - ny \in A_{(s,t)}$. Hence $A_{(s,t)}$ is ideal of E. Equally, let $A_{(s,t)}$ is an ideal of E for all $s \in (0, 0.5], t \in [0.5, 1)$. Now make it possible for $x, y \in E, \mu_A(x-y) < s < \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\nu_A(x-y) > t > \nu_A(x) \vee \nu_A(y) \vee 0.5$. Then $x, y \in A_{(s,t)}$ but $x - y \notin A_{(s,t)}$. Which is a contradiction. Again for $n \in N, x \in E$ if it is assumed that $\mu_A(nx) < s < \mu_A(x) \wedge 0.5$ and $\nu_A(nx) > t > \nu_A(x) \vee 0.5$, then this gives a contradiction that

$x \in A_{(s,t)}$ but $nx \notin A_{(s,t)}$. Similarly, we can show that $\mu_A(y+x-y) \geq \mu_A(x) \wedge 0.5$ and $\nu_A(y+x-y) \leq \nu_A(x) \vee 0.5$ and $\mu_A(n(y+x)-ny) \geq \mu_A(x) \wedge 0.5$ and $\nu_A(n(y+x)-ny) \leq \nu_A(x) \vee 0.5$. Hence $A+ < \mu_A, \nu_A >$ is a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E. □

Remark 3.2. *The result may not be correct for $s \in (0.5, 1]$ and $t \in [0, 0.5)$. For example, let us consider $E = S_3 = \{i, \rho_1, \rho_2, \tau_1, \tau_2, \tau_3\}$ (expressed additively) to be a Z-group. Define IFS $A =< \mu_A, \nu_A >$ as $\mu_A(i) = 1, \mu_A(\rho_1) = \mu_A(\rho_2) = \mu_A(\tau_2) = \mu_A(\tau_3) = 0.6, \mu_A(\tau_1) = 0.8$ and $\nu_A(i) = 0, \nu_A(\rho_1) = \nu_A(\rho_2) = \nu_A(\tau_2) = \nu_A(\tau_3) = 0.3, \mu_A(\tau_1) = 0.1$. Then $A =< \mu_A, \nu_A >$ is a $(\in, \in \vee q)$-intuitionistic fuzzy left ideal of E but $A_{(0.8, 0.1)} = \{i, \tau_1\}$ is not an ideal of E.*

The following result gives a necessary and sufficient condition for $A_{(s,t)}$ to be an ideal of E when $s \in (0.5, 1]$ and $t \in [0, 0.5)$.

Theorem 3.4. *Let $A =< \mu_A, \nu_A >$ be an IFS of N-group E. Then $A_{(s,t)} \neq \phi$ for $s \in (0.5, 1]$ and $t \in [0, 0.5)$ is an ideal of E if and only if A fulfils the subsequent situations:*

(i) $\mu_A(x-y) \vee 0.5 \geq \mu_A(x) \wedge \mu_A(y)$ *and* $\nu_A(x-y) \wedge 0.5 \leq \nu_A(x) \vee \nu_A(y)$.
(ii) $\mu_A(y+x-y) \vee 0.5 \geq \mu_A(x)$ *and* $\nu_A(y+x-y) \wedge \leq \nu_A(x)$.
(iii) $\mu_A(nx) \vee 0.5 \geq \mu_A(x)$ *and* $\nu_A(nx) \wedge 0.5 \leq \nu_A(x)$.
(iv) $\mu_A(n(y+x)-ny) \vee 0.5 \geq \mu_A(x)$ *and* $\nu_A(n(y+x)-ny) \wedge 0.5 \leq \nu_A(x), x, y \in E, n \in N$.

Proof. Suppose $A_{(s,t)} \neq \phi$ for $s \in (0.5, 1]$ and $t \in [0, 0.5)$ is an ideal of E. Let $\mu_A(x-y) \vee 0.5 < \mu_A(x) \wedge \mu_A(y) = s$ and $\nu_A(x-y) \wedge 0.5 > \nu_A(x) \vee \nu_A(y) = t$. Then $x, y \in A_{(s,t)}$ and as $A_{(s,t)}$ is an ideal so $x - y \in A_{(s,t)}$, which implies $\mu_A(x-y) \geq s > \mu_A(x-y) \vee 0.5$ and $\nu_A(x-y) \leq t < \nu_A(x-y) \wedge 0.5$, which is a contradiction. Thus (i) holds. Likewise we can show that (ii) holds. Again $x \in E, n \in N$, let us assume $\mu_A(nx) \vee 0.5 < \mu_A(x) = s$ and $\nu_A(nx) \wedge 0.5 > \nu_A(x) = t$. Then $x \in A_{(s,t)}$ and as $A_{(s,t)}$ is an ideal so $nx \in A_{(s,t)}$, which implies $\mu_A(nx) \geq s > \mu_A(nx) \vee 0.5$ and $\nu_A(nx) \leq t < \nu_A(nx) \wedge 0.5$, a contradiction. Thus (iii) holds. Likewise ,we can show that (iv) holds. Conversely, let $x, y \in A_{(s,t)}$. Then $0.5 < s \leq \mu_A(x) \wedge \mu_A(y) \leq \mu_A(x-y) \vee 0.5 = \mu_A(x-y)$ and $0.5 > t \geq \nu_A(x) \vee \nu_A(y) \geq \nu_A(x-y) \wedge 0.5 = \nu_A(x-y)$ gives $x - y \in A_{(s,t)}$. Also for $x \in A_{(s,t)}, y \in E, 0.5 < s \leq \mu_A(x) \leq \mu_A(y+x-y) \vee 0.5 = \mu_A(y+x-y)$ and $0.5 > t \geq \nu_A(x) \geq \nu_A(y+x-y) \wedge 0.5 = \nu_A(y+x-y)$ gives $y + x - y \in A_{(s,t)}$.

Now for $x \in A_{(s,t)}, n \in N, 0.5 < s \leq \mu_A(x) \leq \mu_A(nx) \vee 0.5 = \mu_A(nx)$ and $0.5 > t \geq \nu_A(x) \geq \nu_A(nx) \wedge 0.5 = \nu_A(nx)$ implies that $nx \in A_{(s,t)}$. Moreover $x \in A_{(s,t)}, n \in N, y \in E, 0.5 < s \leq \mu_A(x) \leq \mu_A(n(y+x)-ny) \vee 0.5 = \mu_A(n(y+x)-ny)$ and $0.5 > t \geq \nu_A(x) \geq \nu_A(n(y+x)-ny) \wedge 0.5 = \nu_A(n(y+x)-ny)$ implies that $n(y+x)-ny \in A_{(s,t)}$. Hence $A_{(s,t)}$ is an ideal of E. □

4. N-HOMOMORPHISHM AND $(\in, \in \vee q)$-INTUITIONISTIC FUZZY IDEAL

In this segment,we discuss the homomorphic image and pre-image of a $(\in, \in \vee q)$-intuitionistic fuzzy ideal under an N-homomorphism.

Theorem 4.1. *Let E and F be two N-groups and $f : E \to F$ be an onto homomorphism.Then if $A =< \mu_A, \nu_A >$ is a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E then $f(A)$ is also a $(\in, \in \vee q)$ intuitionistic fuzzy ideal of F.*

Proof. Assume $A =< \mu_A, \nu_A >$ be $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E. Now for any $x, y \in F, \mu_{f(A)}(x-y) = f(\mu_A)(x-y) = Sup_{x-y=f(u)}\{\mu_A(u) : u \in E\} \geq Sup_{f(a)=x,f(b)=y}\{\mu_A(a-b) : a, b \in E\} \geq Sup_{f(a)=x,f(b)=y}\{\mu_A(a) \wedge \mu_A(b) \wedge 0.5\} = Sup_{f(a)=x}\{\mu_A(a)\} \wedge Sup_{f(b)=y}\{\mu_A(b)\} \wedge 0.5 = f(\mu_A)(x) \wedge f(\mu_A)(y) \wedge 0.5 = \mu_{f(A)}(x) \wedge \mu_{f(A)}(y) \wedge 0.5and\nu_{f(A)}(x-y) = f(\nu_A)(x-y) = inf_{x-y=f(u)}\{\nu_A(u) : u \in E\} \leq inf_{f(a)=x,f(b)=y}\{\nu_A(a-b) : a, b \in E\} \leq inf_{f(a)=x,f(b)=y}\{\nu_A(a) \vee \nu_A(b) \vee 0.5\} = inf_{f(a)=x}\{\nu_A(a)\} \vee inf_{f(b)=y}\{\nu_A(b)\} \vee 0.5 = f(\nu_A)(x) \vee f(\nu_A)(y) \vee 0.5 = \nu_{f(A)}(x) \vee \nu_{f(A)}(y) \vee 0.5$.Also $\mu_{f(A)}(y+x-y) = f(\mu_A)(y+x-y) = Sup_{y+x-y=f(u)}\{\mu_A(u) : u \in E\} \geq Sup_{f(a)=x,f(b)=y}\{\mu_A(b+a-b) : a, b \in E\} \geq Sup_{f(a)=x}\{\mu_A(a) \wedge 0.5\} = Sup_{f(a)=x}\{\mu_A(a)\} \wedge 0.5 = f(\mu_A)(x) \wedge 0.5 = \mu_{f(A)}(x) \wedge 0.5and\nu_{f(A)}(y+x-y) = f(\nu_A)(y+x-y) = inf_{y+x-y=f(u)}\{\nu_A(u) : u \in E\} \leq inf_{f(a)=x,f(b)=y}\{\nu_A(b+a-b) : a, b \in E\} \leq inf_{f(a)=x}\{\nu_A(a) \vee 0.5\} = inf_{f(a)=x}\{\nu_A(a)\} \vee 0.5 = f(\nu_A)(x) \vee 0.5 = \nu_{f(A)}(x) \vee 0.5$.Again for $n \in N, x \in F$we have $\mu_{f(A)}(nx) = f(\mu_A)(nx) = Sup_{nx=f(u)}\{\mu_A(u) : u \in E\} \geq Sup_{f(a)=x}\{\mu_A(na) : a \in E\} \geq Sup_{f(a)=x}\{\mu_A(a) \wedge 0.5\} = Sup_{f(a)=x}\{\mu_A(a)\} \wedge 0.5 = f(\mu_A)(x) \wedge 0.5 = \mu_{f(A)}(x) \wedge 0.5and\nu_{f(A)}(nx) = f(\nu_A)(nx) = inf_{nx=f(u)}\{\nu_A(u) : u \in E\} \leq inf_{f(a)=x}\{\nu_A(na) : a \in E\} \leq inf_{f(a)=x}\{\nu_A(a) \vee 0.5\} = inf_{f(a)=x}\{\nu_A(a)\} \vee 0.5 = f(\nu_A)(x) \vee 0.5 = \nu_{f(A)}(x) \vee 0.5$Lastly,for $n \in N, x, y \in F$, $\mu_{f(A)}(n(y+x)-ny) = f(\mu_A)(n(y+x)-ny) = Sup_{n(y+x)-ny=f(u)}\{\mu_A(u) : u \in E\} \geq Sup_{f(a)=x,f(b)=y}\{\mu_A(n(b+a)-nb) : a, b \in E\} \geq Sup_{f(a)=x}\{\mu_A(a) \wedge 0.5\} = Sup_{f(a)=x}\{\mu_A(a)\} \wedge 0.5 = f(\mu_A)(x) \wedge 0.5 = \mu_{f(A)}(x) \wedge 0.5and\nu_{f(A)}(n(y+x)-ny) = f(\nu_A)(n(y+x)-ny) = inf_{y+x-y=f(u)}\{\nu_A(u) :$

$u \in E\} \leq inf_{f(a)=x,f(b)=y}\{\nu_A(n(b+a)-nb) : a,b \in E\} \leq inf_{f(a)=x}\{\nu_A(a) \vee 0.5\} = inf_{f(a)=x}\{\nu_A(a)\} \vee 0.5 = f(\nu_A)(x) \vee 0.5 = \nu_{f(A)}(x) \vee 0.5$. Thus $f(A)$ is $(\in, \in \vee q)$-intuitionistic fuzzy ideal of F. □

Theorem 4.2. *Let E and F be two N-groups and $f : E \to F$ be an N-homomorphism. If $B =< \mu_B, \nu_B >$ be $(\in, \in \vee q)$-intuitionistic fuzzy ideal of F then $f^{-1}(B)$ is also a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E.*

Proof. Assume $x, y \in E$. Then $f^{-1}(\mu_B)(x-y) = \mu_B(f(x-y)) = \mu_B(f(x)-f(y)) \geq \mu_B(f(x)) \wedge \mu_B(f(y)) \wedge 0.5 = f^{-1}(\mu_B)(x) \wedge f^{-1}(\mu_B)(y) \wedge 0.5$ and $f^{-1}(\nu_B)(x-y) = \nu_B(f(x-y)) = \nu_B(f(x)-f(y)) \leq \nu_B(f(x)) \vee \nu_B(f(y)) \vee 0.5 = f^{-1}(\nu_B)(x) \vee f^{-1}(\nu_B)(y) \vee 0.5$. Again $f^{-1}(\mu_B)(y+x-y) = \mu_B(f(y+x-y)) = \mu_B(f(y)+f(x)-f(y)) \geq \mu_B(f(x)) \wedge 0.5 = f^{-1}(\mu_B)(x) \wedge 0.5$ and $f^{-1}(\nu_B)(y+x-y) = \nu_B(f(y+x-y)) = \nu_B(f(y)+f(x)-f(y)) \leq \nu_B(f(x)) \vee 0.5 = f^{-1}(\nu_B)(x) \vee 0.5$. Also $f^{-1}(\mu_B)(nx) = \mu_B(f(nx)) = \mu_B(nf(x)) \geq \mu_B(f(x)) \wedge 0.5 = f^{-1}(\mu_B)(x) \wedge 0.5$ and $f^{-1}(\nu_B)(nx) = \nu_B(f(nx)) = \nu_B(nf(x)) \leq \nu_B(f(x)) \vee 0.5 = f^{-1}(\nu_B)(x) \vee 0.5$. Lastly,$f^{-1}(\mu_B)(n(y+x)-ny) = \mu_B(f(n(y+x)-ny)) = \mu_B(n(f(y)+f(x))-nf(y)) \geq \mu_B(f(x)) \wedge 0.5 = f^{-1}(\mu_B)(x) \wedge 0.5$ and $f^{-1}(\nu_B)(n(y+x)-ny) = \nu_B(f(n(y+x)-ny)) = \nu_B(n(f(y)+f(x))-nf(y)) \leq \nu_B(f(x)) \vee 0.5 = f^{-1}(\nu_B)(x) \vee 0.5$. Thus $f^{-1}(B)$ remains to a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E. □

Theorem 4.3. *Assume E and F be N-groups and $f : E \to F$ be an onto homomorphism. If for IFS $B =< \mu_B, \nu_B >$ of F,$f^{-1}(B)$ is $(\in, \in \vee q)$-intuitionistic fuzzy ideal of E, then B is also a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of F.*

Proof. Assume $x, y \in F$. Then since f is onto so, we have $a, b \in E$ such that $x = f(a), y = f(b)$. Now $\mu_B(f(a)-f(b)) = \mu_B(f(a-b)) = f^{-1}(\mu_B)(a-b) \geq f^{-1}(\mu_B)(a) \wedge f^{-1}(\mu_B)(b) \wedge 0.5 = \mu_B(f(a)) \wedge \mu_B(f(b)) \wedge 0.5 = \mu_B(x) \wedge \mu_B(y) \wedge 0.5$ and $\nu_B(f(a)-f(b)) = \nu_B(f(a-b)) = f^{-1}(\nu_B)(a-b) \leq f^{-1}(\nu_B)(a) \vee f^{-1}(\nu_B)(b) \vee 0.5 = \nu_B(f(a)) \vee \nu_B(f(b)) \vee 0.5 = \nu_B(x) \vee \nu_B(y) \vee 0.5$. Similarly we can show that $\mu_B(y+x-y) \geq \mu_B(x) \wedge 0.5$ and $\nu_B(y+x-y) \leq \nu_B(x) \vee 0.5$.Also for $n \in N, x \in F, \mu_B(nx) = \mu_B(nf(a)) = \mu_B(f(na)) = f^{-1}(\mu_B)(na) \geq f^{-1}(\mu_B)(a) \wedge 0.5 = \mu_B(f(a)) \wedge 0.5 = \mu_B(x) \wedge 0.5$ and $\nu_B(nx) = \nu_B(nf(a)) = \nu_B(f(na)) = f^{-1}(\nu_B)(na) \leq f^{-1}(\nu_B)(a) \vee 0.5 = \nu_B(f(a)) \vee 0.5 = \nu_B(x) \vee 0.5$. Similarly we can show that $\mu_B(n(y+x)-ny) \geq \mu_B(x) \wedge 0.5$ and $\nu_B(n(y+x)-ny) \leq \nu_B(x) \vee 0.5$. Hence $B =< \mu_B, \nu_B >$ is a $(\in, \in \vee q)$-intuitionistic fuzzy ideal of F. □

5. Conclusion

In this research work, we look at $(\in, \in \vee q)$-intuitionistic fuzzy ideals of a near-ring. Studies can be executed in the direction of the perception of $(\in, \in \vee q)$-intuitionistic fuzzy prime and semi prime ideals of an N-group. Furthermore one can take a look at the other (ϕ, ψ)-structures(stated in [20]) of ideals of N-group.

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