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# SOFT IRRESOLUTE AND SOFT $\alpha$ TOPOLOGICAL VECTOR SPACES

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ABSTRACT. The focus of this work is to investigate the idea of soft irresolute and soft $\alpha$ topological vector spaces. This space is determined by using the notion of soft irresolute mappings and soft semi open sets ($\tilde{S}S$-open).

## 1. INTRODUCTION

The soft set Molodtsov [7] is the one of the best mathematical tool to deal with uncertainties, which the generalization of fuzzy set Zadeh [9]. It has many application in different fields such as game theory, Riemann-Integration, probability and so on. The algebraic operations over the soft sets were given by Maji et.al [5]. The algebraic-topological aspects of soft set has widely developed nowadays. Aktag et.al. [2] investigated the mathematical notion of soft groups. The notion of soft topological vector space is introduced by Roy [8] by assuming the parameter set as usual vector space. This paper is an elaborate study of soft irresolute and soft $\alpha$ topological vector spaces.

## 2. PRELIMINARIES

In every part of this paper, we mention soft irresolute topological vector space as $\tilde{S}ITVS$, soft topological vector space as $\tilde{S}TVS$ and $\tilde{S}$-set means soft set, $\tilde{K}$ is the field of complex or real number which is endowed with usual topology $\sigma$.

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**Definition 2.1.** *[8] The* $\tilde{S}TVS(\tilde{W_\tau}, P, K)$ *is defined as follows: The mappings* $\tilde{h} : \tilde{S}(\tilde{W_\tau}) \times \tilde{S}(\tilde{W_\tau}) \to \tilde{S}(\tilde{W_\tau})$ *defined by* $\tilde{h}(\tilde{w_{1p}}, \tilde{w_{2p}}) = \tilde{w_{1p}} + \tilde{w_{2p}}$ *and* $\tilde{f} : \tilde{S}(\tilde{W_\tau}) \times \tilde{S}(\tilde{W_\tau}) \to \tilde{S}(\tilde{W_\tau})$ *defined by* $\tilde{f}(\hat{\zeta}, \tilde{w_p}) = \hat{\zeta}\tilde{w_p}$ *are both* $\tilde{S}$*-continuous. The domain of* $\tilde{h}$ *and* $\tilde{f}$ *are endowed with* $\tilde{S}$*-product topologies.*

**Definition 2.2.** *[3] A* $\tilde{S}$*-set* $\tilde{B}_P$ *in* $\tilde{S}VS(\tilde{W}, P)$ *is said to be* $\tilde{S}$*-absorbing if for every* $\tilde{w}_p \tilde{\in} \tilde{B}_P$*, there exists a* $\tilde{S}$*-real number* $\hat{\eta}$*, where* $\hat{\eta}(\lambda) > 0$*, for all* $\lambda \tilde{\in} P$ *such that* $\hat{\eta}^{-1}\tilde{w}_p \tilde{\in} \tilde{B}_P$*.*

**Definition 2.3.** *[1, 4] A* $\tilde{S}$*-set* $\tilde{B}_P$ *of a* $\tilde{S}TS(\tilde{W_\tau}, P)$ *is called*

(1) $\tilde{S}\alpha$*-open if* $\tilde{B}_P \tilde{\subseteq} \tilde{S}\text{-}int(\tilde{S}\text{-}cl(\tilde{S}\text{-}int(\tilde{B}_P)))$*.*
(2) $\tilde{S}S$*-open if* $\tilde{B}_P \tilde{\subseteq} \tilde{S}\text{-}cl(\tilde{S}\text{-}int(\tilde{B}_P))$*.*

**Definition 2.4.** *[1, 6] Let* $(\tilde{V_\tau}, P)$ *and* $(\tilde{W_\tau}, P)$ *be two* $\tilde{S}TS$ *and* $\tilde{f} : (\tilde{V_\tau}, P) \to (\tilde{W_\tau}, P)$*. Then* $\tilde{f}$ *is called*

(1) $\tilde{S}$*-irresolute if for every* $\tilde{S}S$*-open set* $\tilde{A}_P$ *in* $\tilde{W_\tau}$*,* $\tilde{f}^{-1}(\tilde{A}_P) \tilde{\in} \tilde{V_\tau}$ *is* $\tilde{S}S$*-open in* $\tilde{V_\tau}$*.*
(2) $\tilde{S}\alpha$*-irresolute for every* $\tilde{S}\alpha$*-open set* $\tilde{B}_P$ *in* $\tilde{W_\tau}$*,* $\tilde{f}^{-1}(\tilde{B}_P) \tilde{\in} \tilde{V_\tau}$ *is* $\tilde{S}\alpha$*-open in* $\tilde{V_\tau}$*.*

## 3. Soft irresolute and soft $\alpha$ topological vector spaces

In this section, we elucidate and investigate the notions of $\tilde{S}ITVS$, $\tilde{S}\alpha TVS$ and its rudimentary properties.

**Definition 3.1.** *A* $\tilde{S}TVS(\tilde{W_\tau}, P, K)$ *is said to be* $\tilde{S}ITVS$ *with the field* $K$ *(complex or real) if the following conditions hold:*

(1) *for any two soft points* $\tilde{v}_1, \tilde{v}_2 \tilde{\in} \tilde{W}$ *and for every soft semi open neighborhood* $\tilde{D}_P$ *of* $\tilde{v}_1 + \tilde{v}_2 \tilde{\in} \tilde{W}$ *we have a* $\tilde{S}S$*-open neighborhoods* $\tilde{B}_P$ *and* $\tilde{C}_P \tilde{\in} \tilde{W}$ *of* $\tilde{v}_1, \tilde{v}_2$ *respectively, such that* $\tilde{B}_P + \tilde{C}_P \tilde{\subseteq} \tilde{D}_P$*.*
(2) *for any* $\tilde{v} \tilde{\in} \tilde{W}$ *and* $\delta \tilde{\in} \tilde{K}$ *for any* $\tilde{S}S$*-open neighborhood* $\tilde{D}_P$ *of* $\dot{\delta W}$ *in* $\tilde{W}$*, we have* $\tilde{S}S$*-open neighborhoods* $\tilde{B}_P$ *of* $\delta$ *in* $\tilde{K}$ *and* $\tilde{C}_P$ *of* $\tilde{v}$ *in* $\tilde{W}$ *such that* $\tilde{B}_P \dot{} \tilde{C}_P \tilde{\subseteq} \tilde{D}_P$*.*

**Definition 3.2.** *In a* $\tilde{S}TVS(\tilde{W_\tau}, P, K)$*:*

(1) *The soft right translation* $T_{\tilde{v}} : (\tilde{W_\tau}, P, K) \to (\tilde{W_\tau}, P, K)$ *is defined by* $T_{\tilde{v}} = \tilde{x} + \tilde{v} \ \forall \ \tilde{x}, \tilde{v} \tilde{\in} \tilde{W}$*.*

(2) *The soft left translation* ${}_{\tilde{v}}T : (\tilde{W}_\tau, P, K) \rightarrow (\tilde{W}_\tau, P, K)$ *is defined by* ${}_{\tilde{v}}T = \tilde{v} + \tilde{x}\ \forall\ \tilde{x}, \tilde{v} \tilde{\in} \tilde{W}$.

(3) *The soft multiplication* $M_{\hat{\zeta}} : (\tilde{W}_\tau, P, K) \rightarrow (\tilde{W}_\tau, P, K)$ *is defined by* $M_{\hat{\zeta}} = \hat{\zeta}\tilde{v}$, $\tilde{v} \tilde{\in} \tilde{W}$ *and* $\hat{\zeta} \tilde{\in} \tilde{K}$.

**Theorem 3.1.** *For a* $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$ *over the field* $\tilde{K}$

(1) *the soft(left)right translation is soft irresolute.*

(2) *the soft multiplication is soft irresolute.*

*Proof.*

(1) Define soft right translation $T_{\tilde{v}_p} : (\tilde{W}_\tau, P, K) \rightarrow (\tilde{W}_\tau, P, K)$ by $T_{\tilde{v}_p}(\tilde{x}_p) = \tilde{x}_p + \tilde{v}_p$ here $\tilde{x}_p, \tilde{v}_p \tilde{\in} \tilde{W}$. Let $\tilde{B}_P \tilde{\in} \tilde{W}$ be a $\tilde{S}S$-open neighborhood of $\tilde{x}_p + \tilde{v}_p$. There exists $\tilde{S}S$-open neighborhoods $\tilde{C}_P, \tilde{D}_P \tilde{\in} \tilde{W}$ of $\tilde{x}_p$ and $\tilde{v}_p$ respectively such that $\tilde{C}_P + \tilde{D}_P \tilde{\subseteq} \tilde{B}_P$, by Definition 3.1.

(2) Define soft multiplication $M_{\hat{\zeta}} : (\tilde{W}_\tau, P, K) \rightarrow (\tilde{W}_\tau, P, K)$ by $M_{\hat{\zeta}}(\tilde{x}_p) = \hat{\zeta}.\tilde{x}_p$. □

**Theorem 3.2.** *For* $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$ *over the field* $\tilde{K}$ *if* $\tilde{G}_P \tilde{\in} \tilde{S}SO(\tilde{W}_\tau, P, K)$*, then*

(1) $\tilde{G}_P + \tilde{y}_p \tilde{\in} \tilde{S}SO(\tilde{W}_\tau, P, K)$, $\tilde{y}_p \tilde{\in} \tilde{W}$.

(2) $\tilde{\zeta} G_P \tilde{\in} \tilde{S}SO(\tilde{W}_\tau, P, K), \hat{\zeta} \tilde{\in} \tilde{K}$.

*Proof.*

(1) Assume that $\tilde{w}_p, \tilde{x}_p \tilde{\in} \tilde{W}$. Let $\tilde{x}_p \tilde{\in} \tilde{G}_P + \tilde{w}_p$. Now $\tilde{x}_p = \tilde{y}_p + \tilde{w}_p$. Then we have $\tilde{x}_p \tilde{\in} \tilde{G}_P + \tilde{w}_p - \tilde{w}_p = \tilde{G}_P$, where $\tilde{y}_p \tilde{\in} \tilde{G}_P$. Define soft right translation $T_{-\tilde{w}_p}$ by image of $\tilde{x}_p$ under $T_{-\tilde{w}_p}$ is equal to $\tilde{x}_p + (-\tilde{w}_p) = \tilde{y}_p$. Hence $T_{-\tilde{w}_p}$ is soft irresolute because the space $(\tilde{W}_\tau, P, K)$ is $\tilde{S}ITVS$ by the above theorem. Thus for a $\tilde{S}S$-open neighborhood $\tilde{G}_P$ containing $T_{-\tilde{w}_p}(\tilde{x}_p) = \tilde{y}_p$, a $\tilde{S}S$-open neighborhood $\tilde{C}_P$ of $\tilde{x}_p$ exists with the condition $T_{-\tilde{w}_p}(\tilde{C}_P) = \tilde{C}_P - \tilde{w}_p \tilde{\in} \tilde{G}_P$, which implies $\tilde{C}_P \tilde{\subseteq} \tilde{G}_P + \tilde{w}_p$.

(2) Let $\hat{\zeta} \tilde{\in} \tilde{K}$, $(\hat{\zeta} \neq \tilde{0})$ and $\tilde{x}_p \tilde{\in} \tilde{\zeta} G_P$. That is $\tilde{x}_p = \tilde{\zeta} y_p$ where $\tilde{y}_p \tilde{\in} \tilde{G}_P$. Since $\tilde{x}_p \tilde{\in} \hat{\zeta}.\tilde{G}_P$, we have $\hat{\zeta}.\tilde{y}_p \tilde{\in} \hat{\zeta}.\tilde{G}_P \Rightarrow \tilde{y}_p \tilde{\in} \tilde{G}_P$. Define soft multiplication $M_{\hat{\zeta}^{-1}} : (\tilde{W}_\tau, P, K) \rightarrow (\tilde{W}_\tau, P, K)$ by image of $\tilde{x}_p$ under $M_{\hat{\zeta}^{-1}}$ is equal to $\hat{\zeta}^{-1}.\tilde{x}_p = \tilde{y}_p$. Now $M_{\hat{\zeta}^{-1}}$ is $\tilde{S}ITVS$, because $(W_\tau, P, K)$ is $\tilde{S}ITVS$ and by the above theorem. Therefore for any $\tilde{S}S$-open neighborhood $\tilde{G}_P$ containing $M_{\hat{\zeta}^{-1}}(\tilde{x}_p) = \tilde{y}_p$ there exists $\tilde{S}S$-open neighborhood $\tilde{D}_P$ of $\tilde{x}_p$ such that image of $\tilde{D}_P$ under $M_{\hat{\zeta}^{-1}}$ is equal to $\hat{\zeta}^{-1}.\tilde{D}_P \tilde{\subseteq} \tilde{G}_P$. Now we have $\tilde{D}_P$ is contained in $\hat{\zeta}.\tilde{G}_P$. Hence $\hat{\zeta}.\tilde{G}_P$ is an element of $\tilde{S}SO(\tilde{W}_\tau, P, K)$. □

**Theorem 3.3.** *For a* $\tilde{S}S$*-open set* $\tilde{G}_P \tilde{\in} \tilde{S}SO(\tilde{W})$ *in a* $\tilde{S}ITVS$, $\tilde{G}_P + \tilde{H}_P \tilde{\in} \tilde{S}SO(\tilde{W})$ *where* $\tilde{H}_P$ *in a soft subset of* $\tilde{W}$.

*Proof.* Let $\tilde{C}_P\tilde{\subseteq}\tilde{W}$ and $\tilde{G}_P\tilde{\in}\tilde{S}SO(\tilde{W})$. Now for every soft point $\tilde{v}_p\tilde{\in}\tilde{H}_P$, $\tilde{G}_P + \tilde{v}_p\tilde{\in}\tilde{S}SO(\tilde{W})$, by Theorem. For every soft point $\tilde{v}_p\tilde{\in}\tilde{H}_P$,

$$\begin{aligned}\tilde{G}_P + \tilde{H}_P &= \tilde{G}_P + \{\tilde{v}_{p_2} + \tilde{v}_{p_1} + ...\}\\ &= \tilde{G}_P + \tilde{\bigcup}\tilde{v}_{p_i}, i\tilde{\in}\triangle\\ &= \tilde{\bigcup_{\tilde{v}_{p_i}\tilde{\in}\tilde{H}_P}} \tilde{G}_P + \tilde{v}_{p_i}\end{aligned}$$

Hence $\tilde{G}_P + \tilde{H}_P\tilde{\in}\tilde{S}SO(\tilde{W})$. □

**Theorem 3.4.** *Let $(\tilde{W}_\tau, P, K)$ be a $\tilde{S}ITVS$ over the field $\tilde{K}$, where $\tilde{K}$ is endowed with soft topology $\sigma$. Then $\tilde{\phi} : (\tilde{K}, \sigma)\times(\tilde{W}_\tau, P, K) \to (\tilde{W}_\tau, P, K)$ defined by $\tilde{\phi}(\hat{\zeta}, \tilde{v}_p) = \hat{\zeta}.\tilde{v}_p$ where $\hat{\zeta}\tilde{\in}\tilde{K}$ and $\tilde{v}_p\tilde{\in}\tilde{W}_P$ is soft irresolute.*

*Proof.* Assume $\tilde{B}_P\tilde{\in}\tilde{W}$ is a $\tilde{S}S$-open neighborhood of $\hat{\zeta}.\tilde{v}_p$ in $\tilde{W}$. There exist a $\tilde{S}S$-open neighborhoods $\tilde{C}_P$ of $\hat{\zeta}$ in $\tilde{K}$ and $\tilde{D}_P$ of $\tilde{v}_p$ in $\tilde{W}$ such that $\tilde{C}_P.\tilde{D}_P$ is contained in $\tilde{B}_P$ that is $\tilde{\phi}(\tilde{C}_P \times \tilde{D}_P) = \tilde{C}_P.\tilde{D}_P$. Then we have $\tilde{\phi}(\tilde{C}_P \times \tilde{D}_P)$ is contained in $\tilde{B}_P$, since $\tilde{W}$ is $\tilde{S}ITVS$. Therefore $\tilde{C}_P \times \tilde{D}_P$ in a $\tilde{S}S$-open neighborhood of $\hat{\zeta} \times \tilde{v}_p$ in $\tilde{K} \times \tilde{W}$. Hence $\tilde{\phi}$ is soft irresolute. □

**Theorem 3.5.** *Let $(\tilde{W}_\tau, P, K)$ be a $\tilde{S}ITVS$ over the field $\tilde{K}$. Then $\hat{\eta} : (\tilde{W}_\tau, P, K) \times (\tilde{W}_\tau, P, K) \to (\tilde{W}_\tau, P, K)$ defined by $\tilde{x}_p, \tilde{v}_p$ is soft irresolute.*

*Proof.* Consider any two soft points $\tilde{x}_p, \tilde{v}_p$ in $\tilde{W}$. Let $\hat{\eta}(\tilde{x}_p, \tilde{v}_p) = \tilde{x}_p + \tilde{v}_p$. Assume $\tilde{C}_P\tilde{\in}\tilde{W}$ is a $\tilde{S}S$-open neighborhood of $\tilde{x}_p + \tilde{v}_p$ in $\tilde{W}$. Since $\tilde{W}$ is $\tilde{S}ITVS$, there exist $\tilde{S}S$-open neighborhoods $\tilde{M}_P, \tilde{N}\tilde{\in}\tilde{W}$ of $\tilde{x}_p$ and $\tilde{v}_p$ respectively with the condition $\tilde{M}_P + \tilde{N}_P\tilde{\subseteq}\tilde{C}_P$. That is $\hat{\eta}(\tilde{M}_P, \tilde{N}_P) = \hat{\eta}(\tilde{M}_P \times \tilde{N}_P) = \tilde{M}_P + \tilde{N}_P\tilde{\subseteq}\tilde{C}_P$. Therefore $\tilde{M}_P \times \tilde{N}_P$ is a $\tilde{S}S$-open neighborhood of $\tilde{x}_p \times \tilde{v}_p$ in $(\tilde{W}_\tau, P, K) \times (\tilde{W}_\tau, P, K)$, since $\tilde{M}_P$, $\tilde{N}_P$ are the $\tilde{S}S$-open neighborhoods of $\tilde{x}_p$, $\tilde{v}_p$ in $(\tilde{W}_\tau, P, K)$ respectively. Hence $\hat{\eta}$ is soft irresolute. □

**Definition 3.3.** *A $\tilde{S}$-function $g : (\tilde{W}_\tau, P, K) \to (\tilde{W}_\tau, P, K)$ is said to be $\tilde{S}I$-homeomorphism if $g$ is*

(1) *$\tilde{S}$-bijective.*
(2) *$\tilde{S}$-irresolute.*
(3) *$\tilde{S}S$-open.*

**Theorem 3.6.** *For a $\tilde{S}ITVS$, the $\tilde{S}$-translation $T_{\tilde{v}_p}(\tilde{x}_p) = \tilde{x}_p + \tilde{v}_p$ and $\tilde{S}$-multiplication $M_{\hat{\zeta}}(\tilde{y}_p) = \hat{\zeta}.\tilde{y}_p$ where $\tilde{x}_p, \tilde{v}_p, \tilde{y}_p \tilde{\in} \tilde{W}$ and $\hat{\zeta} \tilde{\in} \tilde{K}$ are $\tilde{S}I$-homeomorphism onto itself.*

*Proof.* Define $\tilde{S}$-translation $T_{\tilde{v}_p}$ by image of $\tilde{x}_p$ under $T_{\tilde{v}_p}$ is equal to $\tilde{x}_p+\tilde{v}_p \,\forall\, \tilde{x}_p, \tilde{v}_p \tilde{\in} \tilde{W}$. Obviously, $T_{\tilde{v}_p}$ is $\tilde{S}$-bijective. $T_{\tilde{v}_p}$ is $\tilde{S}$-irresolute, by theorem. Also for any $\tilde{S}S$-open set $\tilde{B}_P \tilde{\in} \tilde{W}$, $T_{\tilde{v}_p}(\tilde{B}_P) = \tilde{B}_P + \tilde{v}_p$ is $\tilde{S}S$-open. $\tilde{S}I$-homeomorphism for $\tilde{S}$-multiplication can be proved in the similar manner. □

**Definition 3.4.** *A $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$ over the field $\tilde{K}$ is said to be $\tilde{S}I$-homogeneous space, there exists a $\tilde{S}I$-homeomorphism $\tilde{g} : (\tilde{W}_\tau, P, K) \to (\tilde{W}_\tau, P, K)$ such that $\tilde{g}(\tilde{B}_P) = \tilde{C}_P$ for each $\tilde{B}_P, \tilde{C}_P \tilde{\in}\, \tilde{W}$.*

**Proposition 3.1.** *Every $\tilde{S}ITVS$ is $\tilde{S}I$-homogeneous space.*

*Proof.* Let $\tilde{v}_p, \tilde{w}_p \tilde{\in} \tilde{W}$ and $\tilde{v} = \tilde{x}_p + \tilde{w}_p$ where $\tilde{x}_p \tilde{\in} \tilde{W}$. Define a $\tilde{S}$-left translation $\tilde{x}_pT : (\tilde{W}_\tau, P, K) \to (\tilde{W}_\tau, P, K)$ by $\tilde{x}_pT(\tilde{w}_p) = \tilde{x}_p + \tilde{w}_p = \tilde{v}_p$. By Theorem 3.6, $\tilde{x}_pT$ is $\tilde{S}I$-homeomorphism. Hence $(\tilde{W}_\tau, P, K)$ is $\tilde{S}I$-homogeneous space. □

**Theorem 3.7.** *In a $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$, for any $\tilde{S}$-subspace $\tilde{V}_1$ of $\tilde{W}$ and a non-null $\tilde{S}S$-open subset $\tilde{V}_2$ of $\tilde{W}$, if $\tilde{V}_2 \tilde{\subseteq} \tilde{V}_1$ then $\tilde{V}_1$ is $\tilde{S}S$-open subset of $\tilde{W}$.*

*Proof.* Let $\tilde{B}_P$ be a non-null $\tilde{S}S$-open in $\tilde{W}$ and $\tilde{B}_P \tilde{\subseteq} \tilde{V}_1$. By Theorem 3.1 $T_{\tilde{B}_P} = \tilde{B}_P + \tilde{v}_p$ is $\tilde{S}S$-open subset of $\tilde{W}$ for all $\tilde{v}_p \tilde{\in} \tilde{V}_1$. Hence $\tilde{V}_1 = \bigcup_{\tilde{v}_{p_i} \tilde{\in} \tilde{V}_1} (\tilde{B}_P + \tilde{v}_p)$ is $\tilde{S}S$-open in $\tilde{W}$ being the arbitrary union of $\tilde{S}S$-open sets. □

**Proposition 3.2.** *For any two $\tilde{S}$-subsets $\tilde{B}_P, \tilde{C}_P$ of $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$, $\tilde{S}$-$scl(\tilde{B}_P)$+$\tilde{S}$-$scl(\tilde{C}_P)$ is contained in $\tilde{S}$-$(\tilde{B}_P + \tilde{C}_P)$.*

*Proof.* Assume $\tilde{x}_p \tilde{\in} \tilde{S}$-$scl(\tilde{B}_P)$ and $\tilde{y}_p \tilde{\in} \tilde{S}$-$scl(\tilde{C}_P)$. Let $\tilde{G}_P$ be a $\tilde{S}S$-open neighborhood of $\tilde{x}_p + \tilde{y}_p$. Then there exist $\tilde{S}S$-open neighborhoods $\tilde{H}_P$ and $\tilde{I}_P$ of $\tilde{x}_p$ and $\tilde{y}_p$ respectively, such that $\tilde{H}_P + \tilde{I}_P \tilde{\subseteq} \tilde{G}_P$. By assumption $\tilde{x}_P \tilde{\in} \tilde{S}$-$scl(\tilde{B}_P)$ and $\tilde{y}_p \tilde{\in} \tilde{S}$-$scl(\tilde{C}_P)$ there exist $\tilde{v}_p + \tilde{w}_p \tilde{\in} (\tilde{B}_P + \tilde{C}_P) \cap (\tilde{H}_P + \tilde{I}_P) \tilde{\subseteq} (\tilde{B}_P + \tilde{C}_P) \cap \tilde{G}_P$. That is $\tilde{x}_p + \tilde{y}_p \tilde{\in} \tilde{S}$-$scl(\tilde{B}_P + \tilde{C}_P)$. □

**Theorem 3.8.** *Every $\tilde{S}S$-open subspace of a $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$ is $\tilde{S}S$-closed in $(\tilde{W}_\tau, P, K)$.*

*Proof.* Consider a $\tilde{S}S$-open subspace $\tilde{V}_1$ of a $\tilde{W}$, $\tilde{S}I$-homeomorphism, $\tilde{V}_1 + \tilde{v}_p$ is $\tilde{S}S$-open for any $\tilde{v}_p \tilde{\in} \tilde{W} \setminus \tilde{V}_1$. Therefore $\tilde{V}_2 = \bigcup_{\tilde{v}_{p_i} \tilde{\in} \tilde{W} \setminus \tilde{V}_1} (\tilde{V}_1 + \tilde{v}_p)$ is also $\tilde{S}S$-open. Thus $\tilde{V}_1 = \tilde{W} \setminus \tilde{V}_2$ is $\tilde{S}S$-closed. □

**Theorem 3.9.** *For any two $\tilde{S}$-subsets $\tilde{G}_P$ and $\tilde{H}_P$ of $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$, $\tilde{G}_P + \tilde{H}_P = \tilde{S}$-$scl(\tilde{G}_P + \tilde{H}_P$, where $\tilde{H}_P$ is $\tilde{S}S$-open and $\tilde{G}_P$ is any $\tilde{S}$-set.*

*Proof.* Since $\tilde{G}_P \tilde{\subseteq} \tilde{S}$-$scl(\tilde{G}_P$, we have $\tilde{G}_P + \tilde{H}_P \tilde{\subseteq} \tilde{S}$-$scl(\tilde{G}_P + \tilde{H}_P$. To prove the converse, let $\tilde{x}_p \tilde{\in} \tilde{S}$-$scl(\tilde{G}_P + \tilde{H}_P$ and $\tilde{x}_p = \tilde{y}_p + \tilde{w}_p$ where $\tilde{w}_p \tilde{\in} \tilde{H}_P$ and $\tilde{y}_p \tilde{\in} \tilde{S}$-$scl(\tilde{G}_P$. Then $\tilde{S}S$-open neighborhood $\tilde{M}_P$ of $\theta$ ($\theta$ being the zero element of $\tilde{W}$) exists with the condition, image of $\tilde{M}_P$ under $T_{\tilde{w}_p}$ is equal to $\tilde{M}_P + \tilde{w}_p$ which is contained in $\tilde{H}_P$. Since $\tilde{M}_P$ is a $\tilde{S}S$-open neighborhood of $\theta$ in $\tilde{W}$, we have $-\tilde{M}_P$ is also the $\tilde{S}S$-open neighborhood $\theta$ of $\tilde{W}$. By assumption $\tilde{y}_p \tilde{\in} \tilde{S}$-$scl(\tilde{G}_P), \tilde{v}_p \tilde{\in} \tilde{G}_P \cap (\tilde{y}_p \setminus \tilde{M}_P)$. Now

$$\begin{aligned} \tilde{x}_p &= \tilde{y}_p + \tilde{w}_p \\ &\tilde{\in} \tilde{v}_p + \tilde{M}_P + \tilde{w}_p \\ &\tilde{\subseteq} \tilde{G}_P + \tilde{H}_P. \end{aligned}$$

Therefore $\tilde{S}$-$scl(\tilde{G}_P) + \tilde{H}_P$ is contained in $\tilde{G}_P + \tilde{H}_P$. Thus $\tilde{G}_P + \tilde{H}_P = \tilde{S}$-$scl(\tilde{G}_P) + \tilde{H}_P$. □

**Theorem 3.10.** *In a $\tilde{S}ITVS(\tilde{W}_\tau, P, K)$, each $\tilde{S}$-open subspace $\tilde{V}$ in $\tilde{S}ITVS$.*

*Proof.* Let $(\tilde{V}_\tau, P)$ be an $\tilde{S}$-topological subspace of $(\tilde{W}_\tau, P, K)$. Now it satisfies the below properties:

(1) for each $\tilde{v_{1p}}, \tilde{v_{2p}} \tilde{\in} \tilde{V}, \tilde{v_{1p}} + \tilde{v_{2p}} \tilde{\in} \tilde{V}$,
(2) for $\tilde{v}_p \tilde{\in} \tilde{V}$ and $\hat{\eta} \tilde{\in} \tilde{K}, \hat{\eta}\tilde{v}_p \tilde{\in} \tilde{V}$.

Let $\tilde{v_{1p}}, \tilde{v_{2p}} \tilde{\in} \tilde{V}$ and $\tilde{v_{1p}} + \tilde{v_{2p}}$ has a $\tilde{S}S$-open neighborhood $\tilde{B}_P$ in $\tilde{V}$. Then $\tilde{B}_P$ is a $\tilde{S}S$-open neighborhood in $\tilde{W}_\tau$. Therefore there is $\tilde{S}S$-open neighborhoods $\tilde{C}$ of $\tilde{v_{1p}}$ and $\tilde{D}_P$ of $\tilde{v_{2p}}$ such that $\tilde{C}_P + \tilde{D}_P \tilde{\subseteq} \tilde{B}_P$, since $\tilde{W}_\tau$ is $\tilde{S}ITVS$. Also $\tilde{C}_P \tilde{\cap} \tilde{V}$ and $\tilde{D}_P \tilde{\cap} \tilde{V}$ are both $\tilde{S}S$-open in $\tilde{W}_\tau$ which contains $\tilde{v_{1p}}$ and $\tilde{v_{2p}}$ respectively. Thus $\tilde{C}_P \tilde{\cap} \tilde{V} + \tilde{D}_P \tilde{\cap} \tilde{V} = (\tilde{C}_P + \tilde{D}_P) \tilde{\cap} \tilde{V} \tilde{\subseteq} \tilde{B}_P$. Now for any $\hat{\eta} \tilde{\in} \tilde{K}$ and $\tilde{v}_p \tilde{\in} \tilde{W}_\tau$, consider a $\tilde{S}S$-open neighborhood $\tilde{B}_P$ of $\hat{\eta}\tilde{v}_p$ in $\tilde{V}$ which is also $\tilde{S}S$-open in $\tilde{W}_\tau$. Hence there exists $\tilde{S}S$-open neighborhood $\tilde{H}_P$ of $\hat{\eta}$ in $\tilde{K}$ and $\tilde{C}_P$ of $\tilde{v}_p$ in $\tilde{W}_\tau$ such that $\tilde{H}_P \tilde{C}_P \tilde{\subseteq} \tilde{B}_P$, as $\tilde{W}_\tau$ is $\tilde{S}ITVS$. Also, $\tilde{H}_P \tilde{\cap} \tilde{K}$ and $\tilde{C}_P \tilde{\cap} \tilde{V}$ are $\tilde{S}S$-open in $\tilde{K}$ and $\tilde{W}_\tau$ respectively. Thus the space $(\tilde{W}_\tau, P, K)$ is $\tilde{S}ITVS$. □

**Definition 3.5.** *Let $(\tilde{W}_\tau, P, K)$ be a $\tilde{S}TVS$. If the $\tilde{S}$-addition map $\tilde{f} : \tilde{W}_\tau \times \tilde{W}_\tau \to \tilde{W}_\tau$ defined by $\tilde{f}(\tilde{v_{1p}}, \tilde{v_{2p}}) = \tilde{v_{1p}} + \tilde{v_{2p}}$ and the $\tilde{S}$-multiplication map $\tilde{g} : \tilde{K} \times \tilde{W}_\tau \to \tilde{W}_\tau$ defined by $\tilde{g}(\hat{\eta}, \tilde{v}_p) = \hat{\eta}\tilde{v}_p$ are both $\tilde{S}\alpha I$ (soft $\alpha$-irresolute), then $(\tilde{W}_\tau, P, K)$ is called $\tilde{S}STVS$ and denoted by $({}_\alpha\tilde{W}_\tau, P, K)$.*

**Theorem 3.11.** *Let* $({}_{\alpha}\tilde{W}_{\tau}, P, K)$ *be a* $\tilde{S}\alpha TVS$. *Then*

(1) *Let* $\tilde{B}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$ *be a* $\tilde{S}\alpha$*-neighborhood of* $\tilde{v}_p \tilde{\in} \tilde{W}_{\tau}$ *and* $\tilde{C}_P$ *be a* $\tilde{S}$*-neighborhood of* $\tilde{v}_p$*, then* $\tilde{B}_P \tilde{\cap} \tilde{C}_P$ *is* $\tilde{S}\alpha$*-neighborhood of* $\tilde{v}_p$.

(2) *Let* $\tilde{B}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$ *be a* $\tilde{S}\alpha$*-neighborhood of* $\tilde{v}_p \tilde{\in} \tilde{W}_{\tau}$*, then* $\tilde{v}_p \tilde{\in} \tilde{B}_P$.

(3) *Let* $\tilde{B}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$ *be a* $\tilde{S}\alpha$*-neighborhood of* $\tilde{v}_p \tilde{\in} \tilde{W}_{\tau}$*, then there is a* $\tilde{S}\alpha$*-neighborhood* $\tilde{C}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$ *of* $\tilde{v}_p$ *such that* $\tilde{B}_P \tilde{\in} \tilde{u}_p N(\tilde{W}_{\tau})$ *is a* $\tilde{S}\alpha$*-neighbor- hood of* $\tilde{u}_p$ *for all* $\tilde{u}_p \tilde{\in} \tilde{C}_p$.

(4) *Let* $\tilde{B}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$ *be a* $\tilde{S}\alpha$*-neighborhood of* $\tilde{v}_p \tilde{\in} \tilde{W}_{\tau}$ *and* $\tilde{B}_P \tilde{\subseteq} \tilde{C}_P$*, then* $\tilde{C}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$

*Proof.* (1) Let $\tilde{B}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$ and $\tilde{C}_P$ is a $\tilde{S}$-neighborhood of $\tilde{v}_p$. Then $\tilde{v}_p \tilde{\in} \tilde{D}_P \tilde{\subseteq} \tilde{C}_P$ we have $\tilde{v}_p \tilde{\in} \tilde{F}_P \tilde{\subseteq} \tilde{B}_P$, where $\tilde{F}_P$ is a $\tilde{S}\alpha$-open set and $\tilde{D}_P$ is a $\tilde{S}$-open set. Then $\tilde{F}_P \tilde{\cap} \tilde{D}_P \tilde{\subseteq} \tilde{B}_P \tilde{\cap} \tilde{C}_P$ is $\tilde{S}\alpha$-open. Hence $\tilde{B}_P \tilde{\cap} \tilde{C}_P \tilde{\in} \tilde{v}_p N(\tilde{W}_{\tau})$ is a $\tilde{S}\alpha$-neighborhood of $\tilde{v}_p$.

Proof of (2), (3) and (4) can be derived in the similar manner. □

**Theorem 3.12.** *Let* $\tilde{f} : {}_{\alpha}\tilde{V}_{\tau} \to {}_{\alpha}\tilde{W}_{\tau}$ *be a* $\tilde{S}\alpha$*-homeomorphism between* $\tilde{S}\alpha TVS$*s. A* $\tilde{S}$*-subset* $\tilde{Y}$ *of* ${}_{\alpha}\tilde{V}_{\tau}$ *is of* $\tilde{S}\alpha$*-neighborhood of* $\tilde{y}_p \tilde{\in} \tilde{V}$ *if and only if* $\tilde{f}(\tilde{Y})$ *is* $\tilde{S}\alpha$*-neighborhood of* $\tilde{f}(\tilde{y}_p)$.

*Proof.* Let $\tilde{Y}_P$ be a $\tilde{S}\alpha$-neighborhood of $\tilde{y}_p \tilde{\in} {}_{\alpha}\tilde{V}_{\tau}$. Then $\tilde{y}_p \tilde{\subseteq} \tilde{Z}_P \tilde{\subseteq} \tilde{Y}_P$. Hence $\tilde{f}(\tilde{y}_p) \tilde{\in} \tilde{f}(\tilde{Z}_P) \tilde{\subseteq} \tilde{f}(\tilde{Y}_P)$ and $\tilde{f}(\tilde{Z}_P)$ is $\tilde{S}\alpha$-open in ${}_{\alpha}\tilde{W}_{\tau}$, since $\tilde{f}$ is $\tilde{S}p\alpha$-open. Thus $\tilde{f}(\tilde{Y}_P)$ is a $\tilde{S}\alpha$-neighborhood of $\tilde{f}(\tilde{y}_p)$.

Conversely, consider $\tilde{f}(\tilde{Y}_P)$ is a $\tilde{S}\alpha$-neighborhood of $\tilde{f}(\tilde{y}_p)$. Then there exists a $\tilde{S}\alpha$-open $\tilde{B}_{\alpha}$ in ${}_{\alpha}\tilde{W}_{\tau}$ with the condition $\tilde{f}(\tilde{y}_p) \tilde{\in} \tilde{B}_{\alpha} \tilde{\subseteq} \tilde{f}(\tilde{Y}_P)$. Since $\tilde{f}$ is $\tilde{S}\alpha$-irresolute, $\tilde{f}^{-1}(\tilde{B}_{\alpha})$ is $\tilde{S}\alpha$-open and $\tilde{y}_p \tilde{\in} \tilde{f}^{-1}(\tilde{B}_{\alpha}) \tilde{\subseteq} \tilde{Y}_P$. Thus $\tilde{Y}_P$ be a $\tilde{S}\alpha$-neighborhood of $\tilde{y}_p$. □

**Theorem 3.13.** *Let* $({}_{\alpha}\tilde{W}_{\tau}, P, K)$ *be a* $\tilde{S}\alpha TVS$. *Then every* $\tilde{B}_P \tilde{\in} {}_{\theta}N({}_{\alpha}\tilde{W}_{\tau})$ *is* $\tilde{S}$*-absorbing.*

*Proof.* Assume $\tilde{B}_P \tilde{\in} {}_{\theta}N({}_{\alpha}\tilde{W}_{\tau})$. Then $\tilde{C}_P \tilde{\subseteq} \tilde{B}_P$ we have $\tilde{C}_P \tilde{\in} {}_{\theta}N({}_{\alpha}\tilde{W}_{\tau})$, where $\tilde{C}_P$ is a $\tilde{S}\alpha$-open set. Since the space is $\tilde{S}\alpha TVS$, $\tilde{S}$-multiplication is $\tilde{S}\alpha$-irresolute. So there exists $\tilde{S}\alpha$-open sets $\tilde{G}_P \tilde{\in} {}_{\theta}N({}_{\alpha}\tilde{W}_{\tau})$ and $\tilde{H}_P \tilde{\in} {}_{\theta}N({}_{\alpha}\tilde{W}_{\tau})$ with the condition $\tilde{M}_{\hat{\zeta}}(\tilde{G}_P \times \tilde{H}_P) \tilde{\subseteq} \tilde{C}_P$ and hence $\hat{\zeta}\tilde{w}_p \tilde{\in} \tilde{C}_P \ \forall \ \hat{\zeta}(\lambda) > 0, \lambda \tilde{\in} P$ and $\tilde{w}_p \tilde{\in} \tilde{H}_P$. Thus $\tilde{C}_P$ is $\tilde{S}$-absorbing. □

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