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MIXED ANTI-NEWTONIAN-GAUSSIAN RULE FOR REAL DEFINITE INTEGRALS

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ABSTRACT. In this paper, real definite integrals are approximated with the anti-Newtonian and anti-Gaussian rule. The anti mixed rule contributes a better approximation to that of individual anti-Gaussian and anti-Newtonian rule for the numerical treatment of real definite integral. The validity and applicability of the proposed the scheme is illustrated through six tests and compared with an absolute error of proposed rule with constituent rules and to the analytical solutions.

1. INTRODUCTION

The numerical computation of an integral is performed with numerical quadrature techniques. On the abscissa, an approximate value of an integral is obtained by Newtona's quadrature. In this work, we have suggested anti-Newtoni-an with anti Gaussian quadrature rule and Gaussian type rules for the construction of the mixed rule that is compared with Singh and Dash [3]. The idea of anti-Gaussian quadrature was first thought by Dirk P.Laurie [5].The error equalin magnitude but of opposite sign to that of Gaussian n point formula is obtained in an anti-Gaussian rule with points of precision $(2n-1)$ integrates the polynomial $(n+1)$ is of precision up to $(2n-1)$. Das and Pradhan [7], have taken the initiative to construct mixed rules of higher precision with hybridization of lower precision

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rules. Many researchers have come forward in this field in order to evaluate real definite integrals Jena and Dash [4, 12, 23, 37, 40], Dash and. Das [13], Dash and Jena [8, 34] [35, 36], Davis and Rabinowitz [15], Jena et al. [1, 14, 27, 31] developed to approximate real definite integrals via hybrid quadrature domain Richardson extrapolation and applied mixed quadrature rule on electromagnetic field problems . J. Ma et al. [10] proposed to generalize Gaussian rules for systems of arbitrary functions. Jena and Nayak [2, 18, 19, 32], Nayak et al. [39] implemented in the field of electrical sciences to obtain the instantaneous current in the RLC- circuit and applied hybrid quadrature rule to find the approximate solution of nonlinear Fredholm integral equation with the separable kernel. Patra et al. [16] used a mixed quadrature rule with Gaussian quadrature for approximate evaluation of real definite integrals. The authors Jena and Mishra [11], Mishra and Jena [21] Jena and Singh [9, 20, 33], Meher et al. [22, 38], Singh et al. [24], also suggested mixed rules for approximate evaluation of complex analytic functions. Besides, the others who have come forward to help indirectly to the current methods are Jena and Gebremedhin [28], Gebremedhin and Jena [25, 29], Jena and Mohanty [26], Mohanty and Jena [30]. The highlights of our method is the hybridization of Gaussian, anti-Gaussian, as well as anti-Newtonian rule and a nice comparison to Singh and Dash [3], where they used only the anti-Gaussian with Gaussian rules for the mixed rule. Let $G_w^{(n)}$ is the corresponding Newtona's quadrature formula for n point where q be the weight function on $[m, n]$,

$$G_w^{(n)} = \sum_{j=m}^{n} q_j^{(n)} f(t_j^{(n)}),$$

of degree for the integral $(2n - 1)$,

$$I = \int_{m}^{n} f(t)q(t)dt,$$

$G_w^{(n)}(t) = I(t), \forall t \in P^{2n-1}$, $A^{(n+1)} = \sum_{j=1}^{n+1} \alpha_{j-1} f(\zeta_{j-1})$.

It is an anti Newtonian formula for $(n+1)$ point and $G^n(t)$ be n point Newtonian formula, then $A^{(n+1)}(t) = 2I(t) - G^n(t)$ where t defined as polynomial of degree $\leq 2n + 1$. The paper is synchronized in the following manner. Section 2 deals with anti-Newtonian Simpson's rule . The anti-Gaussian three-point rule is described in Section 3. Section 4 contains the construction of the anti-Newtonian mixed quadrature rule. The error analysis and error bound is investigated in Section 5. The numerical results are verified in section 6. Remarks and conclusions are reported in Section 7.

2. NEWTONIAN AND ANTI-NEWTONIAN RULE

We choose the Simpson's $\frac{1}{3}rd$ rule

$$RS_{\frac{1}{3}rd}(f) = \frac{1}{3}[f(-1) + f(1) + 4f(0)]$$

to develop anti Simpson's $\frac{3}{8}th$ rule $(RS_{\frac{3}{8}th}(f))$. We choose the Simpson's $\frac{1}{3}rd$ rule $(RS_{\frac{1}{3}rd}(f))$ ([5]):

$$RS_{\frac{3}{8}th}(f) = 2\int_{-1}^{1} f(t)dt - RS_{\frac{1}{3}rd}(f),$$

$$\alpha_1 f(-1) + \alpha_2 f(\xi_1) + \alpha_3 f(\xi_2) + \alpha_4 f(1) = 2\int_{-1}^{1} f(t)dt - RS_{\frac{1}{3}rd}(f).$$

A system of six equations in six unknowns is obtained for the integrated of polynomial of degree five. A system of six equations in six unknowns is obtained for the integrated of polynomial of degree five ($\alpha_j (j = 1(1)4)$, $\xi_j (j = 1, 2)$, $f(t) = t^j (j = 0(1)5)$):

$$\begin{aligned}
\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 &= 2\\
-\alpha_1 + \alpha_2\xi_1 + \alpha_3\xi_2 + \alpha_4 &= 0\\
-\alpha_1 + \alpha_2{\xi_1}^2 + \alpha_3{\xi_2}^2 + \alpha_4 &= \frac{2}{3}\\
-\alpha_1 + \alpha_2{\xi_1}^3 + \alpha_3{\xi_2}^3 + \alpha_4 &= 0\\
-\alpha_1 + \alpha_2{\xi_1}^4 + \alpha_3{\xi_2}^4 + \alpha_4 &= \frac{2}{15}\\
-\alpha_1 + \alpha_2{\xi_1}^5 + \alpha_3{\xi_2}^5 + \alpha_4 &= 0.
\end{aligned}$$

The solution of above system of equations is $\alpha_1 = \alpha_4 = -\frac{1}{9}, \alpha_2 = \alpha_3 = \frac{10}{9}$, $\xi_1 = \sqrt{\frac{2}{5}}, \xi_2 = -\sqrt{\frac{2}{5}}$. Hence the anti Simpson's $\frac{3}{8}th$ rule becomes

$$RS_{\frac{3}{8}th}(f) = [\frac{10}{9}\{f(-\sqrt{\frac{2}{5}}) + f(\sqrt{\frac{2}{5}})\} - \frac{1}{9}\{f(-1) + f(1)\}]. \tag{2.1}$$

The corresponding error is obtained as

$$ES_{\frac{3}{8}th}(f) = \frac{4}{3 \times 5!} f^{iv}(0) - \frac{64}{175 \times 6!} f^{vi}(0) \ldots.$$

3. ANTI-GAUSSIAN AND GAUSSIAN RULE

Let us take Gauss Legendre two point rule,

$$RGL^2(f) = [f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})].$$

In the same vein of (2.1) and referring [5], anti Gaussian three point rule can be expressed as

(3.1)
$$\begin{aligned} RS_{GL}{}^3(f) =& \frac{1}{13}[5f(-\sqrt{\frac{13}{15}}) + f(\sqrt{\frac{13}{15}}) + 16f(0)] \\ RS_{GL}{}^3(f) =& -\frac{1}{135 \times 6!} f^{iv}(0) + \frac{1016}{675 \times 7!} f^{vi}(0) \ldots. \end{aligned}$$

4. MIXED RULE

In this section, paragraph various anti- mixed rules are suggested.

4.1. **Anti-Newtonian And Anti-Gaussian Rule.**

Referring (2.1) and (3.1)

(4.1)
$$I = RS_{\frac{3}{8}th}(f) + ES_{\frac{3}{8}th}(f),$$

(4.2)
$$I = RR_{GL}{}^3(f) + ER_{GL}{}^3(f),$$

where, $(ES_{\frac{3}{8}th}(f))$ and $(ES_{GL}{}^3)$ denote the errors for rules $(RS_{\frac{3}{8}th}(f))$ and $(RR_{GL}{}^3(f))$ respectively, for the evaluation of integrals $I(f)$. Expressions (4.1) and (4.2) with Maclaurin's expansion are

(4.3)
$$ES_{\frac{3}{8}th}(f) = \frac{4}{3 \times 5!} f^{iv}(0) - \frac{64}{175 \times 6!} f^{vi}(0) \cdots$$

(4.4)
$$ES_{GL}{}^3(f) = -\frac{1}{135 \times 5!} f^{iv}(0) + \frac{1016}{675 \times 7!} f^{vi}(0) \cdots.$$

Eliminating $f^{vi}(0)$ from (4.3) and (4.4) by multiplying $\frac{1}{1080}$ with (4.1) and adding it with (4.2) we receive

$$I(f) = RR_{GL}{}^3S_{\frac{3}{8}th}(f) + ES_{GL}{}^3S_{\frac{3}{8}th}(f),$$

where

$$RR_{GL}{}^3S_{\frac{3}{8}th}(f) = \frac{1}{5}[3RR_{GL}{}^3(f) + 2RS_{\frac{3}{8}th}(f)],$$

$$ES_{GL}{}^3(f) = \frac{1}{5}[3ER_{GL}{}^3(f) + 2ES_{\frac{3}{8}th}(f)] = \frac{2744}{2362 \times 6!}f^{vi}(0)\ldots. \tag{4.5}$$

4.2. **Anti-Simpson's $\frac{3}{8}$th Rule With Steffenson's Four Point Rule.**

From (2.1) and Steffenson's four point rule (referring [16])

$$R_{st4}(f) = [\frac{11}{12}\{f(-\frac{3}{5}) + f(\frac{3}{5})\} + \frac{1}{12}\{f(-\frac{1}{5}) + f(\frac{1}{5})\}]$$

$$ES_{st4}(f) = \frac{38}{5625}f^{iv}(0) + \frac{13136}{9375 \times 7!}f^{vi}(0)\ldots$$

where $RS_{\frac{3}{8}th}(f)$ and $R_{st4}(f)$ is of precision three and $ES_{\frac{3}{8}th}(f)$ and $E_{st4}(f)$ is the errors due to the former and later rules respectively

$$I = R_{st4}RS_{\frac{3}{8}th}(f) + ER_{st4}RS_{\frac{3}{8}th}(f),$$

$$R_{st4}RS_{\frac{3}{8}}(f) = \frac{1}{49}[125R_{st4} - 76RS_{\frac{3}{8}}(f)]. \tag{4.6}$$

Here (4.6) is the mixed rule of precision five and the error for this approximation is

$$ER_{st4}RS_{\frac{3}{8}}(f) = \frac{1}{49}[125E_{st4}(f) - 76ES_{\frac{3}{8}}(f)] = \frac{27728}{25725 \times 6!}f^{vi}(0)\ldots. \tag{4.7}$$

5. Error Bounds Of Mixed Rules

In this section we have determined the error analysis and error bound in the form of Theorems 1, 2, 3, 4.

Theorem 5.1. *Let the smooth function $f(t)$ is defined on $-1 \leq t \leq 1$, then the error $ER_{GL}{}^3S_{\frac{3}{8}}(f)$ due to the mixed rule $RR_{GL}{}^3S_{\frac{3}{8}}(f)$ is obtained as $ER_{GL}{}^3S_{\frac{3}{8}}(f) = \frac{2744}{23625\times 6!}f^{vi}(0)\ldots$.*

Proof. Expression (4.5) justifies the proof of this theorem. □

Theorem 5.2. *Let the smooth function $f(t)$ is defined on $-1 \leq t \leq 1$, then the error due to the mixed rule $ER_{st4}RS_{\frac{3}{8}}(f)$ is $ER_{st4}RS_{\frac{3}{8}}(f) = \frac{27728}{25725\times 6!}f^{vi}(0)\ldots$.*

Proof. Expression (4.7) conforms the proof of this theorem. □

Theorem 5.3. *The error bound for $ES_{GL}{}^3S_{\frac{3}{8}th}(f) = I(f) - RR_{GL}{}^3S_{\frac{3}{8}th}(f)$ is evaluated by $|ES_{GL}{}^3S_{\frac{3}{8}th}(f)| \leq \frac{2M}{225}$, $M = \max\limits_{-1\leq x\leq 1}|f^v(x)|$.*

Proof. From $ER_{GL}{}^3(f) = -\frac{3}{5\times 135\times 6!}f^{iv}(\eta_1)$, $\eta_1 \in [-1,1]$, (by Conte and Boor [17]) we have $ES_{\frac{3}{8}th}(f) = \frac{8}{15\times 5!}f^{iv}(\eta_2)$, $\eta_2 \in [-1,1]$ (by Conte and Boor [6, 17]) and

$$|ER_{GL}{}^3S_{\frac{3}{8}th}(f)| \cong \frac{1}{225}[f^{iv}(d) - f^{iv}(c)] = \frac{1}{225}\int_{-1}^{1} f^v(t)dt = \frac{1}{225}(d-c)f^v(\gamma)$$

for some $\gamma \in [-1,1]$, where $|d-c| \leq 2$, and then $|ES_{GL}{}^3S_{\frac{3}{8}th}(f)| \leq \frac{2}{225}f^v(\gamma)$.

Hence $|EG_{GL}{}^3S_{\frac{3}{8}th}(f)| \leq \frac{2M}{225}$, where $M = \max\limits_{-1\leq x\leq 1}|f^v(x)|$. □

Theorem 5.4. *The error bound for $ER_{st4}S_{\frac{3}{8}th}(f) = I(f) - R_{st4}RS_{\frac{3}{8}th}(f)$ is computed as $|ER_{st4}RS_{\frac{3}{8}th}(f)| \leq \frac{76M}{2205}$, where $M = \max\limits_{-1\leq t\leq 1}|f^v(t)|$.*

Proof. From $ER_{st4}(f) = \frac{38}{5625}f^{iv}(\eta_1), \eta_1 \in [-1,1]$, we have $ERS_{\frac{3}{8}th}(f) = \frac{4}{3\times 5!}f^{iv}(\eta_2)$, $\eta_2 \in [-1,1]$, and $ER_{st4}S_{\frac{3}{8}th}(f) = \frac{38}{2205}[f^{iv}(\eta_2) - f^{iv}(\eta_1)]$.

So, $|ER_{st4}RS_{\frac{3}{8}th}(f)| \cong \frac{38}{2205}[f^{iv}(d) - f^{iv}(c)] = \frac{38}{2205}\int_{-1}^{1} f^v(t)dt, |d-c| \leq 2$, i.e., $= \frac{38}{2205}(d-c)f^v(\gamma)$, for some $\gamma \in [-1,1]$. Then

$$|ER_{st4}SR_{\frac{3}{8}th}(f)| \leq \frac{76}{2205}f^v(\gamma), |ER_{st4}RS_{\frac{3}{8}th}(f)| \leq \frac{76M}{2205}.$$

□

6. Numerical Results

The approximate value of the following real integrals are computed and reported in Table 1.

$I_1 = \int_{-1}^{1} e^t dt = 2.3504023872876,\ I_2 = \int_0^1 e^{-t^2} dt = 0.746824132812427$
$I_3 = \int_0^1 e^{t^2} dt = 1.462651745907181,\ I_4 = \int_1^3 \frac{sin^2 t}{t} dt = 0.794825180668111$
$I_5 = \int_0^1 \sqrt{t} dt = 0.666666666666667$ and $I_6 = \int_2^3 \frac{ln\sqrt{t}}{t} dt = 0.181623986723595.$

TABLE 1. Anti mixed quadrature rule with mixed rule of Gaussian and anti-Newtonian and corresponding error

	$SRS_{\frac{1}{3}}(f)$	$ER_{GL}{}^2(f)$	$E_{st4}(f)$	$ES_{GL}{}^3(f)$	$ERS_{\frac{3}{8}}(f)$
I_1	0.011651369	0.007706299	0.007038361	0.007711361	0.011628766
I_2	0.000356296	0.000229445	0.000210774	0.000229898	0.000353997
I_3	0.013078837	0.008483857	0.007774726	0.008505229	0.012984982
I_4	0.005373406	0.003734838	0.003378927	0.003724421	0.054225285
I_5	0.028595479	0.007220672	0.006879202	0.006832563	0.017855491
I_6	0.000053749	0.000035368	0.000032331	0.000035408	0.000053571

7. Conclusion

The mixed rule is an efficient as compared to constituent rules and approximate analytical solutions for different integrals through the present rule are nice agreement with the corresponding exact results. The beneficial approach of the proposed rule is compared with the existing method numerically with minimized errors through error analysis. The proposed method may be extended to the approximate solution of analytic functions in the complex plane.

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