SUPRA CO BT URYSOHN SPACE IN SUPRA TOPOLOGICAL SPACES

K. KRISHNAVENI

ABSTRACT. In this article, the author interpolate prolific notion of co $bT^\mu$ graph and co $bT^\mu$ urysohn space by utilizing the concept of $bT^\mu$ set connected functions in supra topological spaces.

1. INTRODUCTION

Nowadays numerous imminent topologists are focused in compact space, separation axioms, connectedness, graph etc., in their research. In the year 1983, Reilly and Vamanamurthy ([9]) introduced a new concept of Clopen Relation in topological spaces. Dontchev, Ganster and Reilly ([1]) came out with a new function called regular set-connected in the year 1999. The idea behind the set-connected regular function is extended to Clopen Sets by Ekici in the year 2005 [2]. It was Mashhour et. al ([7]) who first came up with supra topology concept by studying the $S^*$-continuous as well as the $S$-continuous maps. A further introduction to this concept was given by introducing S-T0, S-T1, S-T2, S-T2 spaces by discussing its relation to the T0,T1,T2,T2 topological spaces. Takashi Noiri and Sayed ([10]) in their research focused on a method of introducing supra b-continuity and b-open sets in the supra topology space in the year 2010 and also elaborated the supra b-open as well as the continuous maps as well as its interconnectivity. Ganes M.Pandya, C.Janaki and I. Arockiarani ([3]) introduced another functions as a set of connected classes named as the

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π-set connected and investigated a relations among the π-set connected function, covering properties and separation axioms. In 1999, Ramprasad Paul and P. Bhattacharyya ([8]) studied some basic properties of pre-urysohn space and discussed some properties of functions with p-θ-closed graph and pre-θ-closed graph and their relationships with the pre urysohn space.

The author ([6]) introduced bTμ-set connected functions in topological supra spaces. They also discussed the separation axioms using bTμ-set connected functions in supra topological spaces. The author defined the function class under one condition that for each supra clopen there is an inverse image present (like the supra closed and open set) in the codomain is bTμ-clopen (that is, bTμ-open and closed) in the domain. The author investigated the fundamental properties of bTμ-set connected functions.

The idea of this paper rises with a latest idea behind co-bTμ graph and co-bTμ urysohn space by utilizing the concept of bTμ-set connected functions in supra topological spaces.

In the present research the functions (Y, σ) and (X, τ) represent supra topological space in which there is no axiom separations were assumed until they are mentioned. For X let the subset be A. The supra interior as well as the supra closure of this set by intμ(A) and clμ(A) are also denoted.

2. Preliminaries

Definition 2.1. [7, 10] A μ of X subfamily is defined as a supra topology on X, if

(i) X, φ ∈ μ

(ii) if A_i ∈ μ for all i ∈ J then ∪ A_i ∈ μ.

The supra topological space is defined by the pair (X, μ). In this supra open sets are called by μ and supra closed set is defined as the complimentary set of the supra open.

Definition 2.2. [7, 10]

(i) A set A supra interior is defined using the term intμ(A) and intμ(A) = ∪ {B: B is defined as supra open set where A ⊇ B}.

(ii) A set A supra closure is defined using the term clμ(A) and clμ(A) = ∩ {B: B is defined as supra closed set where A ⊆ B}.
Definition 2.3. [4] For a set of \((X, \tau)\) as supra subset \(A\) is defined as, \(bT^\mu\)-closed set for \(bc\mu(A) \subset U\) when \(U\) is \(T^\mu\)-open and \(A \subset U\) in \((X, \tau)\).

Definition 2.4. [4] Assume \((Y, \sigma)\) and \((X, \tau)\) be two supra topological spaces. A relation \(f:(X, \tau) \rightarrow (Y, \sigma)\) is defined as \(bT^\mu\)-continuous if in \((X, \tau)\), \(f^{-1}(V)\) is \(bT^\mu\)-closed for each \(V\) of \((Y, \sigma)\) supra closed set.

Definition 2.5. [5] A relation \(f:(X, \tau) \rightarrow (Y, \sigma)\) is known as \(bT^\mu\)-open map \( (bT^\mu\text{-closed}) \) in case the image \(f(A)\) is \(bT^\mu\)-open \((bT^\mu\text{-closed})\) in \((Y, \sigma)\) for every supra open \((supra\text{ closed})\) set \(A\) in \((X, \tau)\).

Definition 2.6. [6] A relation \(f:(X, \tau) \rightarrow (Y, \sigma)\) is known as \(bT^\mu\)-set connected function in case \(f^{-1}(V)\) in \((X, \tau)\) is \(bT^\mu\) clopen set for each \(V\) in \((Y, \sigma)\) is supra clopen set.

Definition 2.7. [6] A \((X, \tau)\) supra topological space is defined as co \(bT^\mu\) \(T_1\) in case each distinct points pair \(x,y\), \(bT^\mu\) clopen sets exist for \(U\) having \(x\) and \(V\) associated to \(y\) in a way that \(x \in U, y \notin U\) and \(x \notin V, y \in V\).

Definition 2.8. [6] A \((X, \tau)\) supra topological space is known as co \(bT^\mu\) \(T_2\) or co \(bT^\mu\) Hausdorff space in case each two distinct points of \((X, \tau)\) could be separated by disarticulate \(bT^\mu\) clopen sets.

3. \(bT^\mu\) CLOPEN MAP

Definition 3.1. A function \(f:(X, \tau) \rightarrow (Y, \sigma)\) can be defined as \(bT^\mu\) clopen map if \(f(A)\) image is supra clopen in \((Y, \sigma)\) for each set \(A\) of \(bT^\mu\) clopen in \((X, \tau)\).

Example 1. Assume \(X = Y = \{a, b, c\}\) with \(\tau = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{a, b\}, \{b, c\}\}\) and \(\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}\). A relation \(f:(X, \tau) \rightarrow (Y, \sigma)\) is defined as \(f(a) = a, f(b) = c, f(c) = b\). The supra clopen set of \((Y, \sigma)\) are \(\{Y, \phi, \{a\}, \{b\}, \{b, c\}, \{a, c\}\}\) and the \(bT^\mu\) clopen set of \((X, \tau)\) are \(\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{b, c\}\}\). Here \(f\) \(bT^\mu\) clopen map.

4. CO \(bT^\mu\) GRAPH

Definition 4.1. If function \(f:(X, \tau) \rightarrow (Y, \sigma)\) are related, then \(G^\mu(f) = \{(x, y) : x, y \in X\}\) subset of a \((X \times Y, \tau \times \sigma)\) product space is defined as supra graph of \(f\).
Definition 4.2. A $G(f)$ graph of a relation $f : (X, \tau) \rightarrow (Y, \sigma)$ can be defined as co $bT^\mu$ graph, if for every $(x, y) \in (X \times Y) \cap G^\mu(f)$, $bT^\mu$ clopen set $U$ exist for $X$ as well as $V$ be supra clopen of $Y$ in a way that $(x, y) \in U \times V$ and $(U \times V) \cap G^\mu(f) = \phi$.

Theorem 4.1. For function $f : (X, \tau) \rightarrow (Y, \sigma)$ is any relation with the co $bT^\mu$ graph, for every $x \in X$, $f(x) = \bigcap \{bT^{int\mu}(bT^{cl\mu}(f(U))) : U \in bT^\mu \setminus CO(X, x)\}$.

Proof. Let us assume that the theorem is false. Then $y \neq f(x)$ exists in a way that $y \in \bigcap \{bT^{int\mu}(bT^{cl\mu}(f(U))) : U \in bT^\mu \setminus CO(X, x)\}$. Which shows that, $y \in bT^{int\mu}(bT^{cl\mu}(f(U)))$ for every $U \in bT^\mu \setminus CO(X, x)$. So, $V \cap f(U) \neq \phi$ for every $V \in \supra CO(Y, y)$. Hence, $int\mu(cl\mu(V)) \cap f(U) \supset V \cap f(U) \neq \phi$ contradicting the above theorem which states $f$ is related to graph co $bT^\mu$. □

Theorem 4.2. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be co $bT^\mu$ graph $G^\mu(f)$. $X$ co $bT^\mu$ clopen $T_1$. If $f$ injective.

Proof. Assume $x$, $y$ be two distinctive points of $(X, \tau)$. Therefore, $(x, f(y)) \in (X \times Y) \cap G^\mu(f)$ is achieved. By co $bT^\mu$ graph, there exist $bT^\mu$ clopen set $U$ of $X$ as well as the supra clopen set $V$ in a way that $(U \times V) \cap G^\mu(f)$ and $(x, f(y)) \in U \times V = \phi$. Hence, $f(U) \cap V = \phi$ is achieved. Therefore, $U \cap f^{-1}(V) = \phi$. Hence, $y \notin U$ which shows $X$ co $bT^\mu$ clopen $T_1$. □

Theorem 4.3. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be co $bT^\mu$ graph $G^\mu(f)$. If $f$ surjective supra clopen function, then $Y$ co $bT^\mu$.

Proof. Assume that for $(Y, \sigma)$, $a$ and $b$ be the distinct points. For $f$ surjective, $f(x) = a$ for some $x \in X$ and $(x, y) \in (X \times Y) \cap G^\mu(f)$. With co-$bT^\mu$ graph, $bT^\mu$-clopen present for set $U$ of $(X, \tau)$ and supra clopen set $V$ of $(Y, \sigma)$. We know that each supra clopen set is $bT^\mu$ clopen set. Hence $V$ is $bT^\mu$ clopen set in $(Y, \sigma)$ in a way, $(U \times V) \cap G^\mu(f) = \phi$ and $(x, b) \in U \times V$. Hence, $f(U) \cap V = \phi$ is achieved. As, $f$ supra clopen function then $f(U)$ supra clopen in a way $f(x) = a \in f(U)$. Hence $Y$ co $bT^\mu$ $T_2$. □

Theorem 4.4. For $f : (X, \tau) \rightarrow (Y, \sigma)$ $bT^\mu$ connected functions and $(Y, \sigma)$ co $bT^\mu$ Hausdorff space, then $G^\mu(f)$ is co $bT^\mu$ graph in $X \times Y$ product space.

Proof. Assume $(x, y) \in (X \times Y)$ $G^\mu(f)$. Then supra clopen sets $V_1$ and $V_2$ with $y \neq f(x)$ of $Y$ in a way that $f(x) \in V_1$, $y \in V_2$ and $V_1 \cap V_2 = \phi$. From hypothesis, there is $U \in bT^\mu \setminus CO(X, x)$ for $f(U) \subset V_1$. Hence, $f(U) \cap V_2 = \phi$ is obtained. □
5. CO $bT^\mu$ URYSOHN SPACE

**Definition 5.1.** Let $X$ is defined as supra Urysohn if for every points pair $x, y \in X$, $x \neq y$ there exist $U \in$ supra $C(x)$, $V \in$ supra $C(y)$ in a way that $\text{cl}^\mu(U) \cap \text{cl}^\mu(V) = \phi$.

**Definition 5.2.** Let $X$ is called co $bT^\mu$ Urysohn if for each pair $x, y \in X$, $x \neq y$ there is $U \in bT^\mu CO(x)$, $V \in bT^\mu CO(y)$ in a way that $bT^{\mu}(bT \text{cl}^\mu(U)) \cap bT^{\mu}(bT \text{cl}^\mu(V)) = \phi$.

**Theorem 5.1.** A co $bT^\mu$ Urysohn space is co $bT^\mu T_1$.

**Proof.** For $X$ assume $x$ and $y$ as the two distinctive points. As $X$ co $bT^\mu$ Urysohn space. There is $U \in bT^\mu CO(x)$, $V \in bT^\mu CO(y)$ in a way, $bT^{\mu}(bT \text{cl}^\mu(U)) \cap bT^{\mu}(bT \text{cl}^\mu(V)) = \phi$.

Thos indicates that $x \notin bT^{\mu}(bT \text{cl}^\mu(V))$ and $y \notin bT^{\mu}(bT \text{cl}^\mu(U))$. Now, $bT^{\mu}(bT \text{cl}^\mu(U))$, $bT^{\mu}(bT \text{cl}^\mu(V)) \in bT^{\mu} CO(X)$. Therefore $X\cdot bT^{\mu}(bT \text{cl}^\mu(U))$, $X\cdot bT^{\mu}(bT \text{cl}^\mu(V)) \in bT^{\mu} O(X)$ (or) $bT^{\mu} C(X)$ in a way, $x \in X \cdot bT^{\mu}(bT \text{cl}^\mu(V))$ and $y \in X \cdot bT^{\mu}(bT \text{cl}^\mu(U))$ were $x \notin X \cdot bT^{\mu}(bT \text{cl}^\mu(U))$ and $y \notin X \cdot bT^{\mu}(bT \text{cl}^\mu(V))$. Thus $X$ co $bT^\mu T_1$. \hfill \Box

**Theorem 5.2.** If $f:(X, \tau) \to (Y, \sigma)$ bijective $bT^\mu$ clopen map as well as $X$ co $bT^\mu$ Urysohn then $Y$ co $bT^\mu$ Urysohn.

**Proof.** Assume $y_1 \neq y_2$ as $y_1, y_2 \in Y$. Since $f$ bijective $f^{-1}(y_1) \neq f^{-1}(y_2)$ and $f^{-1}(y_1), f^{-1}(y_2) \in X$. The urysohn property co $bT^\mu$ of $X$ proves an existence of sets $U \in bT^\mu CO(f^{-1}(y_1))$, $V \in bT^\mu CO(f^{-1}(y_2))$ with $bT^{\mu}_X(bT \text{cl}^\mu(U)) \cap bT^{\mu}_X(bT \text{cl}^\mu(V)) = \phi$. By bijective and $bT^\mu$ clopen map, $f(bT^{\mu}_X(bT \text{cl}^\mu(U))) \in bT^{\mu} CO(Y)$. Again from $U \subset (bT^{\mu}_X(bT \text{cl}^\mu(U)))$. It follows that

$$f(U) \subset f(bT^{\mu}_X(bT \text{cl}^\mu(U))), (bT^{\mu}_Y(bT \text{cl}^\mu(f(U))))$$

Similarly, $(bT^{\mu}_X(bT \text{cl}^\mu(f(V)))) \subset f(bT^{\mu}_X(bT \text{cl}^\mu(V)))$. Therefore by injectivity of $f$,

$$bT^{\mu}_Y(bT \text{cl}^\mu(f(U))) \cap bT^{\mu}_Y(bT \text{cl}^\mu(f(V)))$$

$$\subset f(bT^{\mu}_X(bT \text{cl}^\mu(U))) \cap f(bT^{\mu}_X(bT \text{cl}^\mu(V))) = f(bT^{\mu}_X(bT \text{cl}^\mu(U))) \cap (bT^{\mu}_X(bT \text{cl}^\mu(V))) = \phi.$$ \hfill \Box

**Theorem 5.3.** If $f:(X, \tau) \to (Y, \sigma)$ $bT^\mu$ set connected and $Y$ co $bT^\mu$ urysohn for $G^\mu(f)$ co $bT^\mu$ graph.
Proof. For $y \neq f(x)$, assume $(x, y) \in (X \times Y) - G(f)$. As, $Y$ co $bT^\mu$ urysohn, so that $V \in bT^\mu \text{CO}(Y, y)$, $W \in bT^\mu \text{CO}(Y, f(x))$ such that

\[(bT\text{int}_X^\mu(bT\text{cl}_X^\mu(U))) \cap (bT\text{int}_X^\mu(bT\text{cl}_X^\mu(V))) = \phi.\]

Since $f$ $bT^\mu$ set connected, so that $U \in bT^\mu \text{CO}(X, x)$ for $f(U) \in \text{supra}\text{CO}(Y)$. It depicts that $f(U) \cap V = \phi$. Hence, $G^\mu(f)$ co $bT^\mu$ graph

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REFERENCES


DEPARTMENT OF MATHEMATICS (PG)
PSGR KRISHNAMMAL COLLEGE FOR WOMEN
COIMBATORE
TAMILNADU
INDIA
ADDRESS
Email address: krishnavenikaliswami@gmail.com