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ON RADIO ANALYTIC MEAN D-DISTANCE NUMBER OF MORE GRAPHS

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ABSTRACT. A Radio Analytic Mean D-distance labeling of associated diagram G is a balanced guide v from the vertex set $V(G)$ to N such that for two distinct vertices u and v of G. $d^D(u,v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + \; diam^D(G)$.

1. INTRODUCTION

All the graphs are Limited, Basic, undirected and associated diagrams. Let $V(G)$ and $E(G)$ denote the vertex set and edge set of G. Radio labeling (multi-level distance labeling) can be regarded as an given to distance two labeling which is motivated by the channel assignment problem introduced by Hale [1]. Chartrand et al [2] introduced the concept of radio labeling of graph. Chartrand [3] gave the upper bound for the radio number of path. See also [3-10],

2. PREMILINEARS

Definition 2.1. *The wheel graph providing the Helm graph H_n, also wheel graph itself having pendent edge at every vertex of the n–cycle.*

Definition 2.2. *The graph (C_n^t) denoting one point collecting of t copies cycle C_n. The graph (C_n^t) is called friendship graph.*

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Definition 2.3. *A fan graph obtained by joining all vertices of F_n is a path P_n to a further vertex called the centre.*

Definition 2.4. *The path graph connecting two fan graph is called double fan graph DF_n that have a General path.*

Definition 2.5. *The flower Fl_n is the graph obtained from a helm graph by joining every pendent vertex to the apex of the Helm.*

Definition 2.6. *The Jelly fish graph have four cycle vertices namely $\{u, v, x, y\}$ also $J(n, n)$ is joining by edge $\{x, y\}$also u_i pendent edges joining u and v_i pendent edges joining v.*

Theorem 2.1. *The Radio Analytic mean D-distance number of a fan graph, ramn$^D(F_n) = 2n + 2,\ n \geq\ 3$.*

Proof. Let V $= \{v_0, v_i,\ 1 \leq i \leq n\}$ and $E = \{v_0\ v_{i,}\ v_iv_j\ , 1 \leq i \leq n\ \ , i + 1 \leq j \leq n\}$. We define v_0 be the centre vertex and $v_1, v_2 \ldots .v_n$ be the path graph. The path vertices are joined to centre vertex v_0. Its diam^D(F$_n$) $= n + 6$. We define the vertex label $f(v_0) =\ n + 2$, $f(v_i) =\ n + 2 + i,\ 1 \leq i \leq n$.

Case (i): Compute the pair (v_0, v_i) are adjacent if v_i is end vertices.
$d^D\ (v_0, v_i) + \left\lceil \frac{|f(v_0)^2 - f(v_i)^2|}{2} \right\rceil \geq n{+}7 = n + 3 + \left\lceil \frac{|(n+2)^2 - (n+2+i)^2|}{2} \right\rceil \geq n + 7$

Case (ii): Compute the pair (v_0, v_i) are adjacent if v_i is intermediate vertices, $2 \leq i \leq n - 1 d^D\ (v_0, v_i) + \left\lceil \frac{|f(v_0)^2 - f(v_i)^2|}{2} \right\rceil \geq n + 7 = n + 4 + \left\lceil \frac{|(n+2)^2 - (n+2+i)^2|}{2} \right\rceil$
$\geq n + 7$.

Case (iii): Compute the pair $(v_i,$ v$_j)$ are both end vertices $1 \leq i \leq n$, $i + 1 \leq i \leq n$,
$d^D\ (v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq n{+}7 \quad = n + 6 + \left\lceil \frac{|(n+2+i)^2 - (n+2+j)^2|}{2} \right\rceil \geq n + 7.$

Case (iv): Compute the pair $(v_i,\ v_j)$ intermediate adjacent vertices,
$d^D\ (v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq n{+}7 \quad = 7 + \left\lceil \frac{|(n+2+i)^2 - (n+2+j)^2|}{2} \right\rceil \geq n + 7$

Therefore, ramn$^D(F_n) = 2n + 2,\ n \geq\ 3$ □

Theorem 2.2. *The radio analytic mean D-distance number of Double fan graph , ramn$^D(DF_n) = 3n + 1,\ n \geq\ 3$.*

Proof. Let $v_1, v_2, \ldots\ldots.v_n$ be the path graph and u, w are two vertex are joined to the end vertex of the path graph. we define the vertex label as $f(u) = 2n$, $f(v_i) = 2n + i$, $1 \leq i \leq n$, $f(w) = 3n + 1$. Its diam^D (DF_n) is $2n + 5$.

Case (i): Compute the pair (u, v_i) , $i = 1, n$

$$d^D(u, v_i) + \left\lceil \frac{\left|f(u)^2 - f(v_i)^2\right|}{2} \right\rceil \geq 1 + \; diam^D\,(G) = 1 + 2n + 5 = 2n + 6$$
$$= n + 4 \; + \left\lceil \frac{\left|(2n)^2 - (2n+i)^2\right|}{2} \right\rceil \geq 2n + 6$$

Case (ii): Compute the pair (v_i, w) , $i = 1, n$

$$d^D(v_i, w) + \left\lceil \frac{\left|f(v_i)^2 - f(w)^2\right|}{2} \right\rceil \geq 2n + 6 = n + 4 \; + \left\lceil \frac{\left|(2n+i)^2 - (3n+1)^2\right|}{2} \right\rceil \geq 2n + 6$$

Case (iii): Compute the pair (v_i, v_j) are both end vertices

$$d^D(v_i, v_j) + \left\lceil \frac{\left|f(v_i)^2 - f(v_j)^2\right|}{2} \right\rceil \geq 2n + 6 = n + 8 \; + \left\lceil \frac{\left|(2n+i)^2 - (2n+j)^2\right|}{2} \right\rceil \geq 2n + 6$$

Case (iv): Compute the pair (u, w) are both end vertices

$$d^D(u, w) \; + \left\lceil \frac{\left|f(u)^2 - f(w)^2\right|}{2} \right\rceil \geq 2n + 6 = 2n + 5 + \left\lceil \frac{\left|(2n)^2 - (3n+1)^2\right|}{2} \right\rceil \geq 2n + 6$$

Case (v): compute the pair (v_i, v_j) are both intermediate adjacent vertices

$$d^D(v_i, v_j) + \left\lceil \frac{\left|f(v_i)^2 - f(v_j)^2\right|}{2} \right\rceil \geq 2n + 6 = 9 + \left\lceil \frac{\left|(2n+i)^2 - (2n+j)^2\right|}{2} \right\rceil \geq 2n + 6$$

Case (vi): Compute the pair (u, v_i) for $i = 2, 3, 4, \ldots n - 1$

$$d^D(u, v_i) + \left\lceil \frac{\left|f(u)^2 - f(v_i)^2\right|}{2} \right\rceil \geq 2n + 6 = \; n + 5 + \left\lceil \frac{\left|(2n)^2 - (2n+i)^2\right|}{2} \right\rceil \geq 2n + 6.$$

Hence, ramn$^D(DF_n) = 3n + 1, \; n \geq 3$ □

Theorem 2.3. *The Radio analytic mean D-distance number of a flower graph, ramn$^D(Fl_n) = 3n + 2, \; n \geq \; 2$.*

Proof. Let $V(Fl_n) = \{w\} \cup \{v_i, u_i, i = 1, 2..n\}$ and $E = \{wv_i, wu_i, v_iu_i \; i = 1, 2 \ldots n\}$. The D-distance is $d^D(w, v_i) = d^D(w, u_i) = \; 2n+5$. $d^D(u_i, u_j) = \; 2n+6$. We construct the label f as follows $f(u_i) = n + i + 2$, $1 \leq i \leq n$, $f(v_i) = 2n + i + 2, \; 1 \leq i \leq n$, $f(w) = n$. Its diam$^D(Fl_n) = 2n + 6$.

Case (i): Compute the pair $(w, v_i), \; 1 \leq i \leq n$

$$d^D(w, v_i) + \left\lceil \frac{\left|f(w)^2 - f(v_i)^2\right|}{2} \right\rceil \geq 2n + 7 = 2n + 5 + \left\lceil \frac{\left|(n)^2 - (2n+i+2)^2\right|}{2} \right\rceil \geq \; 2n + 7$$

Case (ii): Compute the pair $(w, u_i), \; 1 \leq i \leq n$

$d^D(w, u_i) + \left\lceil \frac{|f(w)^2 - f(u_i)^2|}{2} \right\rceil \geq 2n + 7 = 2n + 5 + \left\lceil \frac{|(n)^2 - (n+i+2)^2|}{2} \right\rceil \geq 2n + 7$

Case (iii): Compute the pair (v_i, v_j) are adjacent $1 \leq i \leq n$, $i + 1 \leq j \leq n$,

$d^D(v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq 2n + 7 = 9 + \left\lceil \frac{|(2n+i+2)^2 - (2n+j+2)^2|}{2} \right\rceil \geq 2n + 7$

Case (iv): Compute the pair (u_i, u_j) are not adjacent $1 \leq i \leq n$, $i + 1 \leq j \leq n$

$d^D(u_i, u_j) + \left\lceil \frac{|f(u_i)^2 - f(u_j)^2|}{2} \right\rceil \geq 2n + 7 = 2n + 6 + \left\lceil \frac{|(n+i+2)^2 - (n+j+2)^2|}{2} \right\rceil \geq 2n + 7$

Case (v) : Compute the pair $(v_i, v_j), |i - j| > 1$

$d^D(v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq 2n + 7 = 2n + 10 + \left\lceil \frac{|(2n+i+2)^2 - (2n+j+2)^2|}{2} \right\rceil \geq 2n + 7$

Therefore, ramn$^D(Fl_n) = 3n + 2,\ n \geq 2$ □

Theorem 2.4. . *The Radio analytic mean D-distance number of a Helm graph, ramn*$^D(H_n) = 3n + 2,\ n \geq 2$

Proof. Let x_0 be the centre vertex and $v_0, v_1, \ldots, v_n$ be first boundary vertex set. Let $u_0, u_1, \ldots, u_n$ be pendent vertex from boundary vertex set. We define the vertex label as $f(x_0) = 2$, $f(u_i) = n + 2 + i$, $1 \leq i \leq n$, $f(v_i) = 2n + 2 + i$, $1 \leq i \leq n$. The valid diam^D(H_n)$= n + 14$.

Case(i): Compute the pair (w, u_i), $1 \leq i \leq n$

$d^D(w, u_i) + \left\lceil \frac{|f(w)^2 - f(u_i)^2|}{2} \right\rceil \geq n + 15 = n + 7 + \left\lceil \frac{|(2)^2 - (n+2+i)^2|}{2} \right\rceil \geq n + 15$

Case (ii): Compute the pair (w, v_i), $1 \leq i \leq n$

$d^D(w, v_i) + \left\lceil \frac{|f(w)^2 - f(v_i)^2|}{2} \right\rceil \geq n + 15 = n + 5 + \left\lceil \frac{|(2)^2 - (2n+2+i)^2|}{2} \right\rceil \geq n + 15$

Case (iii): Compute the pair $(u_i, v_j), i = j$

$d^D(u_i, v_j) + \left\lceil \frac{|f(u_i)^2 - f(v_j)^2|}{2} \right\rceil \geq n + 15 = 6 + \left\lceil \frac{|(n+2+i)^2 - (2n+2+j)^2|}{2} \right\rceil \geq n + 15$

Case (iv): Compute the pair (v_i, v_j) are adjacent

$d^D(v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq n + 15 = 9 + \left\lceil \frac{|(2n+2+i)^2 - (2n+2+j)^2|}{2} \right\rceil \geq n + 15$

Case(v): Compute the pair (v_i, v_j) are not adjacent $|i - j| > 1$

$d^D(v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq n + 15 = n + 10 + \left\lceil \frac{|(2n+2+i)^2 - (2n+2+j)^2|}{2} \right\rceil \geq n + 15$

Case(vi): Compute the pair (u_i, u_j) are not adjacent $|i - j| > 1$

$d^D(u_i, u_j) + \left\lceil \frac{|f(u_i)^2 - f(u_j)^2|}{2} \right\rceil \geq n+15 = n+14 + \left\lceil \frac{|(n+2+i)^2 - (n+2+j)^2|}{2} \right\rceil \geq n+15$

Case(vii): Compute the pair (v_i, u_j) are not adjacent $|i-j| > 1$

$d^D(v_i, u_j) + \left\lceil \frac{|f(v_i)^2 - f(u_j)^2|}{2} \right\rceil \geq n+15 = n+12 + \left\lceil \frac{|(2n+2+i)^2 - (n+2+j)^2|}{2} \right\rceil \geq n+15$

Therefore, $\text{ramn}^D(H_n) = 3n+2,\ n \geq 2$ □

Theorem 2.5. *The Radio analytic mean D-distance number of a friendship graph, ramn*$^D(C_3^t) = 5t-3,\ t \geq 2.$

Proof. Let V_0 is the centre vertex $(V_0 = 2)$ and $x_i(1 \leq i \leq t)$, $y_i(1 \leq i \leq t)$ be the outer boundary of the vertex sets. The valid diameter of $(C_3{}^t) = 2t+6$. Define the functions f as $f(v_0) = 2$, $f(x_i) = 3t+i-3$ and $f(y_i) = 4t+i-3$.

Case (i): Compute the pair (v_o, x_i) for $1 \leq i \leq n$

$d^D(v_o, x_i)) + \left\lceil \frac{|f(v_o)^2 - f(x_i)^2|}{2} \right\rceil \geq 1 + diam^D(G) = 1 + 2t+6 = 2t+7$

$2t+3 + \left\lceil \frac{|(2)^2 - (3t+i-3)^2|}{2} \right\rceil \geq 2t+7$

Case (ii) : Compute the pair (v_o, y_i) for $1 \leq i \leq n$

$d^D(v_o, y_i)) + \left\lceil \frac{|f(v_o)^2 - f(y_i)^2|}{2} \right\rceil \geq 2t+7 = 2t+3 + \left\lceil \frac{|(2)^2 - (4t+i-3)^2|}{2} \right\rceil \geq 2t+7$

Case (iii): Compute the pair (x_i, y_i) are adjacent for $1 \leq i \leq n$

$d^D(x_i, y_i)) + \left\lceil \frac{|f(x_i)^2 - f(y_i)^2|}{2} \right\rceil \geq 2t+7 = 5 + \left\lceil \frac{|(3t+i-3)^2 - (4t+i-3)^2|}{2} \right\rceil \geq 2t+7$

Case(iv): Compute the pair (x_i, x_j) for $1 \leq i \leq n$, $i+1 \leq j \leq n$

$d^D(x_i, x_j)) + \left\lceil \frac{|f(x_i)^2 - f(x_j)^2|}{2} \right\rceil \geq 2t+7 = 2t+6 + \left\lceil \frac{|(3t+i-3)^2 - (3t+j-3)^2|}{2} \right\rceil \geq 2t+7$

Case(v): Compute the pair (y_i, y_j) for $1 \leq i \leq n$, $i+1 \leq j \leq n$

$d^D(y_i, y_j) + \left\lceil \frac{|f(y_i)^2 - f(y_j)^2|}{2} \right\rceil \geq 2t+7 = 2t+6 + \left\lceil \frac{|(4t+i-3)^2 - (4t+j-3)^2|}{2} \right\rceil \geq 2t+7$

Therefore, $\text{ramn}^D(C_3^t) = 5t-3,\ t \geq 2.$ □

Theorem 2.6. *The Radio analytic mean D-distance number of a coconut tree, ramn*$^D C0(T) = 2n+1,\ n \geq 2.$

Proof. Let u be the central vertex of the coconut tree and v be the pendent vertices , w is the base vertex joined to centre vertex. Let $v(G) = \{v_i, 1 \leq i \leq n\}$

and $E(G) = \{wu, uv_i,\ 1 \leq i \leq n\}$. We define the vertex label f as follows $f\,(u) = n\ ,\ f\,(w) = n+1,\ f\,(v_i) = n+1+i,\ 1 \leq i \leq n$. The valid diam$^D(C0(G)) =\ n+5$.

Case(i): Compute the pair $(u, v_i), 1 \leq i \leq n$

$$d^D\,(u, v_i) + \left\lceil \frac{\left|f(u)^2 - f(v_i)^2\right|}{2} \right\rceil \geq 1 + n + 5 = n+6 = n+3 + \left\lceil \frac{\left|(n)^2 - (n+1+i)^2\right|}{2} \right\rceil \geq n+6$$

Case(ii): Compute the pair (u, w) $d^D\,(u, w) + \left\lceil \frac{\left|f(u)^2 - f(w)^2\right|}{2} \right\rceil \geq n+6 = n+3+ \left\lceil \frac{\left|(n)^2 - (n+1)^2\right|}{2} \right\rceil \geq n+6$

Case(iii): Compute the pair (v_i, v_j) are adjacent $1 \leq i \leq n$,

$$d^D\,(v_i, v_j) + \left\lceil \frac{\left|f(v_i)^2 - f(v_j)^2\right|}{2} \right\rceil \geq n+6 = n+5+ \left\lceil \frac{\left|(n+1+i)^2 - (n+1+j)^2\right|}{2} \right\rceil \geq n+6$$

Case(iv): Compute the pair (w, v_i) are not adjacent $1 \leq i \leq n$

$$d^D\,(w, v_i) + \left\lceil \frac{\left|f(w)^2 - f(v_i)^2\right|}{2} \right\rceil \geq n+6 = n+5+ \left\lceil \frac{\left|(n+1)^2 - (n+1+i)^2\right|}{2} \right\rceil \geq n+6$$

Case(v): Compute the pair (v_i, v_j) are not adjacent $1 \leq i \leq n$, $i+1 \leq j \leq n$

$$d^D\,(v_i, v_j) + \left\lceil \frac{\left|f(v_i)^2 - f(v_j)^2\right|}{2} \right\rceil \geq n+6 = n+5+ \left\lceil \frac{\left|(n+1+i)^2 - (n+1+j)^2\right|}{2} \right\rceil \geq n+6$$

Therefore, ramn$^D(C0(T)) = 2n+1, n \geq 2$. □

Theorem 2.7. *The Radio Analytic mean D-distance number of Jelly fish graph, ramn*$^D(j(n,n)) = 8n,\ n \geq 2$.

Proof. Let J(n,n) be the graph. $v\,(G) = \{u, v, x, y, u_i, v_i\ 1 \leq i \leq n\}$ *and* $E\,(G) = \{ux, uy, vx, vy, xy, uu_i, vv_i, 1 \leq i \leq n\}$. The D-distance is $d^D\,(u, u_i) = n+4$, $d^D\,(u, x) = d^D\,(u, y) = n+6,\ d^D\,(x, y) = 7$, $d^D\,(u_i, x) = d^D\,(u_i, y) = n+8$.

The valid diam^D(J(n,n)= $15 + 2n - 2$. we are provide the labeling as follows

$$f\,(u_i) = 4n + i$$
$$f\,(u) = 5n + 1$$
$$f\,(x) = 6n$$
$$f\,(y) = 6n + 1$$
$$f\,(v) = 7n$$
$$f\,(v_i) = 7n + i$$

Case(i): Compute the pair (u, u_i), $1 \leq i \leq n$

$d^D(u, u_i) + \left\lceil \frac{|f(u)-f(u_i)^2|}{2} \right\rceil \geq 15+2n-1 = n+4+\left\lceil \frac{|(5n+1)^2-(4n+i)^2|}{2} \right\rceil \geq 15+2n-1$

Case(ii): Compute the pair (u, x),

$d^D(u, x) + \left\lceil \frac{|f(u)-f(x)^2|}{2} \right\rceil \geq 15+2n-1 = n+6+\left\lceil \frac{|(5n+1)^2-(6n)^2|}{2} \right\rceil \geq 15+2n-1$

Case(iii): Compute the pair (u, y)

$d^D(u, y) + \left\lceil \frac{|f(u)-f(y)^2|}{2} \right\rceil \geq 15+2n-1 = n+6+\left\lceil \frac{|(5n+1)^2-(6n+1)^2|}{2} \right\rceil \geq 15+2n-1$

Case(iv): Compute the pair (u, v)

$d^D(u, v) + \left\lceil \frac{|f(u)-f(v)^2|}{2} \right\rceil \geq 15+2n-1 = 10+2n-1+\left\lceil \frac{|(5n+1)^2-(7n)^2|}{2} \right\rceil \geq 15+2n-1$

Case(v): Compute the pair (u, v_i), $1 \leq i \leq n$

$d^D(u, v_i) + \left\lceil \frac{|f(u)-f(v_i)^2|}{2} \right\rceil \geq 15+2n-1 = 10+2n+1+\left\lceil \frac{|(5n+1)^2-(7n+i)^2|}{2} \right\rceil \geq 15+2n-1$

Case(vi): Compute the pair (x, y)

$d^D(x, y) + \left\lceil \frac{|f(x)-f(y)^2|}{2} \right\rceil \geq 15+2n-1 = 7+\left\lceil \frac{|(6n)^2-(7n)^2|}{2} \right\rceil \geq 15+2n-1$

Case(vii): Compute the pair $(u_i, x), 1 \leq i \leq n$

$d^D(u_i, x) + \left\lceil \frac{|f(u_i)-f(x)^2|}{2} \right\rceil \geq 15+2n-1 = n+8+\left\lceil \frac{|(4n+i)^2-(6n)^2|}{2} \right\rceil \geq 15+2n-1$

Case(viii): Compute the pair $(u_i, y), 1 \leq i \leq n$

$d^D(u_i, y) + \left\lceil \frac{|f(u_i)-f(y)^2|}{2} \right\rceil \geq 15+2n-1 = n+8+\left\lceil \frac{|(4n+i)^2-(6n+1)^2|}{2} \right\rceil \geq 15+2n-1$

Case(ix): Compute the pair (u_i, v), $1 \leq i \leq n$

$d^D(u_i, v) + \left\lceil \frac{|f(u_i)-f(v)^2|}{2} \right\rceil \geq 15+2n-1 = 10+2n+1+\left\lceil \frac{|(4n+i)^2-(7n)^2|}{2} \right\rceil \geq 15+2n-1$

Case(x): Compute the pair (u_i, v_i), $1 \leq i \leq n$

$d^D(u_i, v_i) + \left\lceil \frac{|f(u_i)-f(v_i)^2|}{2} \right\rceil \geq 15+2n-1 = 15+2n-2+\left\lceil \frac{|(4n+i)^2-(7n+i)^2|}{2} \right\rceil \geq 15+2n-1$

Case (xi): Compute the pair (v, v_i), $1 \leq i \leq n$

$d^D(v, v_i) + \left\lceil \frac{|f(v)-f(v_i)^2|}{2} \right\rceil \geq 15+2n-1 = n+4+\left\lceil \frac{|(7n)^2-(7n+i)^2|}{2} \right\rceil \geq 15+2n-1$

Case (xii): Compute the pair (u_i, u_j), $1 \leq i \leq n$, $i+1 \leq j \leq n$

$d^D(u_i, u_j) + \left\lceil \frac{\left|f(u_i) - f(u_j)^2\right|}{2} \right\rceil \geq 15 + 2n - 1 = n + 6 + \left\lceil \frac{\left|(4n+i)^2 - (4n+j)^2\right|}{2} \right\rceil \geq 15 + 2n - 1$

Case (xiii): Compute the pair (v_i, v_j), $1 \leq i \leq n$, $i + 1 \leq j \leq n$

$d^D(v_i, v_j) + \left\lceil \frac{\left|f(v_i) - f(v_j)^2\right|}{2} \right\rceil \geq 15 + 2n - 1 = n + 6 + \left\lceil \frac{\left|(7n+i)^2 - (7n+j)^2\right|}{2} \right\rceil \geq 15 + 2n - 1$

Case (xiv): Compute the pair (x, v)

$d^D(x, v) + \left\lceil \frac{\left|f(x) - f(v)^2\right|}{2} \right\rceil \geq 15 + 2n - 1 = n + 6 + \left\lceil \frac{\left|(6n)^2 - (7n)^2\right|}{2} \right\rceil \geq 15 + 2n - 1$

Case (xv): Compute the pair (y, v_i), $1 \leq i \leq n$

$d^D(y, v_i) + \left\lceil \frac{\left|f(y) - f(v_i)^2\right|}{2} \right\rceil \geq 15 + 2n - 1 = n + 8 + \left\lceil \frac{\left|(6n+1)^2 - (7n+i)^2\right|}{2} \right\rceil \geq 15 + 2n - 1$

Therefore, $\text{ramn}^D(j(n,n)) = 8n, \ n \geq 2$. □

CONCLUSION

We have studied some new results of radio analytic mean D-distance number. We have obtained upper bounds for the radio analytic mean D-distance number in various graphs. Above results is useful for the existing radio transmitters network. In the expanded network installed nearby transmitters are connected and interference is also avoided between them.

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