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A METHOD FOR FINDING CRITICAL PATH WITH SYMMETRIC OCTAGONAL INTUITIONISTIC FUZZY NUMBERS

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ABSTRACT. The concept of this paper represents finding fuzzy critical path using octagonal fuzzy number. In project scheduling, a new method has been approached to identify the critical path by using Symmetric Octagonal Intuitionistic Fuzzy Number (SYMOCINTFN). For getting a better solution, we use the fuzzy octagonal number rather than other fuzzy numbers. The membership functions of the earliest and latest times of events are by calculating lower and upper bounds of the earliest and latest times considering octagonal fuzzy duration. The resulting conditions omit the negative and infeasible solution. The membership function takes up an essential role in finding a new solution. Based on membership function, fuzzy number can be identified in different categories such as Triangular, Trapezoidal, pentagonal, hexagonal, octagonal, decagonal, hexa decagonal fuzzy numbers etc.

A suitable numerical illustration is given to understand the superiority of the proposed algorithm and methods.

1. INTRODUCTION

For scheduling problem, critical path method (CPM) takes a vital role for finding the optimum solution. The essential objective of the CPM is to identify and analyse the performance of the activities to minimize the total duration of the

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project. Usually, the CPM requires the time duration of each activity. In real situation problems, the time durations are uncertain. This uncertainty may lead to fuzziness. To overcome the fuzziness, we are considering fuzzy numbers instead of a crisp number as the activities durations. Laterally applying defuzzification process to convert as crisp value. In this paper for finding CPM, we introduced the time taken by the activities are symmetric Octagonal Intuitionistic Fuzzy numbers.

D. Stephen Dinagar and N. Rameshan [6, 7] have proposed the critical path by Octagonal fuzzy number and developed the procedure for finding a critical path using TOPSIS method. P. Rajarajeswari and Menaka [3] have reported arithmetic operations and ranking of octagonal intuitionistic fuzzy numbers. S. Vimala and S. Krishna Prabha [8] proposed a method for finding fuzzy critical path using magnitude method of trapezoidal fuzzy number. N. Rameshan and D. Stephen Dinagar [4] have solved CPM with an octagonal intuitionistic fuzzy number. Pathinathan. T and Ajay Mini [2] & Dhanalakshmi V and Felbin C Kennedy [1] have introduced symmetric pentagonal intuitionistic fuzzy number. G. Uhtra, K. Thangavelu, B. Amutha [9] have discussed arithmetic operations of SYMOCINTFN found ranking of Generalized Intuitionistic fuzzy number. D. Stephen Dinagar and D. Abirami [5] have found CPM by using interval-valued fuzzy numbers.

This work is organized as follows: In Section 2, some basic definitions are reviewed. In Section 3, basic definitions of OCFN, OCINTFN, SYMOCINTFN are discussed. In Section 4, arithmetic operations of Octagonal Intuitionistic Fuzzy Number are discussed. In Section 5, a new ranking function, properties and algorithm have been proposed. In Section 6, a numeric example based on ranking function is given.

2. Basic Concepts

Definition 2.1 (Mapping)**.** *Let X be a set. A fuzzy set $\tilde{A}$ on X is defined to be a function $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.*

Definition 2.2 (Fuzzy Set)**.** *The fuzzy number $\tilde{A}$ is fuzzy set if membership function satisfies*

(i) *A fuzzy set of the universe of discourse X is convex;*

(ii) $\tilde{A}$ *is normal if* $\exists\, x_i \in X$, $\mu_{\tilde{A}}(x_i) = 1$;
(iii) $\mu_{\tilde{A}}(x)$ *is piecewise continuous.*

Definition 2.3 (Intuitionistic Fuzzy Set). *An Intuitionistic fuzzy set* $\tilde{I}'$ *of non-empty set* X *is defined as* $\tilde{I}' = \{t, \mu_{\tilde{I}'}(t), \gamma_{\tilde{I}'}(t) : X \to [0,1]\}$ *and* $\tilde{I}' = \{\langle t, \mu_{\tilde{I}'}(t), \gamma_{\tilde{I}'}(t)\rangle / t \in X\}$, $\mu_{\tilde{I}'}(t) \to$ *membership function ,* $\gamma_{\tilde{I}'}(t) \to$ *non-membership function such that* $\mu_{\tilde{I}'}(t), \gamma_{\tilde{I}'}(t) : X \to [0,1]$ & $0 \le \mu_{\tilde{I}'}(t) + \gamma_{\tilde{I}'}(t) \le 1\ \forall\, t \in X$.

Definition 2.4 (Intuitionistic Fuzzy Number). *A subset* $\tilde{I}' = \{\langle t, \mu_{\tilde{I}'}(t), \gamma_{\tilde{I}'}(t)\rangle / t \in X\}$ *of the real line* R *is said to Intuitionistic Fuzzy number, if*

(i) $\exists\, x \in R$, $\because \mu_{\tilde{I}'}(t) = 1$, $\gamma_{\tilde{I}'}(t) = 0$;
(ii) $\mu_{\tilde{I}'}(t)$ *is continuous from* $R \to [0,1]$;
(iii) $0 \le \mu_{\tilde{I}'}(t) + \gamma_{\tilde{I}'}(t) \le 1\ \forall\, t \in X$.

The membership function

$$\mu_{\tilde{I}'}(t) = \begin{cases} 0, & -\infty < t \le i_1 \\ f(t), & i_1 \le t \le i_2 \\ 1, & t = i_2 \\ g(t), & i_2 \le t \le i_3 \\ 0, & i_3 \le t \le \infty \end{cases}.$$

The non- membership function

$$\gamma_{\tilde{I}'}(t) = \begin{cases} 0, & -\infty < t \le i_1 \\ f'(t), & i'_1 \le t \le i_2 \\ 1, & t = i_2 \\ g'(t), & i_2 \le t \le i'_3 \\ 0, & i'_3 \le t \le \infty \end{cases}.$$

3. Octagonal Intuitionistic Fuzzy Number

In this section, we discussed with notion of OCINTFN and their arithmetic operations.

Definition 3.1 (OCFN). *A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ which are real numbers. The membership function*

$$\mu_{\tilde{I}'}(t) = \begin{cases} 0, & x \leq a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ k, & a_2 \leq x \leq a_3 \\ k+(1-k)\left(\frac{x-a_3}{a_4-a_3}\right), & a_3 \leq x \leq a_4 \\ 1, & a_4 \leq x \leq a_5 \\ k+(1-k)\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \leq x \leq a_6 \\ k, & a_6 \leq x \leq a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \leq x \leq a_8 \\ 0, & x \geq a_8 \end{cases},$$

where $0 < k < 1$.

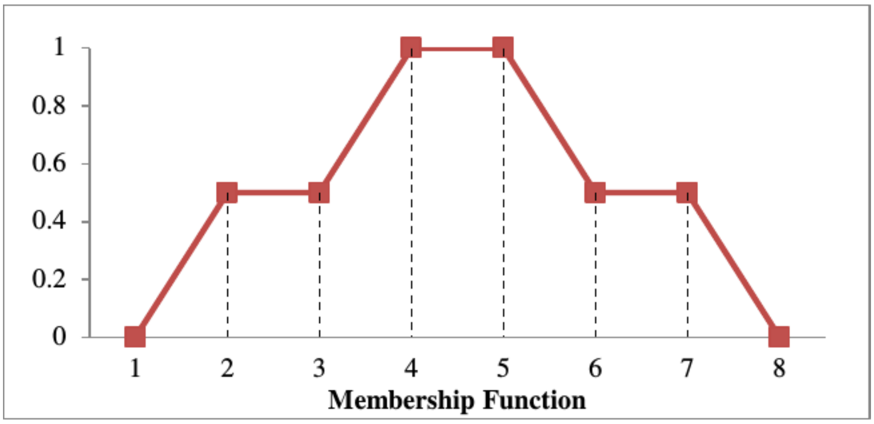

FIGURE 1. Octagonal Fuzzy Number

Definition 3.2 (OCINTFN). *An OCINTFN is* $\tilde{I}'_{OCT} = (io_1, io_2, io_3, io_4, io_5, io_6, io_7, io_8)$, $(io'_1, io'_2, io'_3, io'_4, io'_5, io'_6, io'_7, io'_8)$ *where* $io_1, io_2, io_3, io_4, io_5, io_6, io_7, io_8$ *and* $io'_1, io'_2, io'_3, io'_4, io'_5, io'_6, io'_7, io'_8$ *are real numbers. The membership function*

$$\mu_{\tilde{I}'}(t) = \begin{cases} 0, & t \leq io_1 \\ k\left(\frac{t-io_1}{io_2-io_1}\right), & io_1 \leq t \leq io_2 \\ k, & io_2 \leq t \leq io_3 \\ k+(1-k)\left(\frac{t-io_3}{io_4-io_3}\right), & io_3 \leq t \leq io_4 \\ 1, & io_4 \leq t \leq io_5 \\ k+(1-k)\left(\frac{io_6-t}{io_6-io_5}\right), & io_5 \leq t \leq io_6 \\ k, & io_6 \leq t \leq io_7 \\ k\left(\frac{io_8-t}{io_8-io_7}\right), & io_7 \leq t \leq io_8 \\ 0, & t \geq io_8 \end{cases}.$$

The non-membership function

$$\gamma_{\tilde{I}'}(t) = \begin{cases} 1, & t \leq io'_1 \\ k\left(\frac{io'_1-t}{io'_2-io'_1}\right), & io'_1 \leq t \leq io'_2 \\ k, & io'_2 \leq t \leq io'_3 \\ k+(1-k)\left(\frac{io'_4}{io'_4-io'_3}\right), & io'_3 \leq t \leq io'_4 \\ 0, & io'_4 \leq t \leq io'_5 \\ k+(1-k)\left(\frac{t-io'_5}{io'_6-io'_5}\right), & io'_5 \leq t \leq io'_6 \\ k, & io'_6 \leq t \leq io'_7 \\ k\left(\frac{t-io'_8}{io'_8-io'_7}\right), & io'_7 \leq t \leq io'_8 \\ 1, & t \geq io_8 \end{cases}.$$

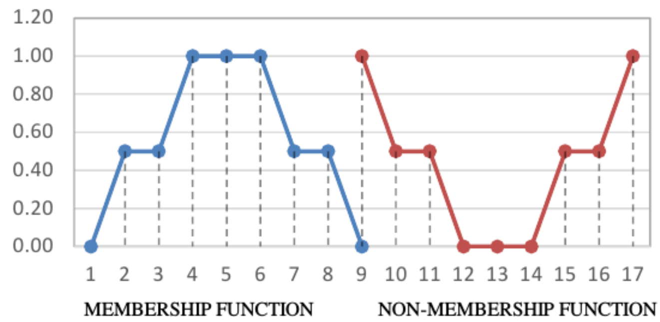

FIGURE 2. Octagonal Intuitionistic Fuzzy Number

Definition 3.3 (SYMOCINTFN). *An SYMOCINTFN is given by*

$$\tilde{SI}'_{OCT} = \begin{bmatrix} (S_L - r - s - t, S_L - r - s, S_L - r, S_L, S_U, S_U + r, S_U + r + s, \\ S_U + r + s + t); \\ (S'_L - r' - s' - t', S'_L - r' - s', S'_L - r', S_L, S_U, S'_U + r', \\ S'_U + r' + s', S'_U + r' + s' + t') \end{bmatrix},$$

where $S_L - r - s - t, S_L - r - s, S_L - r, S_L, S_U, S_U + r, S_U + r + s, S_U + r + s + t$ *and* $S'_L - r' - s' - t', S'_L - r' - s', S'_L - r', S_L, S_U, S'_U + r', S'_U + r' + s', S'_U + r' + s' + t'$ *are real numbers. The membership function is*

$$\mu_{\tilde{SI}'_{OCT}}(\lambda) = \begin{cases} \frac{\lambda - (S_L - r - s - t)}{2t}, & S_L - r - s - t \le \lambda \le S_L - r - s \\ \frac{1}{2}, & S_L - r - s \le \lambda \le S_L - r \\ \frac{1}{2} + \frac{(\lambda - S_L - r)}{2r}, & S_L - r \le \lambda \le S_L \\ 1, & S_L \le \lambda \le S_U \\ \frac{1}{2} + \frac{((S_U + r) - \lambda)}{2r}, & S_U \le \lambda \le S_U + r \\ \frac{1}{2}, & S_U + r \le \lambda \le S_U + r + s \\ \frac{(S_U + r + s + t) - \lambda}{2t}, & S_U + r + s \le \lambda \le S_U + r + s + t \end{cases}.$$

The non-membership function is

$$\gamma_{\tilde{SI}'_{OCT}}(\lambda) = \begin{cases} \frac{1}{2} + \frac{(S'_L - r' - s' - t') - \lambda}{2t'}, & S'_L - r' - s' - t' \le \lambda \le S'_L - r' - s' \\ \frac{1}{2}, & S'_L - r' - s' \le \lambda \le S'_L - r \\ \frac{1}{2}\left(\frac{(S_L - \lambda)}{S_L - (S_L - r)}\right), & S'_L - r' \le \lambda \le S'_L \\ 0, & S_L \le \lambda \le S_U \\ \frac{1}{2}\frac{(\lambda - S_U)}{(S'_U + r') - S_U}, & S_U \le \lambda \le S'_U + r' \\ \frac{1}{2}, & S'_U + r' \le \lambda \le S'_U + r' + s' \\ \frac{1}{2} + \frac{\lambda - (S'_U + r' + s' + t')}{2t'}, & S'_U + r' + s' \le \lambda \le S'_U + r' + s' + t' \end{cases}.$$

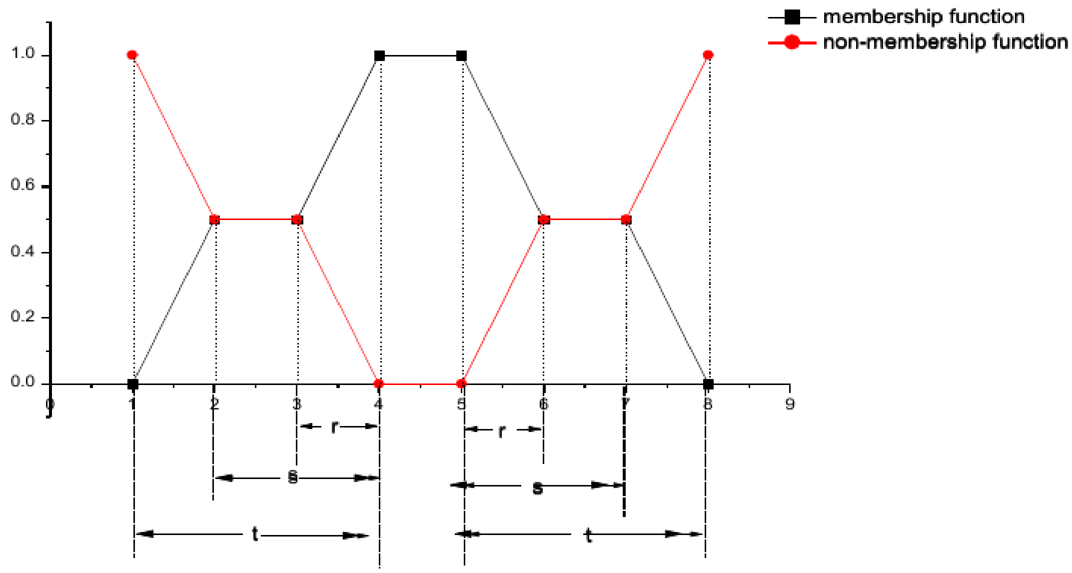

FIGURE 3. Symmetric Octagonal Intuitionistic Fuzzy Number

4. RECKONING OPERATIONS OF SYMOCINTFN

Let

$$\tilde{SI1}'_{OCT} = \begin{bmatrix} (S1_L - r_1 - s_1 - t_1, S1_L - r_1 - s_1, S1_L - r_1, S1_L, S1_U, S1_U + r_1, \\ S1_U + r_1 + s_1, S1_U + r_1 + s_1 + t_1); \\ (S1'_L - r'_1 - s'_1 - t'_1, S1'_L - r'_1 - s'_1, S1'_L - r'_1, S1_L, S1_U, S1'_U + r'_1, \\ S1'_U + r'_1 + s'_1, S1'_U + r'_1 + s'_1 + t'_1) \end{bmatrix}$$

$$\tilde{SI2}'_{OCT} = \begin{bmatrix} (S2_L - r_2 - s_2 - t_2, S2_L - r_2 - s_2, S2_L - r_2, S2_L, S2_U, S2_U + r_2, \\ S2_U + r_2 + s_2, S2_U + r_2 + s_2 + t_2); \\ (S2'_L - r'_2 - s'_2 - t'_2, S2'_L - r'_2 - s'_2, S2'_L - r'_2, S2_L, S2_U, S2'_U + r'_2, \\ S2'_U + r'_2 + s'_2, S2'_U + r'_2 + s'_2 + t'_2) \end{bmatrix}$$

are any two SYMOCINTFNs. Then

(i) Addition of SYMOCINTFN

$$\tilde{SI}1'_{OCT}+\tilde{SI}2'_{OCT}=\left[\begin{array}{l}\left(\begin{array}{l}(S1_L+S2_L)-r_2-s_2-t_2,(S1_L+S2_L)-r_2-s_2,\\(S1_L+S2_L)-r_2,(S1_L+S2_L),(S1_U+S2_U),\\(S1_U+S2_U)+r_2,(S1_U+S2_U)+r_2+s_2,\\(S1_U+S2_U)+r_2+s_2+t_2)\end{array}\right);\\\left(\begin{array}{l}(S1'_L+S2'_L)-r'_2-s'_2-t'_2,(S1'_L+S2'_L)-r'_2-s'_2,\\(S1'_L+S2'_L)-r'_2,(S1_L+S2_L),(S1_U+S2_U),\\(S1'_U+S2'_U)+r'_2,(S1'_U+S2'_U)+r'_2+s'_2,\\(S1'_U+S2'_U)+r'_2+s'_2+t'_2\end{array}\right)\end{array}\right]$$

Here, $r_1+r_2=r$, $s_1+s_2=s$, $t_1+t_2=t$, $r'_1+r'_2=r'$, $s'_1+s'_2=s'$, $t'_1+t'_2=t'$.

(ii) Subtraction of SYMOCINTFN

$$\tilde{SI}1'_{OCT}-\tilde{SI}2'_{OCT}=\left[\begin{array}{l}\left(\begin{array}{l}(S1_L-S2_L)-r_2-s_2-t_2,(S1_L-S2_L)-r_2-s_2,\\(S1_L-S2_L)-r_2,(S1_L-S2_L),(S1_U-S2_U),\\(S1_U-S2_U)+r_2,(S1_U-S2_U)+r_2+s_2,\\(S1_U-S2_U)+r_2+s_2+t_2)\end{array}\right);\\\left(\begin{array}{l}(S1'_L-S2'_L)-r'_2-s'_2-t'_2,(S1'_L-S2'_L)-r'_2-s'_2,\\(S1'_L-S2'_L)-r'_2,(S1_L-S2_L),(S1_U-S2_U),\\(S1'_U-S2'_U)+r'_2,(S1'_U-S2'_U)+r'_2+s'_2,\\(S1'_U-S2'_U)+r'_2+s'_2+t'_2\end{array}\right)\end{array}\right]$$

Here, $r_1-r_2=r$, $s_1-s_2=s$, $t_1-t_2=t$, $r'_1-r'_2=r'$, $s'_1-s'_2=s'$, $t'_1-t'_2=t'$.

iii) Scalar Multiplication of SYMOCINTFN

Let λ be any real and

$$\tilde{SI}'_{OCT}=\left[\begin{array}{l}(S_L-r-s-t,S_L-r-s,S_L-r,S_L,S_U,S_U+r,S_U+r+s,\\S_U+r+s+t);\\(S'_L-r'-s'-t',S'_L-r'-s',S'_L-r',S_L,S_U,S'_U+r',S'_U+r'+s',\\S'_U+r'+s'+t')\end{array}\right]$$

be SYMOCINTFN then scalar multiplication of SYMOCINTFN is defined by

$$\lambda*\tilde{SI}'_{OCT}=\left[\begin{array}{l}(\lambda(S_L-r-s-t),\lambda(S_L-r-s),\lambda(S_L-r),\lambda S_L,\lambda S_U,\lambda(S_U+r),\\\lambda(S_U+r+s),\lambda(S_U+r+s+t));\\(\lambda(S'_L-r'-s'-t'),\lambda(S'_L-r'-s'),\lambda(S'_L-r'),\lambda S_L,\lambda S_U,\\\lambda(S'_U+r'),\lambda(S'_U+r'+s'),\lambda(S'_U+r'+s'+t'))\end{array}\right]$$

(iv) Multiplication of SYMOCINTFN

$$\tilde{SI1}'_{OCT} \times \tilde{SI2}'_{OCT} = \begin{bmatrix} \begin{pmatrix} (S1_L \times S2_L) - r_2 - s_2 - t_2, (S1_L \times S2_L) - r_2 - s_2, \\ (S1_L \times S2_L) - r_2, (S1_L \times S2_L), (S1_U \times S2_U), \\ (S1_U \times S2_U) + r_2, (S1_U \times S2_U) + r_2 + s_2, \\ (S1_U \times S2_U) + r_2 + s_2 + t_2) \end{pmatrix}; \\ \begin{pmatrix} (S1'_L \times S2'_L) - r'_2 - s'_2 - t'_2, (S1'_L \times S2'_L) - r'_2 - s'_2, \\ (S1'_L \times S2'_L) - r'_2, (S1_L \times S2_L), (S1_U \times S2_U), \\ (S1'_U \times S2'_U) + r'_2, (S1'_U \times S2'_U) + r'_2 + s'_2, \\ (S1'_U \times S2'_U) + r'_2 + s'_2 + t'_2 \end{pmatrix} \end{bmatrix}$$

Here, $r_1 \times r_2 = r$, $s_1 \times s_2 = s$, $t_1 \times t_2 = t$, $r'_1 \times r'_2 = r'$, $s'_1 \times s'_2 = s'$, $t'_1 \times t'_2 = t'$.

(v) Division of SYMOCINTFN

$$\frac{\tilde{SI1}'_{OCT}}{\tilde{SI2}'_{OCT}} = \begin{bmatrix} \begin{pmatrix} \left(\frac{S1_L}{S2_L}\right) - r_2 - s_2 - t_2, \left(\frac{S1_L}{S2_L}\right) - r_2 - s_2, \left(\frac{S1_L}{S2_L}\right) - r_2, \left(\frac{S1_L}{S2_L}\right), \\ \left(\frac{S1_U}{S2_U}\right), \left(\frac{S1_U}{S2_U}\right) + r_2, \left(\frac{S1_U}{S2_U}\right) + r_2 + s_2, \left(\frac{S1_U}{S2_U}\right) + r_2 + s_2 + t_2 \end{pmatrix}; \\ \begin{pmatrix} \left(\frac{S1'_L}{S2'_L}\right) - r'_2 - s'_2 - t'_2, \left(\frac{S1'_L}{S2'_L}\right) - r'_2 - s'_2, \left(\frac{S1'_L}{S2'_L}\right) - r'_2, \left(\frac{S1_L}{S2_L}\right), \\ \left(\frac{S1_U}{S2_U}\right), \left(\frac{S1'_U}{S2'_U}\right) + r'_2, \left(\frac{S1'_U}{S2'_U}\right) + r'_2 + s'_2, \left(\frac{S1'_U}{S2'_U}\right) + r'_2 + s'_2 + t'_2 \end{pmatrix} \end{bmatrix}$$

Here, $\frac{r_1}{r_2} = r$, $\frac{s_1}{s_2} = s$, $\frac{t_1}{t_2} = t$, $\frac{r'_1}{r'_2} = r'$, $\frac{s'_1}{s'_2} = s'$, $\frac{t'_1}{t'_2} = t'$.

5. Ranking Function, Properties & Algorithm

Proposition 5.1 (Ranking Function). *Let* $\tilde{SI}'_{oct} = (so_1, so_2, so_3, so_4, so_5, so_6, so_7, so_8), (so'_1, so'_2, so'_3, so'_4, so'_5, so'_6, so'_7, so'_8)$ *be the SYMOCINTFN maps into set of real numbers. We proposed new ranking function of SYMOCINTFN is defined as*

$$R\left[\tilde{SI}'_{oct}\right] = Max\left(\mu_{\tilde{SI}'_{oct}}(\lambda), \gamma_{\tilde{SI}'_{oct}}(\lambda)\right),$$

Here,

$$\mu_{\tilde{SI}'_{oct}}(\lambda) = \frac{1}{2}\left[(so_4 + so_5) - \frac{12}{13}so_1 - \frac{10}{11}so_2 - \frac{8}{9}so_3 + \frac{6}{7}so_6 + \frac{4}{5}so_7 + \frac{2}{3}so_8\right]$$

$$\gamma_{\tilde{SI}'_{oct}}(\lambda) = \frac{1}{2}\left[(so'_4 + so'_5) - \frac{12}{13}so'_1 - \frac{10}{11}so'_2 - \frac{8}{9}so'_3 + \frac{6}{7}so'_6 + \frac{4}{5}so'_7 + \frac{2}{3}so'_8\right]$$

5.1. **Symbolic Representation.**

For Basic computations, Let us use the following notations,

$A_{SYMOCINTFN}(ij)$ = Activity of SYMOCINTFN b/w event i (tail) and event j (head)

$E_{SYMOCINTFN}(i)$ = Earliest occurrence event time i of SYMOCINTFN

$L_{SYMOCINTFN}(j)$ = Latest occurrence event time j of SYMOCINTFN

$ES_{SYMOCINTFN}(ij)$ = Earliest starting time from activity i to j of SYMOCINTFN

$EF_{SYMOCINTFN}(ij)$ = Earliest finishing time from activity i to j of SYMOCINTFN

$LS_{SYMOCINTFN}(ij)$ = Latest starting time from activity i to j of SYMOCINTFN

$LF_{SYMOCINTFN}(ij)$ = Earliest finishing time from activity i to j of SYMOCINTFN

$TF_{SYMOCINTFN}(ij)$ = Total Float time of $A_{SYMOCINTFN}(ij)$

$E_{SYMOCINTFN}(ij)$ = Estimated completion time

5.2. **Some Basic Properties.**

Fix the starting node as zero, i.e.,

$$ES_{SYMOCINTFN}(ij) = (0,0,0,0,0,0,0,0)(0,0,0,0,0,0,0,0)$$

i) $ES_{SYMOCINTFN}(ij) = Max\{ES_{SYMOCINTFN}(ij) + A_{SYMOCINTFN}(ij)\}$

ii) $LS_{SYMOCINTFN}(ij) = Min\{LS_{SYMOCINTFN}(ij) - A_{SYMOCINTFN}(ij)\}$

iii) $TF_{SYMOCINTFN}(ij) = \{LF_{SYMOCINTFN}(ij) - LS_{SYMOCINTFN}(ij)\} - A_{SYMOCINTFN}(ij)$

5.3. **Algorithm.**

Step 1: In project network, identify the SYMOCINTFN activities

Step 2: By using fuzzy ranking function, find the relationship of all activities

Step 3: Draw a project network diagram with SYMOCINTFN as fuzzy activity times

Step 4: Find the earliest fuzzy event time by using property (i) and Find Latest fuzzy event time by using property (ii)

Step 5: To find total float $TF_{SYMOCINTFN}(ij)$ of each activity by property (iii)

Step 6: The $A_{OCINTFN}(ij)$ is critical activity, if $TF_{SYMOCINTFN}(ij) = 0$

Step 7: To identify longest fuzzy critical path from the starting to finishing node of the fuzzy project.

6. Numerical Example

The following table gives list of some SYMOCINTFN activities in network diagram (Figure 3) and basic requirements for the completion of any research work.

TABLE 1. SYMOCINTFN Activity Duration of each Activity

S.No.	Description	Activity	SYMOCINTFN Activity Time
1.	Literature survey	1-2	(42,42,44,44,45,45,47,47) (42,43,43,44,45,46,46,47)
2.	Assumption	2-3	(21,22,23,24,26,27,28,29) (21,22,22,23,23,24,24,25)
3.	Preliminary Ideas	2-4	(33,36,36,37,39,40,40,41) (34,34,36,37,37,38,39,39)
4.	Initial Proposal	3-5	(32,32,33,33,34,34,35,35) (29,31,32,32,33,33,34,36)
5.	Field work	3-6	(25,26,26,27,27,28,28,29) (24,25,25,26,27,28,28,29)
6.	Collections of data & related material	4-6	(38,39,39,41,42,43,43,45) (37,39,40,41,41,42,43,45)
7.	Analysis of Data	5-7	(36,38,39,40,44,45,46,48) (34,37,39,40,42,43,45,46)
8.	Innovative Ideas	6-7	(32,33,34,36,36,38,39,40) (30,31,33,36,38,41,43,44)
9.	Final Result	7-8	(26,28,29,29,31,31,32,34) (25,27,28,29,31,32,33,35)

Calculation

Table 1 shows that activities and the corresponding SYMOCINTFN activity times are given. Let us find the critical path in fuzzy environment by using the proposed ranking procedure SYMOCINTFN. By usual method, we find the fuzzy critical path. In Table 2, expected times of the given activities is represented. Set of all possible paths and their corresponding total completion times are tabulated in table 3.

TABLE 2. Calculation - Expected time

Activity	SYMOCINTFN Activity Time	Duration
1-2	(42,42,44,44,45,45,47,47) (42,43,43,44,45,46,46,47)	40.24
2-3	(21,22,23,24,26,27,28,29) (21,22,22,23,23,24,24,25)	27.52
2-4	(33,36,36,37,39,40,40,41) (34,34,36,37,37,38,39,39)	37.22
3-5	(32,32,33,33,34,34,35,35) (29,31,32,32,33,33,34,36)	30.55
3-6	(25,26,26,27,27,28,28,29) (24,25,25,26,27,28,28,29)	25.81
4-6	(38,39,39,41,42,43,43,45) (37,39,40,41,41,42,43,45)	39.53
5-7	(36,38,39,40,44,45,46,48) (34,37,39,40,42,43,45,46)	44.46
6-7	(32,33,34,36,36,38,39,40) (30,31,33,36,38,41,43,44)	43.83
7-8	(26,28,29,29,31,31,32,34) (25,27,28,29,31,32,33,35)	32.33

Project Network Diagram

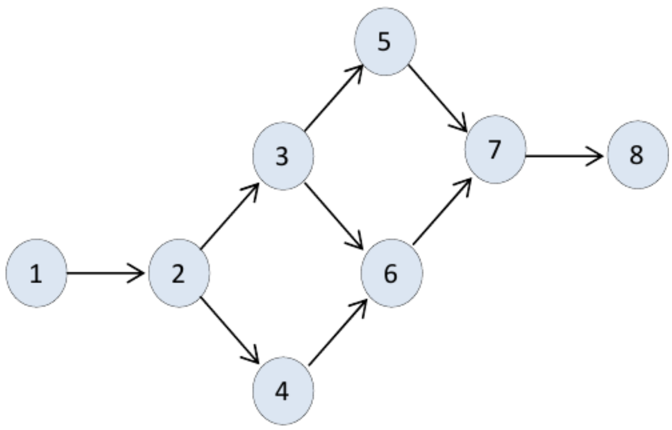

FIGURE 4. Project Network

TABLE 3. Critical Path

Path	Completion Time
1-2-3-5-7-8	175.10
1-2-3-6-7-8	169.73
1-2-4-6-7-8	193.15 *

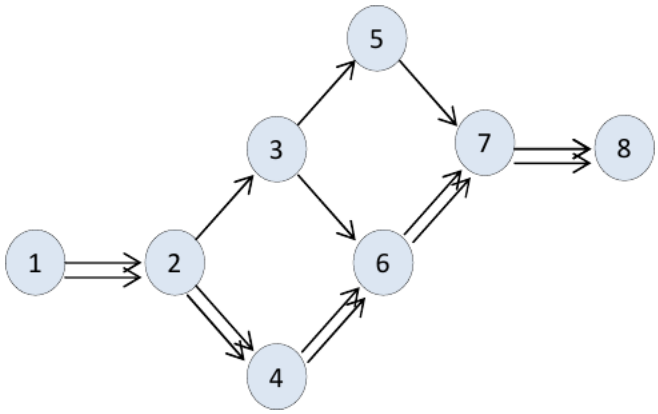

FIGURE 5. Critical Path

From the above, path 1-2-4-6-7-8 is fuzzy critical path as longest path among all other paths the project network with 8 nodes whose activities are in hours.

7. CONCLUSION

In the research work, we represented SYMOCINTFN for solving critical path in project network by using proposed method of ranking and we illustrated fuzzy critical path with suitable example. This approach is very supportive useful and effective in handling the critical problems with symmetric conditions. This kind method can also use in various optimization problem in further research.

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